UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program : Seventh Semester – Fall 2021

Roll No. ……………

Paper: Set Theory **Course Code: MATH-401** **Time: 3 Hrs. Marks: 60**

Q.1. Solve the following:

(i) Determine whether each of the following sets is countable or uncountable (4)

a) $A = \{x \in \mathbb{Q} \mid -100 \leq x \leq 100\}$

b) $B = (0, 0.1]$.

(ii) Prove that a subset of a denumerable set is either finite or denumerable. (3)

(iii) Show that $I = [0,1]$ is non-denumerable set. (4)

(iv) Prove that the set $\mathbb{Q}$ of rational numbers is denumerable. (3)

(v) Let m and n be finite cardinal numbers. Then show that m+n represents usual addition in $\mathbb{N}$; $n+\aleph_0 = \aleph_0$; $\aleph_0 + \aleph_0 = \aleph_0$ and $c + c = c$. (3)

(vi) Prove that $\aleph_0 c = c$. (4)

(vii) Give a bijection to prove that $\mathbb{N} \approx 2\mathbb{N}$. (3)

(viii) Prove that two different initial segments of a well-ordered set cannot be similar. (3)

(ix) Let A be a well ordered set, let B be a subset of A, and let $f : A \rightarrow B$ be a similarity mapping of A on to B. Then for every $a \in A, a \leq f(a)$. (3)

Solve the following: (5x6=30)

Q.2 The union of countable family of countable sets is countable.

Q.3 Prove that cancellation laws under multiplication and addition do not hod for ordinal numbers.

Q.4 Prove that a chain is well ordered if and only if it does not contain an infinite descending sequence.

Q.5 State and prove Cantor's Theorem.

Q.6 Let A and B be well ordered sets such that an initial segment s(a) of A is similar to s(b) of B. Then each initial segment of s(a) is similar to initial segment of s(b).