UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program : Fifth Semester – Fall 2021

Roll No. ……………

Paper: Differential Geometry **Course Code: MATH-306** **Time: 3 Hrs.** **Marks: 60**

Q.1 Give short answers of the following: (5x6=30)

1. Find the equations of binormal line and the osculating plane to the curve represented by $x(u) = 1 + u,\ \ y(u) = -u^2,\ \ z(u) = 1 + u^3$ at the point corresponding to $u = 1$. [06]
2. Show that along a regular curve $\mathbf{x} = \mathbf{x}(s)$ of class ≥ 4, $\left[\mathbf{x}^{(2)}, \mathbf{x}^{(3)}, \mathbf{x}^{(4)}\right] = \kappa^5 \frac{d}{ds}\left(\frac{\tau}{\kappa}\right)$, where the superscripts (2), (3) and (4) in $\mathbf{x}^{(2)}, \mathbf{x}^{(3)}$ and $\mathbf{x}^{(4)}$ denote the respective 2nd, third and fourth derivatives. Also show that it is a general helix if and only if $\left[\mathbf{x}^{(2)}, \mathbf{x}^{(3)}, \mathbf{x}^{(4)}\right] = 0$. [06]
3. Find the singular and non singular points of the epicycloid given by $x = 4\cos\vartheta - \cos 4\vartheta,\ \ y = 4\sin\vartheta - \sin 4\vartheta$ and determine its intrinsic equations. [06]
4. Prove that a curve is uniquely determined except as the position in space, when its curvature and torsion are given functions of its arc length. Derive the equation of a curve whose intrinsic equations are $\kappa(s) = \frac{1}{\sqrt{2as}}$ and $\tau(s) = 0$, where a is a constant. [06]
5. Show that the curvature of the involute $\mathbf{x}^* = \mathbf{x} + (c - s)\,\mathbf{t}$ of the curve $\mathbf{x} = \mathbf{x}(s)$ is $\kappa^{*2} = \frac{\kappa^2 + \tau^2}{(c-s)\kappa^2}$. [06]

Q.2. Answers the following questions. (3x10=30)

1. Find the coefficients g_{jk} and b_{jk} $(j,k = 1,2)$ of the two fundamental forms $I\,(du, d\xi) = g_{11}du^2 + 2g_{12}dud\xi + g_{22}d\xi^2$ and $II\,(du, d\xi) = b_{11}du^2 + 2b_{12}dud\xi + b_{22}d\xi^2$ for the surface $\mathbf{x}\,(u, \xi) = (u\cos\xi, u\sin\xi, c\xi)$. [10]
2. If the parametric curves on a surface are orthogonal, then prove that the differential equation of a line on the surface cutting the parametric curves for $u =$ constant at constant angle β is $\frac{du}{dv} = \sqrt{\frac{G}{E}}\tan\beta$. [10]
3. Derive Gauss-Weingarten equations for a surface $\mathbf{x}\,(u, v)$ and verify these equations for the surface $\mathbf{x}\,(u, v) = (u\cos v, u\sin v, g\,(u))$. [10]