# UNIVERSITY OF THE PUNJAB

**Fifth Semester – 2019**

**Examination: B.S. 4 Years Program**

Roll No. in Fig. ........................

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**PAPER: Topology**
**Course Code: MATH-305 Part–I (Compulsory)**

**MAX. TIME: 30 Min.**
**MAX. MARKS: 10**

**Attempt this Paper on this Question Sheet only.**
**Please encircle the correct option. Division of marks is given in front of each question.**
**This Paper will be collected back after expiry of time limit mentioned above.**

**Q.1. Encircle the right answer, cutting and overwriting is not allowed. (1x10=10)**

(i) In $(\mathbb{R}, \tau)$ with the usual topology $\tau$ or $\mathbb{R}$ the frontier of set $A = \{-\pi, -e, 0, e, \pi\}$ is ...
(a) $\{-4, -2, 0, 2, 4\}$ (b) $\{-\pi, -e, 0, e, \pi\}$ (c) $\mathbb{R}$ (d) $\emptyset$

(ii) In $(\mathbb{R}, \tau)$ with the usual topology $\tau$ or $\mathbb{R}$ if $A = \{1, \frac{1}{2}, \frac{1}{3}, ...\}$ then $\overline{(A^\circ)}$ is ...
(a) $\{1, \frac{1}{2}, \frac{1}{3}, ...\}$ (b) $\{0, 1, \frac{1}{2}, \frac{1}{3}, ...\}$ (c) $\mathbb{R}$ (d) $\emptyset$

(iii) In the real line $\mathbb{R}$ consider $A_n = (\frac{-1}{n}, \frac{1}{n})$, $n \in \mathbb{N}$, then $\cap_{n \in \mathbb{N}} A_n$ is ...
(a) $\emptyset$ (b) $\{0\}$ (c) $(-1, 1)$ (d) $(\frac{-1}{n}, \frac{1}{n})$

(iv) Let $\mathbb{N}$ be the set of natural numbers and $\tau$ be the co-finite topology on $\mathbb{N}$ and if $A_n = \{2, 3, 4, ..., n+1\}, n \in \mathbb{N}$, then $\cup_{n \in \mathbb{N}} \overline{A_n}$
(a) $\{1, 2, 3, ...\}$ (b) $\mathbb{N} \setminus \{2, 3, 4, ...\}$ (c) $\mathbb{N} \setminus \{1\}$ (d) $\emptyset$

(v) Let $X$ be any uncountable set with co-finite topology on $X$, then $(X, \tau)$ is ...
(a) Neither first nor second countable (b) Second countable
(c) First countable but not second countable (d) First countable

(vi) Let $X = \{a, b, c, d, e\}$ and $\tau = P(X)$, then the sub-base for $(X, \tau)$ is ...
(a) $\{\{a, b\}, \{b, c\}, \{c, d\}, \{d, e\}\}$ (b) $\{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}\}$
(c) $\{\{a, b\}, \{b, c\}, \{c, d\}, \{d, e\}, \{e, a\}\}$ (d) $\{\{a, b, c\}, \{b, c, d\}, \{c, d, e\}\}$

(vii) Let $X$ be a $T_1$-space and $A$ a subset of $X$. If $x$ is a limit point of $A$ then every open set containing $x$ contains ...
(a) only one point of $A$ (b) infinite number of distinct points of $A$
(c) no point of $A$ (d) finite points of $A$

(viii) Every compact Hausdorff space is ...
(a) Lindelof (b) $T_3$ (c) regular (d) normal

(ix) If a Hausdorff space $X$ has an open base whose sets are also closed then $X$ is ...
(a) totally disconnected (b) component (c) compact regular (d) locally compact

(x) Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \begin{cases} x & \text{if } x \neq 1 \\ 3 & \text{if } x = 1 \end{cases}$
and let $O = (2, 4)$ be open in $(\mathbb{R}, d)$ with usual metric $d$ on $\mathbb{R}$. Then $f^{-1}(O) = ...$
(a) $O = (2, 4)$ (b) $O = [2, 4]$
(c) $O = (2, 4]$ regular (d) $\{1\} \cup (2, 4)$

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**PAPER: Topology**
**Course Code: MATH-305 Part – II**

**MAX. TIME: 2 Hrs. 30 Min.**
**MAX. MARKS: 50**

**ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED**

**Q.2. Answer the following short questions. (10x2=20)**

(i) Let $X = \{a, b, c\}$ then write any four topologies on $X$.
(ii) Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b, c\}\}, X\}$. Then find interior of $A = \{a, b\}$.
(iii) Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}\}, X\}$. Then write all neighborhoods of point $b$.
(iv) In $(\mathbb{R}, \tau)$ with the usual topology $\tau$ or $\mathbb{R}$ find the boundary of the set of rational numbers $\mathbb{Q}$.
(v) Define metric topology and metrizable spaces.
(vi) Prove that a space $X$ is a $T_1$-space if and only if every singleton subset of $X$ is closed.
(vii) Show that the real line $\mathbb{R}$ with the usual topology is not compact.
(viii) Define component in a topological space.
(ix) Prove that every metric space $(X, d)$ is Hausdorff.
(x) Prove that a space $\mathbb{Q}$ of rational numbers is disconnected.

**Q.3. Answer the following Long questions. (5x6=30)**

i. Prove that any uncountable set $X$ with cofinite topology $\tau$ is neither first countable nor second countable.
ii. Let $X, Y$ be topological spaces. Then prove that a function $f : X \to Y$ is continuous if and only if for each $V$ open in $Y$, $F^{-1}(V)$ is open in $X$.
iii. Let $X$ be a topological space. Then prove that $X$ is Hausdorff if and only if the diagonal $D = \{(x, x) : x \in X\}$ is closed in $X \times X$.
iv. Prove that a Topological space $X$ is disconnected if and only if $X$ contain a non-empty set $A$ which is both open and closed in $X$.
v. Prove that the continuous image of a compact space is compact.