UNIVERSITY OF THE PUNJAB

Seventh Semester – 2019

Examination: B.S. 4 Years Program

Roll No. in Fig. ....

Roll No. in Words. ....

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PAPER: Ring Theory
Course Code: MATH-407 Part–I (Compulsory)

MAX. TIME: 30 Min.
MAX. MARKS: 10

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Signature of Supdt.:

Attempt this Paper on this Question Sheet only.
Please encircle the correct option. Division of marks is given in front of each question.
This Paper will be collected back after expiry of time limit mentioned above.

Q.1. Encircle the right answer, cutting and overwriting is not allowed. (1x10=10)

(i) The Polynomial ring R[x] over R is

(a) PID (b) UFD (c) ED (d) All of these

(ii) The maximal ideal ring in the ring Z of integers is

(a) Z (b) $11Z$ (c) $4Z$ (d) $6Z$

(iii) Which of the following is a prime ideal of the ring Z of integers.

(a) Z (b) $3Z$ (c) N (d) R

(iv) Units of Z are

(a) ± 1 (b) $\pm i$ (c) $\pm 1, \pm i$ (d) None of these

(v) Every ------------------ can be imbedded into a Field.

(a) Integral domain (b) Division Ring (c) subring (d) Ideal

(vi) A ring which is commutative with identity element and having no zero divisor is

(a) Division Ring (b) Integral domain

(c) Prime Ring (d) nilpotent ring

(vii) If R & R' be arbitrary ring $\phi : R \rightarrow R'$ is ring homomorphism such that $\varphi(a) = 0 \ \forall \ a \in R$ then $Ker\phi =$ -------------

(a) R' (b) $\{0\}$ (c) R (d) None of these

(viii) 7π is algebraic over

(a) Q (b) R (c) Z (d) None of these

(ix) If C is a finite extension of R, then $[R : R] =$ -----------

(a) 2 (b) 3 (c) 4 (d) 1

(x) The set $Z_8 = \{0,1,2,3,4,5,6,7\}$ under addition and multiplication modulo 8 forms

(a) Division Ring (b) Integral domain (c) Commutative Ring (d) field

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PAPER: Ring Theory
Course Code: MATH-407 Part – II

MAX. TIME: 2 Hrs. 30 Min.
MAX. MARKS: 50

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

Q.2. Answer these short questions. (5x4=20)

(i) Let R be a ring with identity. Then show that the relation of being associates is an equivalence relation. (4)

(ii) Find all associates of $2+x-3x^2$ in $Z(x)$. (4)

(iii) If R is integral domain then prove that $R[x]$ the polynomial ring over R is integral domain. (4)

(iv) Define the following term: Euclidean Domain, divisor, units, associates, unique factorization domain. (4)

(v) Show that x^3-5 is irreducible polynomial of $Q(\sqrt{5})$. (4)

Answer these long questions. (5x6=30)

Q.3 be an integral domain and *p* be non zero element of *R*. Then prove that *p* is prime in *R* if and only if $\frac{R}{pR}$ is integral domain. (6)

Q.4 Prove that in a unique factorization domain every irreducible element is prime. (6)

Q.5 Prove that every field is an integral domain. Prove or disprove the converse of this statement for infinite integral domain. (6)

Q.6 Differentiate between algebraic and transcendental numbers. Explain with examples. Let F be a subfield of K. Prove that an element $a \in K$ is algebraic over F if and only if *F(a)* is a finite extension of F. (6)

Q.7 Show that the Rings Z_6 and $Z_2 \oplus Z_3$ are isomorphic. (6)