UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program : Fifth Semester – Fall 2021

Roll No. ....

Paper: Vector and Tensor Analysis **Course Code: MATH-304** **Time: 3 Hrs.** **Marks: 60**

Q.1. Solve the following questions: (6x5=30)

i) If $\boldsymbol{F} = 3xy\hat{\imath} - y^2\hat{\jmath}$ find work done over C in the $xy - plane$, where $y = 2x^2$ from (0,0) to (1,2).

ii) Find the volume of the region common to the intersecting cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

iii) Verify Green's theorem in the plane for $M = xy + y^2$ and $N = x^2$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

iv) Express the velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$ of a particle in cylindrical coordinates.

v) Express the determinant $g = \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{vmatrix}$ in terms of the elements of second row and their corresponding cofactors.

vi) a) If A^{pq}_{rs} is a tensor, prove the double contractions results in invariant.
b) state Quotient law for tensors.

Solve the following questions: (3x10=30)

Q NO 2: State Stoke's theorem and verify for $\boldsymbol{A} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (10)

Q NO 3: Derive an expression for the Gradient $\nabla\phi$ in general curvilinear coordinate system. (10)

Q NO 4: (a) Is the function $\vec{f}(t) = ti + t^2j + \frac{1}{t}k$ continuous at $t = 0$?

(b) If $\vec{f}(t)$ is a vector function. Show that $\frac{d}{dt}[\vec{f}.(\vec{f'} \times \vec{f''})] = \vec{f}.(\vec{f''} \times \vec{f'''})$ (10)