UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program : Fifth Semester – Fall 2021

Roll No. ....

Paper: Real Analysis-I **Course Code: MATH-301** **Time: 3 Hrs.** **Marks: 60**

Q.1. Solve the following: **(5x6=30)**

(i) Let $x_n = \frac{(-1)^n}{n^2+1}$, $n \in N$, be a sequence, then find its sup and inf, also discuss its limit and boundedness.

(ii) Discuss the convergence of the sequence $X = (x_n)$ defined by $x_1 = 1$, $x_2 = 2$, and $x_n = \frac{1}{2}(x_{n-2} + x_{n-1})$, for $n > 2$.

(iii) Check the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2-n+1}$

(iv) Discuss the continuity and differentiability at $x = 0$ of the function $f : R \to R$ given by

$$f(x) = \begin{cases} x\sin(\frac{1}{x}), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

(v) Find the local maxima and minima of the function $f(x, y) = x^4 + y^4 - 4xy + 1$.

Solve the following. **(3x10=30)**

Q.2 Let f be defined on $[a, b]$, if f has a local maximum at a point $x \in [a, b]$ and if $f'(x)$ exists then show that $f'(x) = 0$.

Q.3 Prove that a Cauchy sequence of real numbers is bounded. Give an example of bounded sequence which is not Cauchy sequence.

Q.4 Let $u = f(x, y)$, $x = r\cos\theta, y = r\sin\theta$, then prove that $\nabla^2 u = \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$