



Q.1. Solve the following:

(6x5=30)

- (a) Define compact space. Give one example of a compact subspace of \mathbb{R} under usual topology.
- (b) Prove that Unit circle S^1 centered at the origin with radius one is connected under subspace topology of Plane.
- (c) Does the set $\{\{a\}, \{b, c\}, \{d\}\}$ forms a basis for a topology on set $X = \{a, b, c, d\}$, if yes then find the topology.
- (d) Define derived set of a subset in topological space. Find all limit points of the set $X = \{(n, \frac{1}{n}) \mid n \in \mathbb{Z} - \{0\}\}$.
- (e) Define Hausdorff space and give two examples of this space. Given an example of a space in which a sequence can converge to more than one point.
- (f) Prove that union of many connected spaces having non-empty intersection, is also connected.

Solve the following.

(3x10=30)

(Q2) Define usual and Co-finite topology on \mathbb{R} . Find a set (if possible) which is both open and closed and a set (if possible) that is neither open nor closed in both topologies.

(Q3) Prove that continuous image of a compact set is compact. Why $(0, 1]$ is non-compact.

(Q4) Let A be a subset of a topological space X . Then prove A is closed iff (the set of all limit points of A), $A' \subseteq A$.