UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program : Fifth Semester – Fall 2021

Roll No. ....

Paper: Group Theory-I **Course Code: MATH-302** **Time: 3 Hrs.** **Marks: 60**

Q.1. Solve the following: **(6x5=30)**

1. Prove that a finite subset H of a group G is a subgroup of G if and only H is closed.
2. Find all subgroups and all generators of the cyclic group $G = \langle a \mid a^{12} = 1 \rangle$.
3. Prove that the Normalizer of a subset H of a group G is a subgroup of G.
4. Let H, K be normal subgroups of a group G such that $H \cap K = \{e\}$. Prove that H and K commute element wise. That is, $ab = ba,\ a \in A,\ b \in B$.
5. Prove that the relation of conjugacy between the elements of a group G is an equivalence relation.
6. Find all Sylow Subgroups of a group of order 12.

Solve the following questions. **(3x10=30)**

Q.2.

(a) State and Prove Cayley's Theorem. **(05)**

(b) Prove that a group G is abelian if and only if the factor group $G/Z(G)$ is cyclic, where $Z(G)$ is the centre of G. **(05)**

Q.3. Prove that if a prime number p divides the order of a finite group G, then G has an element of order p. Using Sylow's Theory, find all elements of order 5 in a group of order 15. **(10)**

Q.4.

(a) Define Derived subgroup of a group G. Let K be a normal subgroup of a group G Such that G/K is abelian then prove that $K \supseteq G'$, where G' is the derived sub-group of G. Find first derived subgroup of a cyclic group of order 6. **(05)**

(b) Let G be a group with $\zeta(G)$ as its centre and $I(G)$, the group of its inner automorphisms. Then show that $G/\zeta(G) \cong I(G)$. Also find $A(C_4)$. **(05)**