Q.1.   Solve the following: (6x5=30)

i.   Plot the following points (given in polar coordinates). Then find all polar coordinates of each given point
(a) $(2, \pi/2)$
(b) $(-2, 0)$

ii.   Find the focus $(p, 0)$ and directrix of the parabola $y^2 = 20\,x$.

iii.   Show that the straight line $\dfrac{x+3}{2} = \dfrac{y-4}{7} = \dfrac{z}{3}$ is parallel to the plane $4x + 2y + z = 6$.

iv.   Find the equation of cylinder by line parallel to x-axis that passes through a parabola $z = y^2$ in yz-plane and sketch it in the right-handed cartesian coordinate system.

v.   Find an equation for the line perpendicular to the tangent to the curve $y = x^3 - 4x + 1$ at the point $(2, 1)$.

vi.   Find the center and radius of the sphere $x^2 + y^2 + z^2 + 3x - 4z + 10 = 0$.

Solve the following: (3x10=30)

Question 2:

(a) Find the angle of rotation for the curve $2x^2 + \sqrt{3}xy + y^2 - 10 = 0$ when coordinate axes are to be rotated through an angle α. Find α and the new equation. Identify the curve.

(b) Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.

Question 3:

(a) Find the eccentricity of the hyperbola $9x^2 - 16y^2 - 144 = 0$.

(b) Sketch the surfaces of ellipsoid $4x^2 + 9y^2 + 4z^2 = 36$ and discuss them.

Question 4:

A planet travels about its sun in an ellipse whose semi major axis has length 'a'.

(a) Show that $r = a(1 - e)$ when the planet is closest to the sun and that $r = a(1 + e)$ when the planet is farthest from the sun.

(b) Find a polar equation for an ellipse with semi-major axis 1.0 AU (astronomical units) and eccentricity 0.167.