UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Second Semester – 2019

Roll No. .....

Paper: Mathematics A -II, [Plane Curves & Analytic Geometry]
Course Code: MATH-103 / MTH-12309 Part – II

Time: 2 Hrs. 30 Min. Marks: 50

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

Short Questions $10 \times 2 = 20$

1. Find the focus and directrix of the parabola $x^2 + 6x - 8y + 17 = 0$.
2. Find the radius of curvature on any point on the curve $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$.
3. Identify the surface defined by $z^2 - 4y^2 - 16x - 16y - 2z + 49 = 0$.
4. Show that the pedal equation can be expressed as
$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2.$$
5. Find equation of the cylinder with directrix $C : y - z^2 = 0$ and having elements parallel to the vector $\hat{\mathbf{n}} = [2, 3, 4]$.
6. Find the points of relative extreme of the curve $y = 2x^3 - 15x^2 + 36x + 10$.
7. Find the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
8. Find equation of the plane through the origin and perpendicular to the straight line $x = 2 + t, y = 2 - 3t, z = -2 + 2t$.
9. Find the position and nature of the multiple points on the curve $y^3 = x^3 + ax^2$.
10. Show that the equation $z(x^2 + y^2) = 1$ represents a surface of revolution. Also find its generatix in xz-plane and its axis of revolution.

Subjective Questions $6 \times 5 = 30$

1. Find the equation of straight line which is perpendicular to the straight lines
$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \text{ and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}.$$
Also find its points of intersection with the given lines.

Find the equation of the plane through $(5, -1, 4)$ and perpendicular to each of the following planes

$x + y - 2z - 3 = 0$ and $2x - 3y + z = 0$

P.T.O.

2. Show that the pedal equation of the curve

$$x = ae^{\theta}(\sin\theta - \cos\theta), \qquad y = ae^{\theta}(\sin\theta + \cos\theta)$$

is $r = \sqrt{2}p$.

3. Find the envelop of the family of lines $bx + ay - ab = 0$ where the parameters a and b are connected by the relation $a + b = c$.

4. Find the area of the region bounded by the loop of curve $3ay^2 = x(x-a)^2$.

5. Find the equations of the tangent and normal at $\theta = \frac{\pi}{2}$ to the cycloid

$$x = a(\theta - \sin\theta), \qquad y = a(1 - \cos\theta).$$

6. Find the direction of Qibla of Badshahi Mosque, Lahore, latitude = 31°35′.4N and longitude = 74°18′.7E. The latitude and longitude of the Khana-e-Kaba (Qibla) are 21°25′.2N and 39°49′.2E, respectively.

UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Second Semester – 2019

Paper: Mathematics A -II, [Plane Curves & Analytic Geometry]

Course Code: MATH-103 / MTH-12309 Part – I (Compulsory) Time: 30 Min. Marks: 10

Roll No. in Fig. .....

Roll No. in Words. .....

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Signature of Supdt.:

ATTEMPT THIS PAPER ON THIS QUESTION SHEET ONLY.

Division of marks is given in front of each question.

This Paper will be collected back after expiry of time limit mentioned above.

Q.1. Encircle the right answer cutting and overwriting is not allowed. (10x1=10)

1. A pair of straight lines is given by the equation $6x^2 + xy - y^2 - 21x - 8y + 9 = 0$. The angle between these lines is
   (a) $\frac{\pi}{2}$ (b) 0 (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

2. The equation $r^2 = a^2 \sin 2\theta$ can be expressed in rectangular co-ordinates as
   (a) $4xy(x^2 - y^2)$ (b) $(x^2 + y^2)^2 - 2a^2xy = 0$ (c) $(x^2 + y^2) = 2a^2x^2y^2$
   (d) $y^2 = 4ax$

3. The equation $r = \frac{4}{1+\cos\theta}$ represents
   (a) ellipse (b) parabola (c) hyperbola (d) circle

4. The curve $x^2 + y^2 = 14$ is symmetric about
   (a) line x−axis (b) line $x = y$ (c) line y-axis (d) both x and y axes

5. The locus of centers of curvatures for a given curve is called its
   (a) involute (b) envelope (c) diameter (d) evolute

6. The parametric equations of the curve $r = e^\theta$ are
   (a) $x^2 = e^\theta$, $y^2 = e^\theta$ (b) $x^2 + y^2 = e^\theta$, $x^2 - y^2 = 0$ (c) $x = e^\theta \cos\theta$, $y = e^\theta \sin\theta$ (d) None of these

7. The curve $2x^3 - 15x^2 + 36x + 10$ has relative minimum at $x =$
   (a) -1 (b) 3 (c) 2 (d) 0

8. A point through which there pass two branches of a curve is called
   (a) simple point (b) ordinary point (c) double point (d) corner point

9. A surface defined by an equation of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ is called
   (a) ellipsoid (b) hyperboloid of one sheet (c) hyperboloid of two sheets
   (d) paraboloid

10. A surface defined by an equation of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$ is called
    (a) ellipsoid (b) cylinder (c) sphere (d) elliptic paraboloid