Seventh Semester – 2019
Examination: B.S. 4 Years Program

Roll No. in Words. ....

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PAPER: Number Theory-I
Course Code: MATH-408 **Part–I (Compulsory)**

MAX. TIME: 30 Min.
MAX. MARKS: 10

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Signature of Supdt.:

Attempt this Paper on this Question Sheet only.
Please encircle the correct option. Division of marks is given in front of each question.
This Paper will be collected back after expiry of time limit mentioned above.

Q. 1	MCQs (1x10 = 10 Marks)
(i)	641 divides a) F_5 b) F_3 c) F_2 d) F_0
(ii)	The least common multiple of 60 and N is 1260. Which of the following could be the prime factorization of N? a) $2.3^2.5.7$ b) $2^3.5.7$ c) $3^2.5.7$ d) $2.5.7^2$
(iii)	What is the remainder when 2^{15} is divided by 15? a) 10 b) 13 c) 8 d) none
(iv)	The sum of positive divisors of 28 is a) 28 b) 56 c) 60 d) None of (a),(b),(c)
(v)	The number of primitive root mod 55 are (a) 2 (b) - 1 (c) 1 (d) 0
(vi)	If 15x+7y = 210, then a) x=2, y=5 b) x=7, y=15 c) x=2, y=15 d) x=7, y=16
(vii)	If 2 has exponent 3 mod 7, then 2^6 has exponent (a) 1 (b) 3 (c) 5 (d) 7
(viii)	If $\sigma(n) = 2n$, then n is called (a) Composite (b) Perfect (c) Prime (d) Mersenn
(ix)	If p is a prime number and d is a factor of p-1 then the number of solutions of the congruence $x^{d-1} \equiv 0 \pmod{p}$ is a) p-1 b) p c) d-1 d) d
(x)	$\tau(25) =$ (a) 2 (b) 3 (c) 5 (d) 25

UNIVERSITY OF THE PUNJAB

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PAPER: Number Theory-I **MAX. TIME: 2 Hrs. 30 Min.**

Course Code: MATH-408 Part – II **MAX. MARKS: 50**

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

Q. 2	Short Questions (5x4 = 20 Marks)
(i)	Solve the linear congruence $36x \equiv 7 \pmod{157}$ (4)
(ii)	Prove that there are infinitely many primes. (4)
(iii)	Let $(a,51)=1$. Then prove that $a^{32} \equiv 1 \pmod{51}$. (4)
(iv)	By means of theory of exponent, Prove that ${}^{n}p_{r}$ and ${}^{n}c_{r}$ both are integers. (4)
(v)	Using indices, Solve the following congruence $11x^3 \equiv 2 \pmod{23}$ (4)

Long Questions (6x5 = 30 Marks)

Q.3	(i) Let $n>1$ be a composite integer then show that there exists a prime p such that $p \mid n$ and $p \le \sqrt{n}$ (ii) If k integers $a_1, a_2, \ldots, a_k$ form a Complete Residue System modulo m then show that $k=m$. (2+3)
Q.4	State and prove the Chines Remainder theorem. Apply it to solve the following system of linear congruences $x \equiv 2 \pmod{5}$ $x \equiv 3 \pmod{7}$ $x \equiv 5 \pmod{11}$ (3+2)
Q.5	State and prove Wilson Theorem. Apply it to find remainder of 35! when it is divided by 37. (3+2)
Q.6	Define multiplicative arithmetic functions. Let n be an integer > 1. Show that $\sigma(n)$ is odd if and only if n is a perfect square or twice a perfect square. (5)
Q.7	Let a be primitive root modulo n and b,c be integers, then show that $Ind\ bc \equiv Ind\ b + Ind\ c \pmod{\varphi(m)}$. (5)
Q.8	(i) Prove that every prime number has a primitive root. (ii) Find all primitive roots of 50. (3+2)