UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Third Semester – Spring 2022

Roll No. ....

Paper: Mathematics A-III **Course Code: MATH-201** **Time: 3 Hrs.** **Marks: 60**

THE ANSWERS MUST BE ATTEMPTED ON THE ANSWER SHEET PROVIDED

Q.1. Solve the following: **(6x5=30)**

i) If $|A| \neq 0$, then $|A^t| \neq 0$ for non-zero integer n and $(A^t)^{-1} = (A^{-1})^t$.

ii) Show that a square matrix A satisfies the equation $A^2 + 2A + I = 0$ has $|A| \neq 0$.

iii) Using row operation find A^{-1} if $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$.

iv) Find domain and range $T(x_1, x_2, x_3, x_4) = (x_1, x_2)$.

v) Find area of the triangle with vertices $(3,3), (4,0), (-2,-1)$.

vi) Show $u = (-2,3,1,4)$ & $v = (1,2,0,-1)$ are orthogonal vectors in $\mathbb{R}^4$. Also find angle b/w u & v.

Q.2. Solve the following: **(5x6=30)**

i) Show that the following system by inverting the coefficient matrix has non trivial solution if $\alpha = \beta$.

$$x_1 + x_2 + \alpha x_3 = 0; \quad x_1 + \beta x_3 + x_2 = 0; \quad \alpha x_1 + \beta x_2 + x_3 = 0$$

ii) Check either $T(x, y, z) = (x, z, x + y)$ is a matrix transformation also find $R(T)$ and $N(T)$.

iii) A square matrix A is invertible iff $\lambda = 0$ is not an eigen value of A.

iv) Find the bases for the subspace of $\mathbb{R}^4$ that is spanned by the vectors:
$(1,1,-4,-3)$, $(2, 0, 2,-2)$ & $(2,-1, 3, 2)$

v) What condition must a, b, c and d satisfy so that the following matrices are linearly independent in M_{22} $\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$, $\begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix}$ and $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.