UNIVERSITY OF THE PUNJAB

Seventh Semester – 2019

Examination: B.S. 4 Years Program

Roll No. in Fig. ....

Roll No. in Words. ....

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PAPER: Partial Differential Equations
Course Code: MATH-402 Part–I (Compulsory)

MAX. TIME: 30 Min.
MAX. MARKS: 10

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Signature of Supdt.:

Attempt this Paper on this Question Sheet only.
Please encircle the correct option. Division of marks is given in front of each question.
This Paper will be collected back after expiry of time limit mentioned above.

Q.1. Encircle the right answer, cutting and overwriting is not allowed. (1x10=10)

1. $c^2\nabla^2 u = \frac{\partial^2 u}{\partial t^2}$ is called

   (a) heat equation for u.

   (b) Laplace equation for u.

   (c) wave equation for u.

   (d) none of the above

2. If ζ_1 and ζ_2 are two linearly independent solutions of the second order partial differential equation, then which one of the following is also a solution?

   (a) $\zeta_1 + \zeta_2$

   (b) $\zeta_1 - \zeta_2$

   (c) $\zeta_1\zeta_2$

   (d) none of the above

3. To convert $u_{xx} - 5u_{xy} + 6u_{yy} = 0$ into canonical form, we use

   (a) $\xi = 2x + y, \eta = 3x + y$

   (b) $\xi = x + y, \eta = x$

   (c) $\xi = x - y, \eta = y$

   (d) none of the above

4. The heat equation is a ——— partial differential equation

   (a) hyperbolic

   (b) parabolic

   (c) elliptic

   (d) none of the above

5. $\frac{\partial^2 x}{\partial x \partial y}$ =——— (in usual notation)

   (a) p

   (b) q

   (c) r

   (d) s

P.T.O.

6. Solution of $p + q = 1$, is

a) $f(y - x, y + z) = 0$ b) $f(x - y, x - z) = 0$

c) $f(x - y, y - z) = 0$ d) $f(z - x, x + y) = 0$

7. If $p = \frac{\partial}{\partial x}$ *and* $q = \frac{\partial}{\partial y}$, the PDE for $z = f(\frac{y}{x})$ is

a) $xp - yq = 1$ b) $xp + yq = 0$ c) $xp - p = 0$ d) $xp + yp = 0$

8. The solution of PDE $u_{xx} + u_{yy} = 0$, is defined by

a) $u = e^y \cos x$ b) $u = e^x \cos y$ c) $u = \log(x^2 + y^2)$ d) $u = x^2 + y^2$

9. The boundary conditions $u_x(0, t) = \gamma(t)$, $u_x(a, t) = \delta(t)$, is known as--- condtion

a) Neumann b) Mixed c) Dirichlet d) Robin

10. Solution of $a(p + q) = z$, is defined by

a) $\phi(x + y, y - az) = 0$ b) $\phi(x - y, y - az) = 0$

c) $\phi(x - y, y + az) = 0$ d) $\phi(x + y, y + az) = 0$

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PAPER: Partial Differential Equations
Course Code: MATH-402 Part – II

MAX. TIME: 2 Hrs. 30 Min.
MAX. MARKS: 50

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

SECTION II-Questions with short Answers $(5 \times 4 = 20)$

Q.2 Solve the following Short Questions:

(i) Find the PDE by eliminating the constants a and b from $u = xy + y\sqrt{x^2 - a^2} + b$.

(ii) Show that $u(x,y) = xf(2x+y)$ is a general solution of PDE

$$x\frac{\partial u}{\partial x} - 2x\frac{\partial u}{\partial y} = u.$$

(iii) Define linear, non-linear and quasi linear partial differential equation with atleast one example.

(iv) Find the integral surface of the PDE

$$\frac{dx}{4yu} = \frac{dy}{1} = \frac{du}{-2y}.$$

(v) Find the particular integral of the PDE $(D_x^2 + 2D_xD_y + D_y^2)u = e^{2x+3y}$.

SECTION III-Questions with Brief Answers $(6 \times 5 = 30)$

Q.3 Find the surface which intersect orthogonally the family of surfaces $u^2 = c(x^2 + y^2)$ and which passes through the circle $\Gamma : x^2 + y^2 = 9,\ u = 3$.

Q.4 Reduce the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar form.

Q.6 Interpret and solve the following problem.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{k}\frac{\partial u}{\partial t}, \quad 0 \le x \le a,\ 0 \le y \le b, \quad t \ge 0$$

$$u(0,y,t) = 0, \qquad u(a,y,t) = 0$$
$$u(x,0,t) = 0, \qquad u(x,b,t) = 0$$
$$u(x,y,0) = f(x,y).$$

Q.7 Solve the following initial-boundary value problem:

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \qquad (0 \le x < \infty, t > 0)$$

$$u(x,0) = 6e^{-3x}, \qquad \lim_{x \to \infty} u(x,t) = 0$$

Q.8 Reduce the equation $4u_{xx} + 12u_{xy} + 9u_{yy} - 2u_x + u = 0$ to canonical form.