UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program : Fifth Semester – Fall 2021

Roll No. ....

Paper: Complex Analysis-I **Course Code: MATH-303** **Time: 3 Hrs.** **Marks: 60**

Q.1. Give short answers to the following questions. **(6x5=30)**

(i) Show that $\left(-1+\sqrt{3}i\right)^{3/2} = \pm 2\sqrt{2}.$

(ii) Evaluate the integral $\int\limits_{|z-2|=\frac{1}{2}} \left(\cos ec(z)+\sin(z)\right) dz.$

(iii) Find the upper bound of the modulus of the integral $\int\limits_{C:|z|=R>1} \frac{Logz}{z^2} dz.$

(iv) Show that the real and imaginary parts of analytic functions are harmonic functions.

(v) Find the Taylor series of the function $f(z)=\frac{1}{1-z}$ at $z_0 = 2i$. Also find its region of convergence.

(vi) Find the Mobius transformation which maps the points $z_1 = 1, z_2 = 0, z_3 = -1$ to the points $w_1 = i, w_2 = \infty, w_3 = 1.$

Q.2. Answer the following questions in detail. **(3x10=30)**

(i) Evaluate the integral $\int\limits_C \sqrt{z}\, dz$. The contour C is given as

$$C : z = \sqrt{4-y^2} + iy, \quad -2 \le y \le 2.$$

(ii) Show that the function $u(r,\theta) = \sin(r\cos(\theta))\cosh(r\sin(\theta))$ is harmonic. Find its harmonic conjugate.

(iii) Find the image of transformation $f(z)=\frac{1}{z}$ of the lines $y = x-1$ and $y = 0$. Sketch all four curves, determine corresponding directions along them, and verify the conformality of the mapping at the point z = 1.