UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program : First Semester – Fall 2021

Roll No. ……………

Paper: Mathematics B-I [Vectors & Mechanics (1)]
Course Code: MATH-102

Time: 3 Hrs. **Marks: 60**

Q.1. Solve the following: (6x5=30)

1. Show that $\nabla\psi$ is vector perpendicular to the surface $\psi(x, y, z) = c$ where c is a constant.
2. Evaluate (i) $\nabla \cdot (\mathbf{B} \times r)$ if $\nabla \times \mathbf{B} = 0$ (ii) $\nabla \times (\frac{\mathbf{r}}{r^2})$.
3. If $\mathbf{P}$ and $\mathbf{Q}$ are irrotational, prove that $\mathbf{P} \times \mathbf{Q}$ is solenoidal.
4. Define vortex field. If $\mathbf{B} = \mathbf{w} \times \mathbf{r}$, prove $\nabla \times \mathbf{B} = 2\mathbf{w}$ where $\mathbf{w}$ is a constant vector.
5. State and prove Varigon's theorem.
6. Define limiting equilibrium. State and prove the condition for the limiting equilibrium.

Q.2. Solve the following:

1. Prove $\nabla^2 r^n = n(n+1)r^{n-2}$, where n is a constant. (7 marks)
2. Find constants p, q, r so that $V = (x + 2y + pz)\mathbf{i} + (qx - 3y - z)\mathbf{j} + (4x + ry + 2z)\mathbf{k}$ is irrotational. Also prove that V can be expressed as the gradient of a scalar function. (7 marks)
3. If the forces $p\overrightarrow{PQ}$, $q\overrightarrow{RQ}$, $r\overrightarrow{RS}$, $s\overrightarrow{PS}$ acting along the sides of a plane quadrilateral are in equilibrium, show that $pr = qs$. (8 marks)
4. A uniform ladder rests in limiting equilibrium with one end on a rough floor whose coefficient of friction is μ and with the other against a smooth vertical wall. Show that its inclination to the vertical is $\tan^{-1}(2\mu)$. (8 marks)