# UNIVERSITY OF THE PUNJAB

## B.S. 4 Years Program : Third Semester – Fall 2021

Roll No. ................

**Paper: Differential Equations** **Course Code: MATH-2003** **Time: 3 Hrs. Marks: 60**

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**Q.1. Solve the following:** **(6x5=30)**

**(i)** Verify that the indicated function is a solution of the given differential equation on the interval $(-\infty, \infty)$.

(a) $y' = x\, y^{1/2}$; $y = \frac{1}{16}x^4$ (b) $y'' - 2y' + y = 0$; $y = xe^x$

**(ii)** Show that the first order linear nonhomogeneous equation, $y' + P(x)y = f(x)$ has the solution in the following form

$$y_c = ce^{-\int P(x)dx} + e^{-\int P(x)dx}\int e^{\int P(x)dx} f(x)dx$$

where c is an arbitrary constant.

**(iii)** Verify that the given differential operator annihilates the indicated functions.

(a) $(D-2)^2$; $y = 4e^{2x} - 10xe^{2x}$ (b) $D^4$; $y = 1 - 5x^2 + 8x^3$

**(iv)** Guess the form of particular solution $y_p$ for each of the given nonhomogeneous differential equations.

(a) $y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$ (b) $y''' + y'' = e^x \cos x$

**(v)** The homogeneous second-order Cauchy-Euler equation $ax^2y'' + bxy' + cy = 0$ has an auxiliary equation of the form $am^2 + (b-c)m + c = 0$, with two roots $m_1 = \frac{-(b-a)+\sqrt{(b-a)^2-4ac)}}{2a}$ and $m_2 = \frac{-(b-a)-\sqrt{(b-a)^2-4ac)}}{2a}$. Write the general solution when

(a) roots are real and distinct (b) roots are real and repeated

**(vi)** Find the value of **k** so that the given differential equation is exact.

$$(6xy^3 + \cos y)\, dx + (2kx^2y^2 - x \sin y)dy$$

**Q.2. Solve the following:** **(5x6=30)**

**(i)** Solve the given first-order linear differential equation for the given initial condition.

$$\frac{dy}{dx} + y = f(x), \quad y(0) = 0$$

where $f(x) = \begin{cases} 1, & 0 \le x \le 1, \\ 0, & x > 1. \end{cases}$

**(ii)** Solve the given Bernoulli's equation by an appropriate substitution .

$$x\frac{dy}{dx} + y = y^{-2}$$

**(iii)** Using an appropriate annihilator operator for the function g(x)= $8e^{3x} + 4\sin x$ in the given second order nonhomogeneous equation $y'' - 3y' = 8e^{3x} + 4\sin x$, generate the particular solution in the following form

$$y_p = Axe^{3x} + B\cos x + C\sin x$$

Also find the values of arbitrary constants A, B, and C using method of undetermined coefficient.

**(iv)** The standard form of the homogeneous linear second-order differential equation is given as

$$y'' + P(x)y' + Q(x)y = 0$$

where $P(x)$ and $Q(x)$ are continuous on some interval $I$. Suppose that $y_1(x)$ is the known solution of the given equation on $I$ and that $y_1(x) \neq 0$ for every $x$ in the interval. Define $y_2(x) = u(x)y_1(x)$ and use method of reduction of order to show that

$$y_2 = y_1(x)\int \frac{e^{-\int P(x)dx}}{y_1^2(x)}dx$$

**(v)** Evaluate

$$\mathcal{L}^{-1}\left\{\frac{s^2 + 6s + 9}{(s-1)(s-2)(s+4)}\right\}$$