UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Sixth Semester – 2019

Paper: Real Analysis-II
Course Code: MATH-307   Part – I (Compulsory)   Time: 30 Min. Marks: 10

ATTEMPT THIS PAPER ON THIS QUESTION SHEET ONLY.
Division of marks is given in front of each question.
This Paper will be collected back after expiry of time limit mentioned above.

Q.1.   Encircle the correct option.   **(10x1=10)**

1. If $f \in \mathcal{R}(\alpha)$ on $[a,b]$ and $f \geq 0$ on $[a,b]$, then   (1 mark)

   (a) $\int_a^b f d\alpha \leq 0$.

   (b) $\int_a^b f d\alpha \geq 0$.

   (c) $\int_a^b f d\alpha = 0$.

   (d) All of the above

2. Which one of the following is a correct statement.   (1 mark)

   (a) A constant function $f(x) = 6x$ is not integrable on $[0,2]$.

   (b) A constant function $f(x) = 6x$ is integrable on $[0,2]$.

   (c) A function $f(x) := 12x^3$ is not integrable on $[0,2]$.

   (d) None of these.

3. A function f is said to be Riemann-Stieltjes integrable with respect to α on $[a,b]$, if f is bounded on $[a,b]$, and ——————   (1 mark)

   (a) $\int_a^b f d\alpha = \int_a^b f d\alpha$.

   (b) $\int_a^b f d\alpha \neq \int_a^b f d\alpha$.

   (c) $\int_a^b f d\alpha > \int_a^b f d\alpha$.

   (d) None of these

4. The upper and lower Riemann-Stieltjes sums are ——————   (1 mark)

   (a) unbounded.

   (b) bounded.

   (c) both (a) & (b)

   (d) None of these

5. If P^* is the refinement of the partition P, then ——————   (1 mark)

   (a) $L(P^\star, f, \alpha) \leq L(P, f, \alpha)$.

   (b) $L(P, f, \alpha) < L(P^\star, f, \alpha)$.

   (c) $L(P^\star, f, \alpha) = L(P, f, \alpha)$.

   (d) Both (b) & (c)

P.T.O.

6. Let $a < c < b$ and f is of bounded variations on $[a, b]$, then f is of bounded variations on (1 mark)

    (a) $[a, c]$.

    (b) $[c, b]$.

    (c) Both (a) & (b).

    (d) None of these.

7. Let f and g are of bounded variations on $[a, b]$, then (1 mark)

    (a) fg is not of bounded variations on $[a, b]$.

    (b) $\lambda g; \lambda \in \mathbb{R}$ is not of bounded variations on $[a, b]$.

    (c) fg is of bounded variations on $[a, b]$.

    (d) $\lambda f; \lambda \in \mathbb{R}$ is not of bounded variations on $[a, b]$.

8. Let h is of bounded variations on $[a, b]$, then (1 mark)

    (a) h is not of bounded variations on $[a, b]$.

    (b) h is of bounded variations on $[a, b]$.

    (c) Both (a) & (b).

    (d) None of these.

9. A sequence of function $f_n(x) := \frac{x}{n}; n \in \mathbb{N}, x \in \mathbb{R}$ is .... (1 mark)

    (a) not pointwise convergent on $[0, 1]$

    (b) not pointwise convergent on $[0, 1]$

    (c) More information is required

    (d) None of the above.

10. Which one of the following is not a true statement about the function f defined by (1 mark)

$$f(x) = \begin{cases} 0 & \text{if x is irrational,} \\ 1 & \text{if x is rational,} \end{cases}$$

    (a) f is not Riemann integrable on any interval.

    (b) f is not continuous on any point of $\mathbb{R}$.

    (c) is not a measureable function.

    (d) All of the above.

UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Sixth Semester – 2019

Roll No. ....

Paper: Real Analysis-II
Course Code: MATH-307   Part – II

Time: 2 Hrs. 30 Min.   Marks: 50

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

Q #2

SECTION II-Questions with Short Answers

1. Let $f \in \mathcal{R}(\alpha)$ on $I := [a, b]$, then prove that $|f| \in \mathcal{R}(\alpha)$ on I.   (4 marks)

2. Test the uniform convergence of $\int_1^\infty \exp(-xy) \sin x \, dx$.   (4 marks)

3. Let f be a real valued bounded function on $I := [a, b]$. Show that the upper and lower Riemann integrals are bounded on I.   (4 marks)

4. Let f be a real valued bounded function on $I := [a, b]$. Let α be a monotonically increasing function and P^* be the refinement of a partition P of I, then prove that $L(P^*, f, \alpha) \geq L(P, f, \alpha)$.   (4 marks)

5. Check whether or not the following integral converges or diverges   (4 marks)

$$\int_0^\infty \frac{x}{1 + x^2 \sin^2 x} dx.$$

SECTION III-Questions with Brief Answers

Q 3 Let $f : [a, b] \to \mathbb{R}$ be a continuous function. Show that there exists $c \in (a, b)$ such that

$$\frac{1}{b - a} \int_a^b f(x) dx = f(c).$$

Is this still true for Riemann integrable functions.   (6 marks)

Q 4 Show that the following integral is conditionally convergent.   (6 marks)

$$\int_2^\infty \frac{\cos x}{\log x} dx.$$

Q 5 Test for uniform convergence of the sequence $\{f_n(x)\}$, where   (6 marks)

$$f_n(x) = \frac{nx}{1 + n^2 x^2}, \quad \text{for all real } x.$$

Q 6 Let f be a real valued continuous function defined on $[a, b]$. If f' exists and is bounded on $[a, b]$, then f is a function of bounded variations on $[a, b]$. But its converse may not be true.   (6 marks)

Q 7 State and prove Cauchy's criterion of convergence for infinite integrals.   (6 marks)