



**THE ANSWERS MUST BE ATTEMPTED ON THE ANSWER SHEET PROVIDED**

**Q.1. Solve the following.**

**(6x5=30)**

- (i) Find equation of normal to the curve  $x(x^2 + y^2) - ay^2 = 0$  at  $x = \frac{a}{2}$ .
- (ii) Identify and graph the given polar equation  $r = \frac{6}{1-2\sin\theta}$ .
- (iii) Find  $a$  and  $b$  so that the function  $f$  given by  $f(x) = ax^3 + bx^2$  has  $(1,6)$  as point of inflection.
- (iv) Find the area of the region bounded by  $y = \ln x, x - axis, x = a, x = b$ .
- (v) Find an equation of the plane which passes through the point  $(3,4,5)$ , has an  $x$ -intercept equal to  $-5$  and is perpendicular to the plane  $2x + 3y - z = 8$ .
- (vi) Prove that in a spherical triangle  $ABC$ , with  $a, b, c$  are lengths of the sides  $BC, CA, AB$  respectively

$$\frac{\sin \frac{A+B}{2}}{\cos \frac{C}{2}} = \frac{\cos \frac{a-b}{2}}{\cos \frac{c}{2}}$$

**Q.2. Solve the following.**

**(3x10=30)**

- i. Prove that if  $P$  is any point on a hyperbola with foci  $F_1$  and  $F_2$ , the tangent at  $P$  bisects the angle  $F_1PF_2$ .
- ii. Find the radius of curvature of the curve  $r = a(1 + \cos\theta)$  at the point where the tangent is parallel to the initial line.
- iii. Find the equation of the cone whose directrix is  $y^2 = x, z = 4$  and whose vertex is at  $A(0,2,0)$ .