UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program : Seventh Semester – Fall 2021

Roll No. ....

Paper: Number Theory-I **Course Code: MATH-408** **Time: 3 Hrs. Marks: 60**

Q.1. Solve the following: (6x5=30)

(I)	Show that if p is prime and $p\|a_1a_2\cdots a_n$, then $p\|a_k$ for some k, where $1 \le k \le n$.
(II)	State and prove Wilson theorem.
(III)	Find the last two digits of $(2022)^{2023}$.
(IV)	Solve the system of linear congruence $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$, $x \equiv 3 \pmod 7$.
(V)	Show that if p is a prime, then are exactly $\varphi(p-1)$ incongruent primitive roots of p.
(VI)	For $n = 2005060$, find $\varphi(n), \tau(n), \sigma(n)$. Where φ, τ, σ are Euler Phi, number of positive divisors, and sum of positive divisors functions respectively.

Solve the following: (5x6=30)

Q2	Show that a linear congruence $ax \equiv b \pmod n$ has a solution if and only if $d\|b$, where $\gcd(a,n) = d$. Further, if $d\|b$ then it has d mutually incongruent solutions modulo n.
Q3	Prove that if p is a prime number and $d\|(p-1)$, then there are exactly $\varphi(d)$ incongruent integer having order d.
Q4	If the integer $n > 1$ has prime factorization $n = p_1^{k_1}p_2^{k_2}\cdots p_r^{k_r}$, then show that $\tau(n) = (k_1+1)(k_2+1)\cdots(k_r+1)$.
Q5	If n has a primitive root r and ind a denotes the index of a relative to r, then (a) $\text{ind}(ab) \equiv \text{ind}\, a + \text{ind}\, b \pmod{\varphi(n)}$, (b) $\text{ind}(a^k) \equiv k\, \text{ind}\, a \pmod{\varphi(n)}$.
Q6	Show that if F is a multiplicative function and $F(n) = \sum_{d\|n} f(d)$, then f is also multiplicative.