UNIVERSITY OF THE PUNJAB

Part-II A/2015
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: I (Advanced Analysis)

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

SECTION-I

Q.1 (a) Suppose A is any uncountable set and B is a denumerable subset of A. Show that $A \setminus B \approx A$. (10)

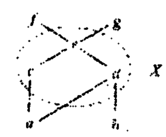
(b) Prove the Cantor's Theorem. For any set A we have $|A| \lhd |P(A)|$. (10)

Q.2 (a) State and Prove the Schroder Bernstein Theorem. (10)

(b) Let A be ordered set and $p\ A$ denote the collection of all predecessors sets of the elements of A then show that $A \approx p\ A$. (10)

Q.3 (a) Let $A = 1,2,3,5,6,9,10,15,18$ with the Order xRy defined by x is a multiple of y. Show that it is Partial order relation also find the Minimal, Maximal. (10)

(b) Let $S = a,b,c,d,e,f,g$ be ordered as in the figure and $X = c,d,e$ be subset of S. Find the minimal and maximal elements of S. Also find the supremum and infimum of X. (10)



Q.4 (a) State and prove the principle of transfinite induction. (10)

(b) Prove the well-ordering theorem (Zermelo): Every nonempty set can be well ordered. (10)

SECTION-II

Q.5 (a)

i) Verify that the class $\mathcal{A} = A \subset N : A$ is finite or A^c is finite is algebra of sets but not $\sigma-$algebra. (5+5)

ii) Let $X = N. \mathrm{A} = 2^N, \mu : \mathrm{A} \to \overline{R}$, $\mu\ E = \begin{cases} \text{number of points, if A is finite} \\ \infty, \quad \text{if A is infinite} \end{cases}$

Show that μ is a measure on A.

(b) If E_1 and E_2 are Lebesgue measurable sets, then show that (10)

$m\ E_1 \cup E_2 = m\ E_1 + m\ E_2 - m\ E_1 \cap E_2$.

Hence calculate the Lebesgue measure of $A = (-3,4) \cup\ 1,6$.

P.T.O.

Q.6 (a) Prove that the Borel algebra B is generated by each of the following collection of sets. (10)

(i) The collection C_1 of all closed subsets of R.

(ii) The collection C_2 of all sub-intervals of the form $(-\infty, b]$.

(iii) The collection C_3 of all sub-intervals of the form $(a, b]$.

(b) Prove that a continuous function defined on a measurable set is measurable. (10)

Q.7 (a) Let f be a measurable function and G an open set. Then $\{x : f(x) \in G\}$ is a measurable set. (10)

(b) Let $f : R \to R$ be a measurable function and $g : R \to R$ a Borel measurable function. Then prove that $g \circ f$ is measurable function. (10)

Q.8 (a) Let f be a bounded function defined on $[a, b]$ and f is Riemann integrable on $[a, b]$, then show that (i) f is measurable (10)

(ii) $R\int_a^b f = \int_a^b f$ (the two integrals are equal)

(b) Let f and g be bounded and measurable functions defined on a set E of finite measure. Then prove that (i) $\int_E af + bg = a\int_E f + \int_E g$ (ii) $\left|\int_E f\right| \le \int_E |f|$ (10)

(ii) If A and B are disjoint measurable subsets of E, then $\int_{A\cup B} f = \int_A f + \int_B f$

Q.9 (a) Let $\langle G_n \rangle$ be sequence of integrable function which converges $a.e.$ on E to an integrable function g. Let $\langle F_n \rangle$ be a sequence of measurable functions such that $|F_n| \le G_n$ and $\langle F_n \rangle$ converges to f $a.e.$ on E. If $\underset{n}{Lim}\int_E G_n = \int_E g$ then prove that $\underset{n}{Lim}\int_E F_n = \int_E f$. (10)

(b) Minkowski's inequality: Prove that $f, g \in L^p \Rightarrow f + g \in L^p$ and $\|f + g\|_p \le \|f\|_p + \|g\|_p$ (10)

UNIVERSITY OF THE PUNJAB

Part-II A/2015
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: II (Methods of Mathematical Physics) **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all, selecting at least TWO questions from each section.

	Section-I	
1(a)	Define ordinary and singular points of the differential equation $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$. When a singular point is said to be regular and irregular? Find regular and irregular singular points of the differential equation $(x^2-4)^2y'' + (x-2)y' + y = 0$.	10
1(b)	Use the method of Frobenius to obtain two linearly independent series solutions about the singular point $x_0 = 0$ of the differential equation $xy'' + (1-x)y' - y = 0$.	10
2(a)	Show that for hypergeometric function $$F\left[-\frac{1}{2}n, -\frac{1}{2}n + \frac{1}{2}, b + \frac{1}{2}; 1\right] = \frac{2^n(b)_n}{(2b)_n}.$$	10
2(b)	Obtain the ortho-normalization relation for the Legendre polynomials.	10
3(a)	Prove that the eigenvalues of a regular Sturm Liouville (SL) system are real.	10
3(b)	Determine the eigenvalues and the corresponding eigenfunctions for the following S-L problem: $$u''(x) + \lambda u(x) = 0,$$ $$u(0) = 0, \quad u'\left(\frac{\pi}{2}\right) = 0.$$	10
4(a)	Show that $$J_{-5/2} = \sqrt{\frac{2}{\pi x}}\left[\frac{3}{x}\sin x + \left(\frac{3}{x^2} - 1\right)\cos x\right].$$	10
4(b)	Derive heat equation for a rod of constant thermal conductivity K. Write Neumann and Dirichlet boundary conditions for the problem.	10
5(a)	Obtain the general solution of the following first order partial differential equation $$(z^2 - 2yz - y^2)z_x + (xy + xz)z_y = xy - xz.$$	10
5(b)	A hot spherical ball is immersed in ice-cold water. Formulate and solve the problem when the initial temperature is given by $f(r)$, where r is radial distance.	10
	Section-II	
6(a)	State and prove Laplace convolution theorem.	10
6(b)	Use Fourier transform method to solve the wave equation $u_{tt} = c^2u_{xx}, \quad -\infty < x < \infty, \quad t > 0,$ subject to the boundary conditions $u(x,0) = f(x), \quad u_t(x,0) = 0,$ $u \to 0, \ u_x \to 0, \ as \ \lvert x\rvert \to \infty \quad \forall t,$ where f is assumed to have a Fourier transform.	10
7(a)	Compute the green's function associated with the boundary value problem $y''(x) = f(x), \quad y(0) = 0, \quad y'(1) = 0,$ where $f(x)$ is a well-behaved arbitrary function.	10
7(b)	Solve the initial value problem by using the Laplace transform $$y''(t) + ty'(t) - y(t) = 0, \quad y(0) = 0, \quad y'(0) = 1.$$	10
8(a)	Describe all the steps (properties) to construct the modified Green's function associated with a boundary value problem.	10
8(b)	Find the curve of shortest length on the surface of a sphere.	10
9(a)	State and prove the fundamental theorem on variational calculus (one independent variable case only).	10
9(b)	Find the extremal of the functional $$I[y(x)] = \int_0^1 [y'^2 + y^2]dx,$$ subject to the end point conditions $$y(0) = 0, \quad y(1) = 1.$$	10

UNIVERSITY OF THE PUNJAB

Part-II A/2015
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: IV-VI (opt. ix) [Electromagnetic Theory] **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions in all selecting at least TWO questions from each section.

SECTION I

1. (a) Find the magnetic field due to a distant circuit, when the origin is taken near the circuit. (10 marks)
   (b) Derive an expression for the magnetic energy in terms of currents and inductances? (10 marks)

2. (a) Find the capacitance of parallel plate capacitors? And calculate the relationship between capacitance and resistance. (10 marks)
   (b) Work out the expression for the vector potential $\vec{A}$ and the magnetic induction $\vec{B}$ due to current I in a long straight wire. (7 marks)
   (c) Point charges $q_1 = q_2 = 10^{-9}C$ are located at $(1,0,0)$ and $(0,0,1)$ respectively. Find the force on each charge. (3 marks)

3. (a) Calculate the electromotance induced in a fixed as well as in a rotating loop with a time dependent magnetic field. (10 marks)
   (b) Work out the potential energy of a group of point charges. (5 marks)
   (c) Find the work done required to move a point charge of $5C$ from $(0,0,1)$ to $(1,1,1)$ along the arc of parabola $y = x^2$, $z = 1$ in the field $\vec{E} = 2y\hat{i} + 2x\hat{j} + \hat{k}$. (5 marks)

4. (a) Find a relationship between capacitance and resistance. (5 marks)
   (b) Calculate the energy stored in an isolated spherical conductor of radius R and surface charge density σ. (5 marks)
   (c) Calculate the vector potential $\vec{A}$ and use it to find the magnetic induction for a long straight wire. (10 marks)

5. (a) Calculate the electric field due to a uniform spherical charge distribution at an external point. (5 marks)
   (b) An electric field $\vec{E} = E_0(\sin x\hat{i} + \cos y\hat{j})e^{-y}$ exists in free space. Find the volume charge density. (5 marks)
   (c) Calculate the magnetic field at a point on the axis of a circular loop. (10 marks)

SECTION II

6. (a) Discuss the propagation of electromagnetic wave in the region between parallel conducting plates. (10 marks)
   (b) Prove that a plane electromagnetic wave in free space has only transverse components of electric and magnetic fields. (10 marks)

7. (a) Calculate the co-efficient of refraction and reflection for normal incidence. Prove that $R + T = 1$. (10 marks)
   (b) State and Prove Poynting's Theorem. (10 marks)

8. (a) Calculate the Electromagnetic Potentials V and $\vec{A}$ for a line charge moving with a constant velocity along its length. (10 marks)
   (b) If $f_1(x.t) = A_1\cos(kx - \omega t + \theta_1)$ and $f_2 = A_2\cos(kx - \omega t + \theta_2)$ are two sinusoidal waves then show that their sum is also a sinusoidal wave. (10 marks)

9. (a) Verify that the flux of Poynting vector through any closed surface gives the energy flow through the volume enclosed by the surface. (10 marks)
   (b) Work out the Fresnel's equations for the case when its $\vec{E}$ vector is normal to the plane of incident. (10 marks)

UNIVERSITY OF THE PUNJAB

Part-II A/2015

Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics

PAPER: IV-VI (opt. x) [Operations Research]

TIME ALLOWED: 3 hrs.

MAX. MARKS: 100

NOTE: Attempt any FIVE questions selecting atleast TWO questions from each section.

SECTION-I

Marks

Q.1 A lake contains two types of fish, I and II. The owner provides two types of food, A and B for these fish. Species I require 2 units of food A and 4 units of food B; and species II require 5 units of food A and 2 units of food B. If the owner has 800 units of each food, find the maximum number of fish that the lake can support.

(a) Formulate the problem as a Linear Programming Model. (10)

(b) Solve the problem using Graphical Method. (10)

Q.2 **(a)** Write the steps of Simplex Method. (8)

(b) Consider the following LP-Model (12)

Minimize: $Z = 2x_1 + 3x_2 - 5x_3$

Subject to

$x_1 + x_2 + x_3 = 7$

$2x_1 - 5x_2 + x_3 \geq 10$

$x_1, x_2, x_3 \geq 0$

Solve by using M-technique.

Q.3 **(a)** What do you mean by Infeasible solution? (10)

Show that the following LP-Model has no feasible solution.

Maximize: $Z = 3x_1 + 2x_2$

Subject to

$2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

(b) Apply Dual Simplex Algorithm to solve (10)

Minimize: $Z = 5x_1 + 6x_2$

Subject to

$x_1 + x_2 \geq 2$

$4x_1 + x_2 \geq 4$

$x_1, x_2 \geq 0$

Q.4 Compare the starting solution of the following model using (6+6+8)

(i) North West Corner Method

(ii) Least Cost Method

(iii) Vogel's Approximation

					Supply
	10	2	20	11	15
	12	7	9	20	25
	4	14	16	18	10
Demand	05	15	15	15	

(P.T.O.)

Q.5 **(a)** Solve the following transportation model using method of multiplier with northwest corner method as starting solution. (12)

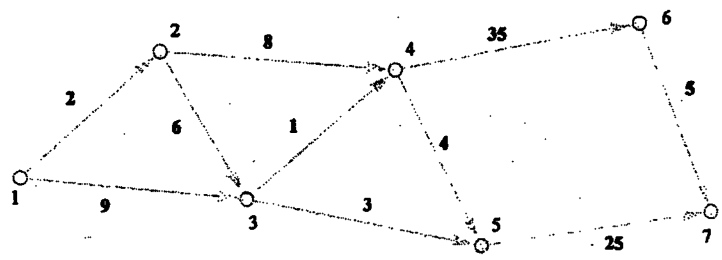
				Supply
	0	2	1	5
	2	1	5	10
	2	4	3	5
Demand	5	5	10	

(b) Find all possible assignments using Assignment Model. (8)

3	9	2	3	7
6	1	5	6	6
9	4	7	10	3
2	5	4	2	1
9	6	2	4	6

SECTION-II

Q.6 **(a)** For the following network, the distance in miles are given between different cities from city 1 to city 7. Find the shortest rout between city 1 and city 7.



(8)

(b) Use Revised Simplex Method to Solve (12)

Maximize: $Z = 4x_1 + 3x_2 + 6x_3$

Subject to

$3x_1 + x_2 + 3x_3 \le 30$

$2x_1 + 2x_2 + 3x_3 \le 40$

$x_1, x_2, x_3 \ge 0$

Q.7 Solve the following LPP using Bounded Variable Algorithm. (20)

Maximize: $Z = 3x_1 + 5x_2 + 3x_3$

Subject to

$x_1 + 2x_2 + 2x_3 \le 14$

$2x_1 + 4x_2 + 3x_3 \le 23$

$0 \le x_1 \le 4,\ 0 \le x_2 \le 5,\ 0 \le x_3 \le 3$

Q.8 Solve the following problem by Cutting Plane Algorithm. (20)

Maximize: $Z = 7x_1 + 10x_2$

Subject to

$-x_1 + 3x_2 \le 6$

$7x_1 + x_2 \le 35$

$x_1, x_2, \ge 0$ and integers.

Q.9 **(a)** Use Dynamic Programming to solve (10)

Maximize: $u_1 u_2 u_3$

Subject to

$u_1 + u_2 + u_3 = 10$

$u_1, u_2, u_3 \ge 0$

(b) Solve by using Parametric Linear Programming (10)

Maximize: $Z = (3-6t)x_1 + (2-2t)x_2 + (5+5t)x_3$

Subject to

$x_1 + 2x_2 + x_3 \le 40$

$3x_1 + 2x_3 \le 60$

$x_1 + 4x_2 \le 30$

$x_1, x_2, x_3 \ge 0$

UNIVERSITY OF THE PUNJAB

Part-II **A/2015**
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt.ii) [Computer Applications]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 50

Note: Attempt any FIVE questions by selecting atleast TWO from each section.

Section 1

1. a) **[3 marks]** Define the following in Fortran language.
   (i) Assignment Statement (ii) Array Variable (iii) Constants

   b) **[2 marks]** Find the value of e after the execution of following statements

```
Program example
Integer :: p,q,i
Real :: a,e,f
a=45.2;e=52.3;f=2.2;p=26;q=52;i=1.
e=a/e*f**2+Mod(q,p)+Log(1)
p=e;e=p
Print*, p
Print*, e
end
```

   c) **[5 marks]** Which of the following are unacceptable as constant or variable names and why?

   (i) 2.345E-345 (ii) 5Asim (iii) 24.657E20 (iv) 43.2E2.3 (v) 34/25
   (vi) fg1 (vii) 92.AE-7 (viii) u25*kj (ix) alpha (x) wxy+35

2. [1.5+1.5+7 marks]

   a) Find and correct errors in the following programs.

   i.
```
If (w>=z)
Then y = x+y
else y = y-x
endif
```

   ii.
```
If (a>b) then
If (c>d) then
x=y
else
x=z
endif
```

   b) Write a Fortran program to arrange an array of ten elements in ascending order.

Please turn →

(P.T.O.)

3. **[10 marks]** Consider two circles with centers (x_1, y_1) and (x_2, y_2) and radii r_1 and r_2 taken as input. Write a Fortran 90 program to check whether these two circles intersect each other, touch each other or do not touch each other by calculating the distance between centers of these circles and comparing it with sum of their radii.

4. **[10 marks]** Write a Fortran 90 program to find the roots of a quadratic equation using case statement.

Section 2

5. **[10 marks]** Write a Fortran 90 program to solve the following system of equations using Gauss Seidel method

$$10x + 2y + z = 9, \quad 2x + 20y - 2z = -44, \quad -2x + 3y + 10z = 22.$$

The program should check for diagonal dominant matrix as well.

6. **[10 marks]** Write a Fortran 90 program to implement Runge–Kutta method of order 4 for the solution of following initial value problem (IVP) at $x = 1$ with stepsize $h = 0.1$ and initial condition $y(0) = 1$

$$\frac{dy}{dx} = x^2 + y^2$$

7. **[10 marks]** The following data give power I and speed V of a ship.

V	8	10	12	14	16
I	1000	1900	3250	5400	8950

Write a Fortran 90 program to find I when $V = 9$ using Newton's forward interpolation formula.

8. **[10 marks]** Write a Fortran 90 program to find real root of $e^x - x^2 - 5 = 0$ using Newton Raphson method correct to four decimal places.

9. Write the Mathematica statements for the following.
   a) **[1.5 marks]** Find the conjugate of $x^2 + 3i$.
   b) **[1.5 marks]** Find intersection of two sets, $A = \{1, 3, 7, 9\}$, $B = \{3, 9\}$.
   c) **[1.5 marks]** Solve numerically $\frac{dy}{dx} = x + 2y$ with $y(0) = 1$.
   d) **[1.5 marks]** Evaluate numerically $\int_0^2 Sin(\frac{1-x}{x})dx$.
   e) **[2 marks]** Evaluate $\prod_{k=1}^{5} x^k + k$.
   f) **[2 marks]** Plot the graph of $x - \frac{1}{x}$, $0 \leq x \leq 4$. Also draw the frame and grid lines in the graph.

UNIVERSITY OF THE PUNJAB

Part-II A/2015
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: IV-VI (opt.v) [Number Theory] **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

SECTION-I

Q.1 **(a)** Let $b > 1$ be a fixed integer. Prove that every positive integer x has a unique representation of the form $x = a_s b^s + a_{s-1} b^{s-1} + \dots + a_1 b + a_0, a_s \neq 0,$ an $0 \leq a_i < b$, for each $i = 0,1,2,\dots,s$. Apply it to write 23414 to the base 13, where $\alpha = 10, \beta = 11$ and $\lambda = 12$. (8+2)

(b) (i) Prove that there exist infinitely many primes of the form $4k-1$. (5+5)

(ii) Write 100 as the sum of two integers one of which is divisible by 7 and other is divisible by 11.

Q.2 **(a)** Let $a, b \neq 0$ and c be any three integers with $d = (a,b)$. Prove that the linear Diophantine equation $ax + by = c$ has a solution if and only if d/c. Further show that, if (x_0, y_0) is a particular solution of $ax + by = c$ then any other solution (x', y') is of the form $x' = x_0 - \frac{b}{d}t$ and $y' = y_0 + \frac{a}{d}t$, where t is an integer. (10)

(b) State and prove the Chinese Remainder Theorem. Apply it to solve the following system of linear congruences (10)

$$x \equiv 2 \pmod 3$$
$$2x \equiv 3 \pmod 5$$
$$3x \equiv 5 \pmod 7$$

Q.3 **(a)** State and prove the Euler's Theorem. Deduce Fermat's Little Theorem as a consequence of Euler's Theorem. (10)

(b) Prove that an integer p is prime if and only if (10)

$$(p-1)! \equiv p-1 \pmod p$$

Prove or disprove, 101 is a prime number.

Q.4 **(a)** Let $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$, p_i are distinct primes and $k_i \geq 1$. (10)

Show that $$\sum_{d/n} \frac{\varphi(d)}{d} = \prod_{i=1}^{r} [1 + k_i \frac{p_i - 1}{p_i}]$$

(b) (i) How many zeros will there be in 50! in its ordinary decimal representation. (10)

(ii) Prove that C_r^n and P_r^n both are integers.

(P.T.O.)

Q.5 **(a)** Prove that there exists at least one primitive root modulo each prime p. (10)

(b) Prove that all primitive solutions of Pythagorean Diophantine equation $x^2 + y^2 = z^2$ subject to the conditions $x > 0, y > 0, z > 0$, $(x, y, z) = 1$ and y is even are given by (10)

$$x = r^2 - s^2$$
$$y = 2rs$$
$$z = r^2 + s^2$$

SECTION II

Q.6 **(a)** Prove that an integer a is a quadratic residue modulo a prime p, if and only if, $a^{\frac{p-1}{2}} \equiv 1 (\mathrm{mod}\, p)$. (10)

(b) Let p and q be distinct odd primes. Prove that (10)

$$\left(\frac{p}{q}\right) = (-1)^{\sum_{k=1}^{\frac{q-1}{2}} [\frac{kp}{q}]}$$

where $[\frac{kp}{q}]$ denotes the greatest integer and $\left(\frac{p}{q}\right)$ denotes the Legendre of p modulo q.

Q.7 **(a)** Define a primitive polynomial. Let p be a prime and $f(x) = a_0 + a_1 x + \dots + a_n x^n$, be a polynomial with integral coefficients such that $p \nmid a_n, p^2 \nmid a_0$ and $p / a_i, i = 0, 1, \dots, n-1$. Show that $f(x)$ is irreducible over R (10)

(b) Prove that an irreducible polynomial of degree n over F has n distinct roots. (10)

Q.8 **(a)** Prove that the totality of algebraic numbers form a number field. (10)

(b) If E over K and K over F are both finite extensions then prove that E over F is also finite. Hence deduce that if K is of degree n over F, then any element of K is algebraic over F of degree dividing n. (10)

Q.9 **(a)** Prove that every algebraic number field has at least one integral basis. (10)

(b) Prove the existence of transcendental numbers. (10)

UNIVERSITY OF THE PUNJAB

Part-II A/2015
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: IV-VI (opt.vi) [Fluid Mechanics] **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions in all, selecting at least TWO questions from each section.

Section I

Q.1(a)	What is a fluid? Why is fluid considered as a continuum? Explain.	(10)
(b)	A flat plate having dimensions of $2m \times 2m$ slides down an inclined plane at an angle of one radian to the horizontal at a speed of 6 m/s. The inclined plane is lubricated by a thin film of oil having a viscosity of 30×10^{-3} $Pa.s$. The plate has a uniform thickness of 20 mm and a density of 40,000 kg/m^3. Determine the thickness of lubricating oil film.	(10)
Q.2(a)	What are the general methods for describing the fluid motion and explain the method which is commonly used in fluid mechanics.	(10)
(b)	If every particle of fluid moves on the surface of a sphere, prove that the equation of continuity is $\frac{\partial \rho}{\partial t}\cos\theta' + \frac{\partial(\rho\omega'\cos\theta')}{\partial\theta'} + \frac{\partial(\rho\omega\cos\theta')}{\partial\varphi} = 0$, ρ being the density, θ', φ the latitude and longitude of any element, and ω', ω the angular velocities of the element in latitude and longitude respectively.	(10)
Q.3(a)	State and prove Kelvin's theorem on the constancy of circulation.	(10)
(b)	The velocity components of a two dimensional fluid flow are given by $$u = 3x + y, \qquad v = 2x - 3y.$$ Calculate the circulation around the circle $(x-1)^2 + (y-6)^2 = 4$.	(10)
Q.4(a)	For $u = -\omega y,\ v = \omega x,\ w = 0$, show that the surfaces intersecting the streamlines orthogonally exist and are planes through z-axis, although the velocity potential does not exist.	(10)
(b)	Find the velocity potential, stream function, stagnation point, speed and pressure distribution at any point in the case when a vortex is placed in a uniform stream.	(10)
Q. 5	For the velocity components of a certain fluid $$v_r = -\left(1 - \frac{a^2}{r^2}\right)\cos\theta, \qquad v_\theta = \left(1 + \frac{a^2}{r^2}\right)\sin\theta,$$ Find (i) The complex velocity $\frac{dw}{dz}$ (ii) The speed V and the complex velocity potential $w(z)$ (iii) The velocity potential φ and stream function ψ (iv) The equipotential lines and streamlines.	(20)

Section II

Q.6(a)	State and prove Milne-Thomson circle theorem.	(10)
(b)	Show that the superposition of a uniform stream over a doublet represents the streaming flow past a circular cylinder of radius a.	(10)
Q. 7	Derive Navier-Stokes' equations of motion of an incompressible viscous fluid and also write the same equations in Cartesian and cylindrical coordinates.	(20)
Q.8(a)	Show that the velocity distribution for the Hagen-Poiseuille flow depends upon the external pressure for its existence.	(12)
(b)	What is the Stokes' first problem? Write its mathematical formulation and solve it for the velocity field.	(08)
Q.9(a)	Define mean motion and fluctuations in a turbulent flow and prove that the mean value of a fluctuating quantity is always zero.	(10)
(b)	Describe Karman' s vortex street and evaluate the expression for the velocities with which the both rows of vortices advance.	(10)

UNIVERSITY OF THE PUNJAB

Part-II **A/2015**
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: IV-VI (opt.xi) [Theory of Approximation & Splines] **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions, select at least TWO questions from each section.

SECTION I

	QUESTION 1	Marks
a)	Determine the image of an ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$, under the transformation of reflection in the line making an angle $\frac{\pi}{4}$, in anticlockwise direction with the x-axis. Also prove that image is still an ellipse.	(10)
b)	Show that the composition of rotation and reflection is a reflection.	(10)
	QUESTION 2	
a)	Determine the image of the circle $x^2+y^2=4$, under the transformation of the shear parallel to x-axis by a factor -2.	(10)
b)	Show that rotation transformation is an isometry.	(10)
	QUESTION 3	
a)	Find the least squares parabola $y=f(x)=Ax^2+Bx+C$ for the given four points (-3, 3), (0, 1), (2, 1) and (4, 3).	(10)
b)	Find the curve fit $y=\frac{1}{(Ax+B)}$ using change of variables $X=x, Y=\frac{1}{y}$, linearizing the following data points.	(10)
	QUESTION 4	
a)	Establish the Pade approximation $\exp(x)\approx R_{2,2}(x)=\frac{12+6x+x^2}{12-6x+x^2}$.	(10)
b)	Determine Lagrange-Chebyshev polynomial for $N=7$ and $f(x)=\exp(x)$.	(10)

Data points for Question 3 b):

x_k	-1	0	1	2	3
y_k	6.62	3.94	2.17	1.35	0.89

SECTION II

	QUESTION 5	
a)	Show that the control point form for k = 2, 3 satisfies an affine invariance property.	(10)
b)	Prove the De-Casteljau Algorithm using symbolic Bernstein Bezier form of degree 'n'.	(10)

PTO

	QUESTION 6	
a)	Discuss the subdivision algorithm to compute the following Bernstein Bezier curve $\underline{P}(\theta) = \sum_{i=0}^{n} B_i^n(\theta)\, \underline{b}_i, \theta \in [0,1]$; for $\alpha, \beta \in [0,1]$. Also write the relation that defines the new control points of the segment of B. B. curve corresponding to $\alpha \le \theta \le \beta$.	(10)
b)	Given a cubic polynomial $\underline{P}(\theta) = \underline{a}_0 + \underline{a}_1\theta + \underline{a}_2\theta^2 + \underline{a}_3\theta^3$, $0 \le \theta \le 1$, derive the Cubic Hermite form using the following conditions $$\underline{P}(0) = \underline{b}_0,\ \underline{P}(1) = \underline{b}_3,\ \underline{P}'(0) = k(\underline{b}_1 - \underline{b}_0),\ \underline{P}'(1) = k(\underline{b}_3 - \underline{b}_2).$$	(10)
	QUESTION 7	
a)	Discuss the Variation Diminishing property of the cubic Bernstein Bezier form.	(10)
b)	Find the natural cubic spline such that $S(0)=0$, $S(1)=0.5$ and $S(2)= 2.0$ with boundary conditions $S''(0) = 0$ and $S''(2) = 0$.	(10)
	QUESTION 8	
a)	If $f \in C^4[a,b]$ and S is the cubic Hermite interpolation spline defined on $[a,b]$ with $a = t_0 < t_1 < t_2 < \cdots < t_i < \cdots < t_{n-1} < t_n = b$. Then show that $$\lvert S(t) - f(t) \rvert \le \frac{h_i}{4} Max\left\{\lvert d_i - f_i^{(1)}\rvert, \lvert d_{i+1} - f_{i+1}^{(1)}\rvert\right\} + \frac{h_i^4}{384}\lVert f^{(4)}\rVert,\ t \in]t_i, t_{i+1}[$$ where, $S^{(1)}(t_i) = d_i$, $f^{(1)}(t_i) = f_i^{(1)}$, $i = 0,1,2,\ldots,n$ and $\lVert . \rVert$ denotes the uniform norm defined on $[a,b]$.	(10)
b)	Determine the basis function $N_0^4(t)$ of uniform B-spline of degree 3, using the basis function $N_0^3(t)$ of uniform B-spline of degree 2.	(10)
	QUESTION 9	
a)	Let S be a spline of degree 'n' with knots at the points $x_0 < x_1 < x_2 < \cdots < x_{k-1} < x_k$. Then show that S has a representation of the form $$S(x) = \sum_{i=0}^{n} a_i x^i + \sum_{i=0}^{k} c_i (x - x_i)_+^n$$	(10)
b)	Verify whether the following function is a spline or not $$S(x) = \begin{cases} x^3 + 2x + 1, & x \in (-\infty, -1) \\ -3x^2 - x, & x \in [-1, 1) \\ 2x^3 - 9x^2 + 5x - 2, & x \in [1, 2) \\ x^3 - 3x^2 - 7x + 6, & x \in [2, \infty) \end{cases}$$	(10)

UNIVERSITY OF THE PUNJAB

Part-II A/2015
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: (IV-VI) (opt.iv) [Rings & Modules]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section I

1. (a) Let R[x] be the ring of polynomials of a ring R and suppose

$$f(x) = a_0 + a_1 x + a_2 x^2 + \dots\dots + a_m x^m \quad \forall a_i \in R,\ a_i = 0\ \forall\, i > m$$

$$g(x) = b_0 + b_1 x + b_2 x^2 + \dots\dots + b_n x^n \quad, \forall b_i \in R \quad, b_j = 0 \ \forall\, j > n$$

be two non zero polynomials of degree m and n respectively then if $f(x)g(x) \neq 0$, deg $(f(x)g(x)) \leq m+n$.

(b) Show that $Z[i] = \{a+ib: a,b \in Z\}$, the ring of Gaussian integers is an Euclidean domain.

10+10

2. (a). Let R be an integral domain. Show that the relation "is an associate of" denote by ~ is an equivalence relation on R. Further, that an equivalence class under ~ containing an element a has the form $\{au: u \in U\}$, where U is the set of units of R.

(b). For an integral domain R and $p \in P^* = R - \{0\}$, prove that p is prime iff R / pR is an integral domain.

10+10

3. (a). Let R be a Euclidean domain (or ring). Then any two elements a and b in R have a greatest common divisor d. Moreover $d = \lambda a + \mu b$ for some $\lambda, \mu \in R$.

(b) Let R be an I.D such that R[x] is a Principal Ideal domain (P.I.D). Then R is a field.

10+10

4. (a) Define a splitting field. Show that the field of complex numbers C is a splitting field for the polynomial x^2+1 over R and C is not a splitting field for x^2+1 over Q.

(b) Let K be an extension of a field F and $a \in K$ be algebraic of degree n over F. Show that $F(a) = \{\beta_0 + \beta_1 a + \dots\dots\dots\dots + \beta_{n-1} a^{n-1} : \beta_1, \beta_2, \dots, \beta_{n-1} \in F\}$.

10+10

5. (a). Let K be an extension field of a field F and $a_1, a_2, \dots, a_n$ be n elements in K, algebraic over F. Show that $F(a_1, a_2, \dots, a_n)$ is a finite extension of F and consequently an algebraic extension of F.

(b). Let F be a field and p(x) is a polynomial in F[x] of degree $n \geq 1$ and is irreducible over F. Then there exists an extension E of F such that $[E, F] = n$ in which p(x) has a root.

10 +10

Section II

6. (a) Let $f: M \to N$ be a homomorphism from M onto N. Then f is an isomorphism if and only if Ker (f) = {0}.

(b) Give an example of a finitely generating module, which has submodule which is not finitely generated.

10+10

7. (a) Define cyclic module. Show that any irreducible R-module, where R is a ring with identity, is a cyclic module.

(b). Define torsion free module. Prove that a vector space V over a field K, considered as a K-module, is torsion free.

10+10

8. (a) Let M be a module over an Integral Domain R and T denotes the set of torsion elements of M. Then T is submodule of M and M/T is torsion free.

10+10

(b) Prove that the Extended Homomorphism is freely generated module and is unique.

9. Let R be a Principal Ideal Domain and F be a free R-module of finite rank s. Prove that every submodule of F is free of rank $\leq s$.

20

UNIVERSITY OF THE PUNJAB

Part-II A/2015
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: IV-VI (opt.iii) [Group Theory] **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions by selecting at least TWO from each section.

SECTION I

Q1.	a)	Let G be direct product of groups A and B i.e. $G = A \otimes B$ and $Z(G)$, $Z(A)$ and $Z(B)$ be the centers of G, A and B, respectively, then show that $Z(G) = Z(A) \otimes Z(B)$.	10
	b)	State and prove Sylow's first Theorem.	10
Q2	a)	Prove that a group of order 15 is always cyclic.	10
	b)	Define normal product of two groups. Determine the normal product of cyclic group of order 4(C_4) by cyclic group of order 2(C_2).	10
Q3	a)	Define normal product of two groups. Determine the normal product of cyclic group of order 4(C_4) by cyclic group of order 3(C_3).	10
	b)	Prove that a p-sub group of a finite group G is contained in some Sylow p-sub group of G.	10
Q4	a)	Let $G=Gl_2(R)$ be the group of all invertible matrices with real entries and $X=R^2$. Let $\bullet: G \times X \to X$ be defined as $A \bullet V = AV$ for $A \in G$, $V \in R^2$. Then show that $\bullet$ is a group action. Also find the orbits of X.	10
	b)	Define group action, G-set, orbit and stabilizer with examples. Also show that stabilizer G_x is a subgroup of G.	10

SECTION II

Q5	a)	State and prove Schreier's refinement Theorem.	10
	b)	Show that lower central series is central series and construct the lower central series for $\langle a,b : a^8 = 1 = b^2, bab = a^{-1} \rangle$	10
Q6	a)	Discuss the solvability of S_n for all n by using derived series.	10
	b)	Prove that group of order 24 is solvable.	10
Q7	a)	Determine Frattini subgroup and construct lower and upper central series for $\langle a,b : a^2 = b^2, a^4 = 1, bab = a^{-1} \rangle$.	20
	b)	Let $G = G_0 \rhd G_1 \rhd \ldots \rhd G_k = E$ be a central series for G. Then Show that *i)* $G_i \supseteq \gamma_i(G)$, $0 \le i \le k$ *ii)* $G_{k-i} \supseteq \xi_i(G)$, $0 \le i \le k$	
	c)	Where $\gamma_i(G)$ and $\xi_i(G)$ are terms of lower central series and upper central series of G, respectively.	
Q8	a)	Define partial complement of a subgroup and prove that a normal subgroup H of G is contained in Frattini subgroup of G if and only if H has no partial complement in G.	10
	b)	Let $\Phi(G)$ and G' denote the Frattini and derived subgroup of and G. Then show that a finite group G is nilpotent if and only if $G' \subseteq \Phi(G)$.	10
Q9	a)	Define special linear group and projective linear groups. Also prove that general linear group is not simple.	10
	b)	Define general linear group $GL(n,q)$, special linear group $SL(n,q)$ and representation of group and prove that $\lvert SL(n,q) \rvert = (q^n - 1)(q^n - q) \ldots\ldots (q^n - q^{n-1}) / (q-1)$	10

UNIVERSITY OF THE PUNJAB

Part-II A/2015
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: IV-VI (opt.viii) [Special Theory of Relativity and Analytical Dynamics] **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions in all selecting at least TWO questions from each section.

SECTION I

Q1(a) Derive the Lorentz transformations for 1-dimension and hence obtain a formula for the general, 3-dimensional Lorentz transformations for any velocity v. [10]

(b) Two events occur simultaneously at points (21, 2, 1) and (1, 0, 0) of a frame O. Determine the time interval between them in a frame O′ moving with speed 0.6c relative to O along the direction of their common x-axis. [10]

Q2(a) Prove Lorentz transformations for motion in arbitrary direction and also deduce these transformations for an observer in uniform circular motion. [10]

(b) A spaceship sends out a scout ship with a velocity $\left(\frac{c}{2},\frac{c}{3},\frac{c}{4}\right)$. The scout ship spies an enemy ship approaching with velocity $\left(\frac{c}{4},\frac{c}{3},\frac{c}{2}\right)$. What velocity should the scout ship tell its mother ship that the enemy ship is approaching at? [10]

Q3(a) Explain, with reference to the null cone, what is meant by timelike, null and spacelike vectors. [10]

(b) A π^+ meson has rest mass in energy units, $140MeV/c^2$. Pions of total energy $200MeV/c^2$ are produced in a high energy accelerator. Calculate the distance they travel if only αN pions survive after a time t, N_o is the number of pions at time t, and $\alpha < 1$. The half-life of π^+ mesons has been found experimentally as 1.8×10^{-8} seconds. [10]

Q4(a) Define electric and magnetic field intensities and show that these quantities satisfy wave equation with the wave speed being the speed of light, i.e. $c = \frac{1}{\sqrt{\mu_o\varepsilon_o}}$ (in the absence of charge density). [10]

(b) Using the Maxwell field tensor, show that

(i) $F^{\alpha\beta} = -F^{\beta\alpha}$ (ii) $F^{\alpha\beta}F_{\alpha\beta} = -2\mu_o\left(\varepsilon_o E\bullet E - \frac{B\bullet B}{\mu_o}\right)$ (iii) $^*F^{\alpha\beta}F_{\alpha\beta} = 4\left(\frac{E\bullet B}{c}\right)$. [10]

Q5(a) Derive a formula for the minimum kinetic energy required to produce a particle of mass M. [10]

(b) A rod of length l makes angles $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with the x-axis and y-axis respectively. What does the length of the rod appear to be to an observer moving with velocity $\left(\frac{c}{\sqrt{3}},\frac{c}{\sqrt{6}},\frac{c}{\sqrt{3}}\right)$. [10]

SECTION II

Q6(a) Derive the expression of kinetic energy of a particle of mass m showing that kinetic energy is homogeneous function of generalized velocities. [10]

(b) Using Poisson bracket, show that the transformation $q = \sqrt{2P}\sin Q$, $p = \sqrt{2P}\cos Q$, is canonical. [10]

Q7(a) State and prove Hamilton's principle. [10]

(b) A particle P moves under a central force f(r) towards a fixed point O, where r is the distance of P from O. Using $\dot{r}$, $\dot{A} = \frac{1}{2}r^2\dot{\theta}$ as quasi-velocities, find the 3-index symbol. [10]

Q8(a) Prove Hamilton's principle for non-holonomic systems. [10]

(b) If F(q, p, t) and G(q, p, t) are two integrals of motion, show that the Poinson bracket [F, G] is also and integral of motion. [10]

Q9(a) State and prove Poisson theorem. [10]

(b) What are generalized coordinates? Generalized velocities? Give example in each case. State and prove D'Alembert principle for a dynamical system. [10]

UNIVERSITY OF THE PUNJAB

Part-II A/2015
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt.vii) [Quantum Mechanics]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions in all, selecting at least TWO questions from each section.

SECTION I

1. (a) Define Hermitian operator and show that eigen values of hermitian operators are always real. (10 marks)

   (b) Show that the de Broglie wavelength of an electron of kinetic energy $E(ev)$ is $\lambda_e = \frac{12.3\times10^{-8}}{\sqrt{E}}$ cm and that of proton is $\lambda_p = \frac{0.29\times10^{-8}}{\sqrt{E}}$ cm. (10 marks)

2. (a) Write a note on wave-particle duality. (10 marks)

   (b) Show that in the nth eigen state of the harmonic oscillator, the average kinetic energy $\langle T\rangle$ is equal to the average potential energy $\langle V\rangle$. (10 marks)

3. (a) Consider a particle incident on a potential step as shown in the Figure 1. Obtain the solution of Schrödinger equation for (i) $V_2 > E > V_1$ (ii) $E > V_2$. (10 marks)

   (b) Derive the operator equation $\frac{d}{dx}x^n = nx^{n-1} + x^n\frac{d}{dx}$ and show that $\left[\frac{d}{dx}, x^n\right] = nx^{n-1}$. (10 marks)

4. (a) Show that the continuous set of eigen functions $\{\delta(x-x')\}$ obeys the "orthonormality" condition
$$\int_{-\infty}^{\infty}\delta(x-x')\delta(x-x'')dx = \delta(x'-x'')$$
(10 marks)

   (b) An electron beam is incident on a barrier of heigh $10eV$. At $E = 10eV$, $T = 3.37\times10^{-3}$. What is the width of the barrier? (10 marks)

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(P.T.O.)

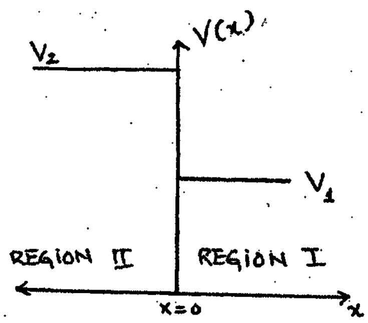

Figure 1:

SECTION II

5. (a) What is $[\hat{\phi}, \hat{L}_z]$? and show that $[\sin\phi, \hat{L}_z] = \iota\hbar\cos\phi$. (10 marks)

   (b) Discuss the scattering of a particle from a time independent potential and obtain formulas for scattering amplitude and differential cross section. Also consider the case of Born approximation. (10 marks)

6. (a) Represent $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$ and $\hat{L}^2$ in spherical polar coordinates? (10 marks)

   (b) What is Schrödinger equation in momentum representation for a free particle moving in one dimension? What are the eigen functions $b(k)$ of this equation? (10 marks)

7. (a) Find the wave functions of 2 systems of identical, noninteracting particles: the first consists of two bosons, and the second of two spin 1/2 fermions. (10 marks)

   (b) Show that $\hat{L}_x$ is Hermitian. (10 marks)

8. Define perturbation theory. Obtain first order correction in energy for time-independent generate perturbation theory. (20 marks)

9. (a) Show that the expression

$$\langle J^2 \rangle = \hbar^2 j(j+1)$$

   is implied by the two assumptions:

   (i) The only possible values that the components of angular momentum can have on any axis are $\hbar(-j,\ ,\dots,+j)$.

   (ii) All these components are equally probable.

   (10 marks)

   (b) Compute the expression for ionization potential for hydrogen, helium and Lithium atoms. (10 marks)

UNIVERSITY OF THE PUNJAB

Part-II A/2015
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt.i) [Mathematical Statistics

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section. All questions carry equal marks.

Q.1	(a)	Find mean deviation of the Normal distribution.	(10)
	(b)	If the moment generating function of X is $M(t)=e^{166t+200t^2}$, Find (i) P(170<X<200), ii) P(148<X<172) Where P stands for probability.	(10)
Q.2	(a)	Bag-I contains 6 white and 5 black balls while Bag-II contains 4 white and 3 black balls. One ball is drawn from the Bag-I and replaced unseen in the Bag-II. What is the probability that a ball now drawn from the Bag-II is (i) black (ii) white.	(10)
	(b)	A continuous r.v has p.d.f $f(x)=\begin{cases} k(2-x)(x-5), & 2\le x\le 5 \\ 0, & elsewhere \end{cases}$ Find Harmonic mean? Calculate the distribution function, and estimate the probabilities $P(\lvert X-\mu\rvert\le 3\sigma)$ and $P(\lvert X-\mu\rvert\ge 3\sigma)$.	(10)
Q.3	(a)	There are three coins identical in appearance, one of which is ideal and the other two biased with probabilities 1/3 and 2/3 respectively for a head. One coin is taken at random and tossed twice. If a head appears both the times, what is the probability that the ideal coin was chosen.	(10)
	(b)	State and prove Baye's theorem.	(10)
Q.4	(a)	Suppose that airplane engines operate independently and fail with probability equal to 0.4. Assuming that a plane makes a safe flight if at least one half of its engine run, determine whether a 4 engine plane is safer than the 2 engine plane justify your answer.	(10)
	(b)	Find first three moments about mean for a Binomial distribution.	(10)
		SECTION-II	
Q.5	(a)	5 If λ, μ are the deviations of the random variables X and Y from their respective means. Then show that $r=1-\frac{1}{2N}\sum\left(\frac{\lambda_i}{\sigma_x}-\frac{\mu_i}{\sigma_y}\right)^2$, also $r=-1+\frac{1}{2N}\sum\left(\frac{\lambda_i}{\sigma_x}+\frac{\mu_i}{\sigma_y}\right)^2$ And deduce that $-1\le r\le 1$	(10)
	(b)	If X_r and X_s are the rth' and sth' random variable of random sample of size n drawn from the finite population $\{C_1,C_2,.....C_N\}$. Then $Cov(X_r,X_s)=\frac{\sigma^2}{N-1}$	(10)
Q.6	(a)	Verify that the Chi-square(χ^2) distribution has the following density	(10)

function $f(\chi^2)=\frac{1}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})}(\chi^2)^{\frac{n}{2}-1}e^{-\frac{\chi^2}{2}}, \quad 0<\chi^2<\infty$

(P.T.O.)

(b) Four pennies were tossed 1000 times and the number of heads were observed as given
Below: **(10)**

No. of Heads	0	1	2	3	4
Frequencies	38	149	352	292	169

Test the Hypothesis whether a Binomial distribution gives the satisfactory fit to the data.

Q.7 (a) If the joint probability density of X_1 and X_2 is given by **(10)**

$$f(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)} & \text{for } x_1 > 0 \text{ and } x_2 > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability density of $Y = \dfrac{X_1}{X_1 + X_2}$.

(b) Prove that all even moments about origin for student's t-distribution is **(10)**

$$\mu'_{2r} = E(t^{2r}) = \frac{n^r \Gamma(n+\frac{1}{2})\Gamma(\frac{n}{2}-r)}{\Gamma(\frac{1}{2})\Gamma(\frac{n}{2})}$$

Q.8 (a) Show that coefficient of correlation is independent of change of origin and scale. **(10)**

(b) Ten individuals are chosen at random from a normal population and the heights are found to be in inches 63, 63, 66, 67, 68, 70, 70, 71 and 71. In the light of these data, discuss the suggestion that mean height in the population is 66 inches. **(10)**

Q.9 (a) If F *follows* $F(\upsilon_1, \upsilon_2)$ then $y = (1 + \frac{\upsilon_1}{\upsilon_2} F)^{-1}$ *follows* $\beta(\frac{\upsilon_1}{2}, \frac{\upsilon_2}{2})$ **(10)**

Where υ_1, υ_2 are the degrees of freedom for F-distribution.

(b) Find the moment generating function for Gamma distribution and use it to find mean and variance of the distribution. **(10)**