UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Sixth Semester – 2019

Roll No. in Fig. .....

Roll No. in Words. .....

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Signature of Supdt.:

Paper: Linear Algebra (MA)
Course Code: IT-312 Part – I (Compulsory) **Time: 30 Min. Marks: 10**

ATTEMPT THIS PAPER ON THIS QUESTION SHEET ONLY.
Division of marks is given in front of each question.
This Paper will be collected back after expiry of time limit mentioned above.

Q.1. Encircle the correct choice. (1x10=10)

(i) If A and B are two square matrices of the same size then
(a) $tr(AB) = tr(A)tr(B)$ (b) $tr(AB) \neq tr(A)tr(B)$
(c) $tr(AB) = tr(A) + tr(B)$ (d) None of these

(ii) A system of m homogeneous linear equations $Ax = 0$ in n variables has a non-trivial solution if and only if the rank of A is ----------
(a) equal to n (b) less than n (c) greater to n (d) None of these

(iii) If A is a matrix of order 3×3 and $\det(A) = 2$, then the value of $\det(3A)$ is --------
(a) -24 (b) -6 (c) -27 (d) None of these

(iv) A group has exactly ---------- identity element.
(a) One (b) Two (c) Three (d) Four

(v) The subspace of R^3 spanned by the vector (a, b, c) is ----------
(a) $x = t, y = bt, z = ct$ (b) $x = -at, y = -bt, z = -ct$
(c) $x = at, y = bt, z = ct$ (d) None of these

(vi) The property $\forall$ a, b, c $\in$ R then $a + (b + c) = (a + b) + c$ is called
(a) Associative property (b) Transitive property
(c) Closure property (d) None of these

(vii) A linear transformation $T : U \to V$ is one-to-one if and only if ----------
(a) $N(T) = \{0\}$ (b) $N(T) \neq \{0\}$ (c) $N(T) = \{1\}$ (d) $N(T) = \{-1\}$

(viii) Let R^3 be the vector space of all ordered triples of real numbers. Then the transformation $T : R^3 \to R^3$ defined by $T(x, y, z) = (x, y, 0)$ is
a) Linear b) Not Linear c) Rational d) None of these

(ix) The dimension of $KerT$ is called .....
(a) Rank (b)Nullity
(c) basis (d)} none of these

(x) The characteristic polynomial of the matrix $\begin{pmatrix} 4 & 0 \\ 0 & 7 \end{pmatrix}$ is.....
(a) $p(\lambda) = (2 - \lambda)^2$ (b) $p(\lambda) = (4 - \lambda)(7 - \lambda)$
(c) $p(\lambda) = (4 + \lambda)(7 + \lambda)$ (d) None of these

UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Sixth Semester – 2019

Roll No. .....

Paper: Linear Algebra (MA)
Course Code: IT-312 Part – II

Time: 2 Hrs. 30 Min. Marks: 50

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

Q. 2 SHORT QUESTIONS (4x5 = 20 Marks)

(i) Find the reduced echelon form of the matrix

$$\begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}$$

(ii) Prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^3 & \beta^3 & \gamma^3 \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

(iii) Show that the vectors $(1-i,i)$, and $(2,-1+i)$ C^2 are linearly independent over C but linearly independent over R.

(iv) Define $T: R^3 \to R^3$ by $T(x_1,x_2,x_3) = (-x_3, x_1, x_1+x_3)$. Find $N(T)$. Is T one-to-one?

(v) Find the Eigen Values and eigen vectors of $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

LONG QUESTIONS (6x5 = 30 Marks)

Q.3 Solve the system of linear equations

$$3x+2y+4z=7$$
$$2x+y+z=4 \quad 3x+2y+4z=7$$
$$x+3y+5z=3$$

Q.4 Show that $\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+y & 1 \\ 1 & 1 & 1 & 1-y \end{vmatrix} = x^2y^2$

Q.5 If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ determine the value of $A^2 - 4A - 5I$

Q.6 Determine whether or not the set of vectors $\{(1,2,-1),(0,3,1),(1,-5,3)\}$ is a basis for R^3?

Q.7 Find the real orthogonal matrix P for which $P^{-1}AP$ is orthogonal where $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$