UNIVERSITY OF THE PUNJAB

Part-II A/2016
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics | **TIME ALLOWED: 3 hrs.**
PAPER: I (Advanced Analysis) | **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

SECTION-I

Q.1 (a) Prove that the union of two denumerable sets is denumerable and deduce that $\aleph_0 + \aleph_0 = \aleph_0$. (10)

(b) Suppose that A, B, C, D are any Sets. Show that (10)

(i) $(A \times B) \times C \approx A \times B \times C \approx A \times (B \times C)$

(ii) If $A \approx B$ and $C \approx D$ then show that $A \times C \approx B \times D$.

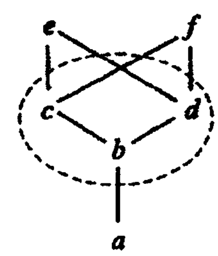
Q.2 (a) (i) Prove that the set $\mathbb{Q}$ of rational numbers is denumerable. (10)

(ii) Prove that the set $\mathbb{Q}'$ of all irrational numbers is non-denumerable.

(b) Consider the set $\mathbb{Q}$ of rational numbers with the usual order and consider the subset $D = \{x \in \mathbb{Q} : 8 < x^3 < 15\}$. Determine D is bounded above or below? (10)

Do $Sup(D)$ and $Inf(D)$ exists?

Q.3 (a) Let $X = \{a, b, c, d, e, f\}$ be ordered as shown in the figure and $A = \{b, c, d\}$ be a subset of X. Find minimal and maximal element of X. Also find $Sup(A)$ and $Inf(A)$ in X. (10)



(b) Let A be a Well ordered set and let B be a subset of A, and let $f : A \to B$ be a similarity mapping. Then show that for every $a \in A$, $a \le f(a)$. (10)

Q.4 (a) Define Zorn's Lemma and show that Axiom of choice is equivalent to Zermelo's postulate. (10)

(b) Let S be a subset of a well-ordered set A with the following property: (10)

If $a \le b$ and $a \in S$, then $a \in S$. Then $S = A$ or S is an initial segment of A.

SECTION-II

Q.5 (a) (i) Let G be the family of subsets of X. Then there is a smallest σ – algebra containing G. (5+5)

(ii) Given any set A and $\varepsilon > 0$ there is an open set O with $A \subseteq O$ then show that $m^*(O) < m^*(A) + \varepsilon$.

(b) Prove that the Lebesgue outer measure of any interval is its length. (10)

Q.6 (a) Let E be a subset of R with $m^*(E) < \infty$. The set E is measurable if and only if given $\varepsilon > 0$, (10)

PTO

there is a finite union H of finite open intervals such that $m^*(E\Delta H) < \varepsilon$.

(b) Demonstrate the existence of a non-Lebesgue measurable set P contained in $A = [0,1)$. **(10)**

Q.7 **(a)** Prove that a continuous function defined on a measurable set is measurable. **(10)**

(b) Let f and g be two measurable functions defined on the same set D. Then prove that the following sets are measurable **(10)**

(i) $\{x : f(x) > g(x)\}$ (ii) $\{x : f(x) < g(x)\}$

(iii) $\{x : f(x) \le g(x)\}$ (iv) $\{x : f(x) = g(x)\}$.

Q.8 **(a)** (i) Let ϕ and ψ be simple functions on D which vanish outside a set E of finite measure and a, b be real numbers. Then prove that $\int a\phi + b\psi = a\int \phi + b\int \psi$ **(10)**

(ii) Let $\phi(x) = \begin{cases} 2, \text{ if } x \in \mathbb{Q}' \cap (-5,5) \\ 3, \text{ if } x \in \mathbb{Q} \cap (-5,5) \end{cases}$ then find the Lebesgue integration $\int_{-5}^{5} \phi\, dm$

(b) Let f be measurable function defined on a measurable set E, then **(10)**

(i) f is integrable on E if and only if $|f|$ is integrable on E

(ii) If f is integrable, then $\left|\int f\right| \le \int |f|$.

Q.9 **(a)** State and prove Bounded Convergence Theorem. **(10)**

(b) Riesz-Fischer: Prove that L^p spaces are complete. **(10)**

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Part-II A/2016
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: II (Methods of Mathematical Physics) **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all, selecting at least TWO questions from each section.

Q. No		Marks
	Section-I	
1(a)	Use the power series method to obtain the general solution for the differential equation $y'' + y = 0$.	10
1(b)	Use the method of Frobenius to find the power series solution of Bessel differential equation of the form $y(x) = c_1 J_\nu(x) + c_2 J_{-\nu}(x)$.	10
2(a)	Prove the identity $$x\frac{d}{dx}P_{k-1}(x) - \frac{d}{dx}P_{k-2}(x) = (k-1)P_{k-1}(x).$$	10
2(b)	Define Hypergeometric functions. Prove that $$F_{21}(\alpha,\beta,\gamma;z) = \frac{\Gamma(\gamma)}{\Gamma(\gamma-\alpha)\Gamma(\alpha)}\int_0^1 t^{\alpha-1}(1-t)^{\gamma-\alpha-1}(1-tz)^{-\beta}dt.$$	10
3(a)	If $u(x)$ and $v(x)$ are periodic solutions of the Mathieu equation with period π having distinct eigenvalues, then prove that the solutions are orthogonal.	10
3(b)	Find the eigen values and eigen solutions for the Sturm-Liouville problem $(xu')' + \lambda\frac{u}{x} = 0, \quad 0 < x < b,$ $u(0) = 0, \quad u(b) = 0.$	10
4(a)	Use Lagrange's method to obtain a complete integral for the linear first order partial differential equation $$x^2 z_x + y^2 z_y = (x+y)z.$$	10
4(b)	Discuss if the following PDE is hyperbolic, parabolic or elliptic $(1-x^2)z_{xx} - 2xyz_{xy} + (1-y^2)z_{yy} + 2xz_x + 3xyz_y + 3z = 0.$	10
5(a)	Find the general solution of the inhomogeneous linear second order PDE $(D^2 - 2DD' - 15D'^2)z = 12xy.$	10
5(b)	Interpret and solve the following problem $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}, \quad 0 < x < a, \quad t > 0,$ $u(0,t) = T_0, \quad u(a,t) = T_1, \quad t > 0,$ $u(x,0) = f(x), \quad 0 < x < a.$	10
	Section-II	
6(a)	Find the Fourier transform of the Gaussian function $g(x) = Ne^{-ax^2}$, where N and α are constants and $\alpha > 0$.	10

PTO

6(b)	Use Fourier transform method to obtain a unique solution of the initial/boundary value problem $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad -\infty < x < \infty, \quad t \geq 0,$ $u(x,0) = e^{-ax^2}, \quad u(x) \text{ and } u_x(x) \to 0 \text{ as } x \to \pm\infty.$	10
7(a)	Solve the problem using the Laplace transform method $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} - g, \quad 0 < x < \infty, \quad t \geq 0,$ $u(x,0) = u_t(x,0) = 0, \quad 0 < x < \infty,$ $u(0,t) = 0, \quad \lim_{x \to \infty} u_x(x,t) = 0.$	10
7(b)	Define convolution of two functions. State and prove convolution theorem of Laplace transform.	10
8(a)	Find the Green's function associated with the boundary value problem $\frac{d}{dx}\left\{(1-x^2)\frac{du}{dx}\right\} - \frac{h^2}{1-x^2}u + \lambda r(x)u = 0,$ with $u(-1)$ and $u(1)$ both being finite.	10
8(b)	Show that a solid of revolution for a given surface area having maximum volume, is a sphere.	10
9(a)	State and prove fundamental theorem of calculus of variation.	10
9(b)	Find the extremal curve $y = y(x)$ of the functional $I[y(x)] = \int_0^{\pi/2} [y''^2 - y^2 + x^2]dx,$ subject to the end point conditions $y(0) = 1, \quad y(\pi/2) = 0, \quad y'(0) = 0, \quad y'(\pi/2) = 1.$	10

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Part-II **A/2016**
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: III (Numerical Analysis) **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions, selecting at least TWO questions from each section.

Section 1

Q1. Use R-K method of fourth order. Solve for $y(0.1), y(0.2), y(0.3)$ and $y(0.4)$, given that $\frac{dy}{dx} = -2xy,\ y(0) = 1.$ (20)

Q2. Solve $\frac{dy}{dx} = x^2(1+y),\ y(1) = 1,\ h = 0.1$ to find $y(1.5)$ by Adams-Bashforth formula. (20)

Q3.

a) Derive the Secant method. Calculate the rate of convergence of secant method. Use Secant method to find to 4 decimal places the root of the equation $1 + cost - 4t = 0$.

b) Define absolute error and relative error. Regard 100 as approximate number 99.76145. Find the absolute error and relative error.

(16+04)

Q4.

a) Define algorithm. Write an algorithm to find a root of nonlinear equation with Newton Raphosn formula.

b) Calculate four iterations of Gauss Seidel method to approximate the solution of

$8x - 3y + 2z = 20, 6x + 3y + 12z = 35, 4x + 11y - z = 33$ (10+10)

Q5.

a) Define Eigen value and Eigen vector. Use power method to find the Eigen values and Eigen vectors of matrix A

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$

PTO

b) Define difference operators. Prove that

$$(E-a)^r y_n = a^{n+r}\Delta^r\left(\frac{y_n}{a^n}\right)$$

(10+10)

Section 2

Q6. Define interpolation. Derive Four points Lagrange's interpolation formula. Use an interpolation formula of appropriate degree to obtain an approximation to the value of function at $x = 1.16$ for the given data: **(20)**

x	1.0	1.2	1.3	1.4
$f(x)$	8.01	11.56	13.61	15.84

Q7. Drive four points Simpson's 3/8th rule of integration. Write an algorithm (Simpson's 3/8th Rule) to approximate the integral of function $f(x)$ over the interval $[a, b]$ using n subintervals. Use Simpson's 3/8th rule to evaluate $\int_{1.0}^{1.5}(e^x + 2x + 1)dx$ **(20)**

Q8. Derive Six points Newton's backward difference interpolation formula. Use Newton's backward difference interpolation to find an approximation to the value of function at $x = 1.86$ for the given data: **(20)**

x	1.0	1.1	1.2	1.3	1.4	1.5
$f(x)$	8.01	9.69	11.56	13.61	15.84	18.26

Q9. Solve

i) $y_{k+2} - 6y_{k+1} + 8y_k = 2.3^k$

ii) $(E^2 - 5E + 6)y_n = n^2 - 1$

iii) $y_{k+3} - y_{k+2} + 3y_{k+1} - y_k = 0$

(07+08+05)

UNIVERSITY OF THE PUNJAB

Part-II A/2016
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt.i) [Mathematical Statistics

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section. All questions carry equal marks.

			Marks
		SECTION-I	
Q.1	(a)	If A and B are any two events and $P(B) \neq 1$, then prove that $$P(A/\bar{B}) = \frac{P(A) - P(A \cap B)}{1 - P(B)}$$	(10)
	(b)	The probability that a doctor correctly diagnoses a particular illness is 0.7. Given that the doctor makes an incorrect diagnosis, the probability that the patient files a lawsuit is 0.9. What is the probability that the doctor makes an incorrect diagnosis and the patient sues?	(10)
Q.2	(a)	For Poisson distribution with parameter m, show that $$\mu_{r+1} = m[r\mu_{r-1} + \frac{d\mu_r}{dm}]$$	(10)
	(b)	Prove that if X and Y are independent Gamma variates, with parameters l and m respectively, then $\frac{X}{Y}$ is a $\beta_2(l, m)$ variate.	(10)
Q.3	(a)	A manufacturer of cotter pins knows that 5% of his product is defective. If he sells cotter pins in boxes of 100 and guarantees that not more than 4 pins will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality?	(10)
	(b)	Prove that for a Hyper-Geometric distribution, the following relation holds: $$P(X = x + 1) = \frac{(k - x)(n - x)}{(x + 1)(N - k - n + x + 1)} P(X = x)$$	(10)
Q.4	(a)	Prove that the mean deviation of the normal distribution is approximately $\frac{4}{5}$ of its standard deviation.	(10)
	(b)	Two caplets are selected at random from a bottle containing three aspirin, two sedative, and four laxative caplets. If X and Y are respectively, the numbers of aspirin and sedative caplets included among the two caplets drawn from the bottle. Find the probabilities associated with all possible pair of values of X and Y. i) Find the joint probability distribution of X and Y. ii) Find the marginal distribution of X and Y.	(10)
		SECTION-II	
Q.5	(a)	Prove that $$R_{2.31} = \sqrt{\frac{r_{23}^2 + r_{12}^2 - 2r_{12}r_{13}r_{23}}{1 - r_{13}^2}}$$	(10)
	(b)	State and prove central limit theorem.	(10)

P.T.O.

Q.6	**(a)**	Given the joint density $f(x,y)=\begin{cases}2 & for\ \ 0<y<x<1\\ 0 & elsewhere\end{cases}$ Show that $\mu_{Y/x}=\frac{x}{2}$ and $\mu_{X/y}=\frac{1+y}{2}$.	(10)
	(b)	If X and Y are two uncorrelated variables and if $U=X+Y$ and $V+X-Y$, find the correlation between U and V in terms of σ_x and σ_y where σ_x and σ_y are standard deviations of X and Y.	(10)
Q.7	**(a)**	If the joint probability density of X_1 and X_2 is given by $f(x_1,x_2)=\begin{cases}e^{-(x_1+x_2)} & for\ x_1>0\ and\ x_2>0\\ 0 & elsewhere\end{cases}$ Find (i) the joint density of $Y_1=X_1+X_2$ and $Y_2=\frac{X_1}{X_1+X_2}$ (ii) the marginal density of Y_2.	(10)
	(b)	If X has the standard normal distribution, find the probability density of X^2.	(10)
Q.8	**(a)**	Derive χ^2-distribution as a limiting form of F-distribution.	(10)
	(b)	State and prove partitioning property of χ^2-distribution.	(10)
Q.9	**(a)**	Prove that student t-distribution approaches to the standard normal distribution as $n\rightarrow\infty$.	(10)
	(b)	If $T\sim t_v$, then show that $T^2\sim F(1,v)$.	(10)

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Part-II A/2016
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt.ii) [Computer Applications]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 50

NOTE: Attempt any FIVE questions by selecting atleast TWO from each section.

Section 1

1. a) [5 marks] Write a Fortran 90 expression corresponding to each mathematical expression
   i. $\frac{sin(2x)+tan^{-1}(x)}{|x^2-5|}$
   ii. $\frac{(\pi-3\alpha)e^x}{\sqrt{2(\gamma+\pi)}}$

   b) [5 marks] Find the value of c after execution of the following statements
   i) Integer :: c, a = 3, b = 4
   $c = 5*a+(b-a)/4+6-b*b*b/7$
   ii) Real :: a = 5.4, b = 0.9; Integer :: c
   $c = 2*b/1.5 - a*4.0/b - 3.2$

2. [10 marks] A list of 50 integers are arranged in serial order. Write a Fortran 90 program to print the serial number of those integers which are negative and count the number of negative integers.

3. [10 marks] A list of 20 integers are given. Write a Fortran 90 program to arrange the elements in a descending order.

4. [10 marks] Write a Fortran 90 program to multiply two matrices.

Section 2

5. [10 marks] Write a Fortran 90 program to implement Simpson's $\frac{3}{8}$ rule to evaluate

$$\int_2^3 \frac{1}{(x-1)^2}\,dx$$

6. [10 marks] Write a Fortran 90 program to use Lagrange interpolation formula to find the value of y at $x = 3$ using the table

x	2	4	6	8
y	1	7	11	13

7. [10 marks] Write a Fortran 90 program to implement Euler method to solve the following differential equation and calculate y(0.5),

$$y' = y^2 \qquad y(0) = 1 \qquad h = 0.25$$

8. [10 marks] Write a Fortran 90 program to use Jacobi method to solve

$$10x + y + z = 24$$
$$-x + 20y + z = 21$$
$$-x - 2y + 100z = 300$$

9. Write the Mathematica statements for the following.

   a) [2 marks] Write a command in Mathematica to find the sum of series $\sum_{j=1}^{\infty} \frac{j}{x^j}$

   b) [2 marks] Write a command in Mathematica to evaluate

$$\int_0^3 \int_0^{\sqrt{9-y^2}} (x^2+y^2)dx\ dy.$$

   c) [2 marks] Write a command in Mathematica to evaluate

$$\frac{\partial^2}{\partial y \partial x}(x\ e^y + sin(xy)).$$

   d) [2 marks] Write a command in Mathematica to find the real root of

$$x^3 - x^2 + 3 = 0$$

   e) [2 marks] Write a command in Mathematica to plot $\cos(\frac{x}{2})$ and $\sin^{-1}(2x)$ on the same graph for $0 < x < 10$.

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Part-II A/2016
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: (IV-VI) (opt.iv) [Rings & Modules]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section I

Q. 1. (a). Let R be a commutative ring with identity that has exactly three ideals 0, I and R, show that if $a \notin I$, then a is a unit R.

b) If R is an integral domain, then ring of polynomials $R[x]$ is an integral domain. 10+10

Q. 2. a) The polynomial $x - c$ is a factor of a polynomial $p(x) \in K[x]$ if and only if $x = c$ is a root of $p(x) = 0$. 10+10

(b) Prove or disprove that every Euclidean domain is Principal Ideal Domain.

Q. 3. (a) Prove or disprove that $\mathbb{Z}[i]$, the ring of Gaussian integers, is Euclidean domain. 10+10

(b) Let F be a field. Then show that the polynomial ring $F[x]$ is a principal ideal domain.

Q. 4. (a) Define extension of a field. Find the smallest extension of Q having a root of $x^3 - 2 \in Q[x]$.

(b) Show that $Q(\sqrt{2}, \sqrt{3}) = Q(\sqrt{2} + \sqrt{3})$, where Q is the field of rational numbers. 10+10

Q. 5. (a) Define an algebraic extension of a field and prove that every finite extension of a field is algebraic. 10+10

(b) Let K be a field, an element a of K is algebraic over F if and only if $[F(a): K]$ is finite.

Section II

Q. 6. (a) Let R be a ring with identity and M be irreducible R-module, then M is cyclic. 10+10

(b) Let M and N be two R-modules, $f: M \to N$ and $g: N \to M$ be two module homomorphisms such that $gof = I_M$ (identity map on M). Show that $N = Kerg \oplus Imf$.

Q. 7. (a) If $M_1, M_2, M_3, \ldots \ldots M_n$ be submodules of M, then the following statements are equivalent:

i. M is direct sum of $M_i, i = 1,2,3, \ldots \ldots, n$ 10+10

ii. Each $m \in M$ can be uniquely expressed as $m = m_1, m_2, \ldots \ldots, m_n$, where $m_i \in M_i, i = 1,2, \ldots \ldots, n$.

(b) Let M be an R- module, where R is a commutative ring with identity. Show that M is simple if and only if $M \cong R/_I$, I is a maximal ideal of R.

Q. 8. (a) Every FG – R-module is homomorphic image of a free module. 10+10

(b) Let M be a module over an integral domain R and let T denote the set of torsion elements of M. Show that T is a submodule of M the quotient module $M/_T$ is torsion free.

Q. 9. Let R be a principal ideal domain and F be a free R-module of finite ranks. Prove that every submodule of F is free of rank $\leq s$. 20

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Part-II A/2016
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: IV-VI (opt.v) [Number Theory] **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

SECTION-I **Marks**

Q.1 **(a)** Let a,b be two non-zero integers. Prove that $\gcd(a,b)lcm(a,b)=ab.$ (6)

(b) (i) State and prove division algorithm for integers. (8+6)
(ii)) Let $n>1$ be a composite integer then show that there exists a prime p such that $p|n$ and $p\le\sqrt{n}$

Q.2 **(a)** Let a,b and $m>0$ be any integers. Prove that the linear congruence $ax\equiv b(\text{mod } m)$ is solvable if and only if $d/b,\ d=\gcd(a,m)$. In this case; prove that the given congruence has d mutually incongruent solutions. (10)

(b) Solve the system of linear congruences. (10)

$$x\equiv 2\ (\text{mod}\,4)$$
$$x\equiv 3(\text{mod}\,7)$$
$$x\equiv 4(\text{mod}\,10)$$

Q.3 **(a)** Define ϕ and σ function. Show that both are multiplicative arithmetic functions. (2+8)

(b) (i) Let p be an odd prime. Prove that the congruence $x^2\equiv -1(\text{mod}\,p)$ has a solution if and only if $p\equiv 1(\text{mod}\,4)$. (5+5)
(ii) Using Euler's Theorem, show that $x^6\equiv 7\ (\text{mod}\,13)$ has no solution.

Q.4 **(a)** For each positive integer $n\ge 1$. Prove that $\phi(n)=\sum_{d/n}\phi(d).$ (10)

P.T.O.

	(b)	State and prove F. Merten's lemma.	(10)
Q.5	(a)	Prove that there exist primitive roots of p^k and $2p^k$, p being an odd prime.	(10)
	(b)	Discuss all solutions of the Diophantine equation $x^2+y^2=z^2$	(10)

Section II

Q.6	(a)	Let p be an odd prime. Prove that $\left(\frac{2}{p}\right)=\begin{cases}1 \text{ if } p\equiv \pm 1 \pmod 8\\ -1 \text{ if } p\equiv \pm 3 \pmod 8\end{cases}$	(10)
	(b)	Define quadratic residue and quadratic non-residues of a prime number. Prove that the product of two quadratic residues or quadratic non-residues of a prime number *p* is always a quadratic residue of *p*.	(10)
Q.7	(a)	Define a primitive polynomial. State and prove Gauss Lemma of primitive polynomials.	(10)
	(b)	Let p be an odd prime. Prove that the polynomials $\frac{x^p-1}{x-1}$ and $\frac{x^{p^2}-1}{x^p-1}$ are irreducible.	(7+3)
Q.8	(a)	Let $f(x)$ and $g(x)$ be non-zero polynomials over F, relatively prime over F. Prove that there exist polynomials $s_0(x)$ and $t_0(x)$ over F, such that $s_0(x)f(x)+t_0(x)g(x)=1$.	(10)
	(b)	Distinguish algebraic and transcendental numbers. Prove that there exist transcendental numbers.	(10)
Q.9	(a)	For $\alpha,\beta\in R[\theta]$, Show that $N\alpha\beta=N\alpha N\beta$, where N is the norm of the algebraic number	(10)
	(b)	Show that an element $\alpha\in R(\theta)$ is a unit if and only if its norm is ± 1.	(10)

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Part-II A/2016
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt.vi) [Fluid Mechanics]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions in all, selecting at least TWO questions from each section.

Section I

Q. No. 1 (a) Define vorticity vector and find its components along e_r, e_θ, e_z in cylindrical coordinate system. (10+10)
(b) A shaft 25 m in radius rotate inside a fixed bearing 25.5 m radius and 3 meter long. The gap between the shaft and the bearing is filled with oil having kinematics viscosity of 8×10^{-4} m^2 per second having specific gravity 0.81. Determine torque require to rotate the shaft with angular velocity 3 rad per second.

Q. No. 2 (a) Derive Euler's equation of motion. (10+10)
(b) Define path line stream line streak line and find differential equation for path line and streamline.

Q. No. 3 (a) Define stream tube and find the strength of vortex tube for an compressible flow. (10+10)
(b) An idealized velocity field is given by the formula $\mathbf{V} = 4tx\mathbf{i} + 2t^2y\mathbf{j} + 4xz\mathbf{k}$. Is this flow field steady or unsteady? Is it two- or three-dimensional? At the point $(x, y, z) = (-1, +1, 0)$, compute (i) the acceleration vector and (ii) any unit vector normal to the acceleration (iii) local acceleration vector

Q. No. 4 (a) State and prove Kelvin's minimum energy. (10+10)
(b) Does the velocity field $\frac{10y}{x^2+y^2}\hat{i} - \frac{10x}{x^2+y^2}\hat{j} + 0\hat{k}$, represent a possible flow? If so, find the pressure gradient ∇p, assuming a frictionless air flow under the influence of body force $(F_x, F_y, F_z) = (0, 0, -g)$. Use $\rho = 1{,}23 kg/m^3$.

Q. No. 5 The velocity potential for certain two-dimensional incompressible fluid flow is $\phi = A\tan^{-1}\left(\frac{y}{x}\right)$, (20)
find the velocity components, speed, stagnation points, stream function.

Section II

Q. No. 6 (a) Define uniform stream flows and draw flow net for it. (10+10)
(b) Define source and sink. Draw flow net for the two dimensional source.

Q. No. 7 (a) Write note on Karman's vortex-street. (10+10)
b) Discuss the pulsating flow of viscous fluid between two parallel plates when both the plates are rest and $\frac{\partial p}{\partial x} = P_x \cos\omega t$. Find the expression for the velocity field .

Q. No. 8 (a) Make a mathematical model for the steady Couttee flow between two parallel plates when one at rest and other is moving with constant velocity. (10+10)
(b) Discuss the steady steady Laminar flow of a viscous fluid through a pipe and find the expression for velocity field.

Q. No. 9 (a) Find the velocity field corresponding to the laminar flow between two parallel plates when at both plates are at rest and the pressure varies linearly in the direction of flow. Also calculate the maximum velocity. (10+10)
(b) Determine whether the following velocity field are steady, 2-dim , 3dim, and find the associated shear stress with p = 18, $\mu = 0.23$ and t= 8 , $\vec{V} = ax\hat{i} - by\hat{j} + (t - cz)\hat{k}$

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Part-II A/2016
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt.vii) [Quantum Mechanics]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions in all, selecting at least TWO questions from each section.

SECTION I

1. (a) Define unbound states. Show that the state function or wave function for a particle in one dimensional box are orthonormal. (10 marks)

   (b) What is the formula for wavelength λ emitted when the hydrogen atom decays from state n to state n'. Give your answer in terms of Rydberg constant, n, n' and Planck constant, only. (10 marks)

2. (a) Define Hermitian operator and show that eigen values of hermitian operators are always real. (10 marks)

   (b) If $\hat{H} = \frac{p^2}{2m} + V$, then find $[\hat{x}, [\hat{x}, \hat{H}]]$. (5 marks)

   (c) Prove that (5 marks)

$$[\hat{A}, \hat{B}^3] = 3\hat{B}^2[\hat{A}, \hat{B}].$$

3. (a) State Heisenberg's uncertainity principle. Show that the uncertainity relation forces us to reject the semiclassical Bohr model for hydrogen atom. (10 marks)

   (b) Show that eigen vectors belonging to two different eigen values are orthogonal. (10 marks)

4. (a) Consider the Hamiltonian for a one dimensional system of two particles of masses m_1 and m_2 subjected to a potential that depends only on the distance between the particles $x_1 - x_2$, (10 marks)

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V(x_1 - x_2),$$

   then write the Schrödinger equation using the new variables x and X, where

$$x = x_1 - x_2 \text{ (relative distance) and } X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}.$$

   (b) Write a note on wave-particle duality. (10 marks)

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SECTION II

5. (a) What is Schrödinger equation in momentum representation for a free particle moving in one dimension? What are the eigen functions $b(k)$ of this equation? (10 marks)

   (b) Prove that if an orthonormal discrete set of kets $\{|u_i\rangle, \; i = 1, \; 2, \; 3, ...\}$ constitutes a basis, then it follows that (10 marks)

$$\sum_i |u_i\rangle\langle u_j| = 1.$$

6. (a) For a system with an angular momentum $l = 1$, find the eigen values and eigen vectors of $L_x L_y + L_y L_x$. (10 marks)

   (b) Starting from the radial part of the Schrödinger equation, determine the scattering amplitude and total scattering cross-section by using partial wave analysis. (10 marks)

7. (a) A particle of mass μ and momentum $\mathbf{p} = \hbar\mathbf{k}$ is scattered by the potential $V(r) = r^{-1}e^{-r/a}V_0 a$, where V_0 and $a > 0$ are real constants. Using Born approximation, calculate the differential cross section. (10 marks)

   (b) Show that $\hat{L}_x$ is Hermitian. (10 marks)

8. Define perturbation theory. Obtain first order correction in energy and eigen functions for time-independent non-degenerate perturbation theory. (20 marks)

9. (a) Compute the matrix elements $\langle n|x^2|m\rangle$ and $\langle n|p^2|m\rangle$ for the one dimensional harmonic oscillator. (10 marks)

   (b) A particle with mass m_1 is scattered elastically by a particle of mass m_2 at rest in the Lab frame. Find the relation between the scattering angle of m_2 in the Lab frame and the scattering angle in the CM frame. Show that in the Lab frame, the particle m_2 will always recoil in the front half of the sphere. (10 marks)

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Part-II *A/2016*

Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics

TIME ALLOWED: 3 hrs.

PAPER: IV-VI (opt.viii) [Special Theory of Relativity and Analytical Dynamics]

MAX. MARKS: 100

NOTE: Attempt any FIVE questions in all, selecting at least TWO questions from each section.

SECTION I

Q1(a) Write a note on the Lorentz and Poincare groups. [10]

(b) A sees B moving with velocity (10c, 20c, 0) while B sees C moving with velocity (– 20c, 10c, 0). What velocity does C see A as moving at? [10]

Q2(a) Explain, in terms of null cone structure, why a massive particle can never go at the speed of light and a mass less particle can never go less than that the speed of light. [10]

(b) Determine the formula for minimum kinetic energy required to produce a particle of mass M. [10]

Q3(a) Discuss the Doppler Effect in light as well as in sound. [10]

(b) A π^+ meson has rest mass in energy units, $180MeV/c^2$. Pions of total energy $240MeV/c^2$ are produced in a high energy accelerator. Calculate the distance they travel if only αN pions survive after a time t, N_o is the number of pions at time t, and $\alpha < 1$. The half-life of π^+ mesons has been found experimentally as 1.9×10^{-8} seconds. [10]

Q4(a) Derive formula for the gravitational red shift and the deflection of light. Under what conditions are the formula valid? [10]

(b) A spaceship sends out a scout ship with a velocity $\left(\frac{c}{4},\frac{c}{5},\frac{c}{6}\right)$. The scout ship spies an enemy ship approaching with velocity $\left(\frac{c}{6},\frac{c}{5},\frac{c}{4}\right)$. What velocity should the scout ship tell its mother ship that the enemy ship is approaching at? [10]

Q5(a) Work out the acceleration 4-vector for a particle in uniform circular motion. [10]

(b) Using the Maxwell field tensor, show that [10]

(i) $F^{\alpha\beta} = -F^{\beta\alpha}$ (ii) $F^{\alpha\beta}F_{\alpha\beta} = -2\mu_c\left(\varepsilon_0 E\bullet E - \frac{B\bullet B}{\mu_0}\right)$ (iii) $^*F^{\alpha\beta}F_{\alpha\beta} = 4\left(\frac{E\bullet B}{c}\right)$.

SECTION II

Q6(a) Derive the expression of kinetic energy of a particle of mass m showing that kinetic energy is homogeneous function of generalized velocities. [10]

(b) Find under what conditions, $Q = \frac{\alpha p}{x}$, $P = \beta x^2$, where α and β are constants, represents a canonical transformation. [10]

Q7(a) Derive Hamilton equations from the variational principle? [10]

(b) What are generalized coordinates? Generalized velocities? Give example in each case. State and prove D'Alembert principle for a dynamical system. [10]

Q8(a) Show that the Hamilton's principle $\delta\int_{t_0}^{t_1} L\,dt = 0$ also holds for the non-conservative system.

(b) Prove that $\{u,u\}[u,u] = -1$, where curly ans square brackets means Lagrange and Poisson's brackets respectively. [10+10]

Q9(a) State and prove Poisson theorem. [10]

(b) A particle of mass m moves in a force field of potential v. If $v = \frac{k\cos\theta}{r^2}$, write down the Hamilton-Jacobi equations. [10]

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Part-II A/2016
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt. ix) [Electromagnetic Theory]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions in all selecting at least TWO questions from each section.

SECTION I

1. (a) Discuss the propagation of electromagnetic wave in the region between parallel conducting plates. (10 marks)
   (b) Prove that a plane electromagnetic wave in free space has only transverse components of electric and magnetic fields. (10 marks)

2. (a) Find electric and magnetic field for the plane electromagnetic waves propagating in free space in the absence of sources. (10 marks)
   (b) Prove that the incident, reflected and transmitted waves are coplanar. (10 marks)

3. (a) Explain the generalized form of Faraday's law. (10 marks)
   (b) Find the relationship between magnetic susceptability, permeability and relative permeability. (10 marks)

4. (a) Verify that the flux of Poynting vector through any closed surface gives the energy flow through the volume enclosed by that surface. (10 marks)
   (b) Prove that the ratio of magnitudes of $\vec{E}$ and $\vec{H}$ is equal to intrinsic impedence of free space. (10 marks)

5. (a) Calculate the electric field due to a uniform spherical charge distribution at an external point. (5 marks)
   (b) An electric field $\vec{E} = E_0(\sin x\hat{i} + \cos y\hat{j})e^{-y}$ exists in free space. Find the volume charge density. (5 marks)
   (c) Calculate the magnetic field at a point on the axis of a circular loop. (10 marks)

SECTION II

6. (a) Prove that transverse electromagnetic waves cannot be transmitted inside a hollow conducting wave guide but can be transmitted in a co-axial line. (10 marks)
   (b) Discuss Poynting vector in conducting media. (10 marks)

7. (a) Find out the wave equations for the electromagnetic potentials V and $\vec{A}$. (10 marks)
   (b) Discuss the Lienard-Wiechert potentials for a moving charge. (10 marks)

8. (a) Discuss the retarded potential for a sinusoidal charge distribution. (10 marks)
   (b) If $f_1(x,t) = A_1\cos(kx - \omega t + \theta_1)$ and $f_2 = A_2\cos(kx - \omega t + \theta_2)$ are two sinusoidal waves then show that their sum is also a sinusoidal wave. (10 marks)

9. (a) Write down the laws of reflection and prove that the incident, reflected and transmitted waves are coplanar. (10 marks)
   (b) Define a wave and all the related terminologies like amplitude, period, frequency, wavelength, phase constant etc. (10 marks)

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Part-II A/2016
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt. x) [Operations Research]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions selecting atleast TWO questions from each section.

SECTION-I **Marks**

Q.1 A company manufactures two types of products, A and B and sells them at a profit of Rs. 4 on type A and Rs. 5 on type B. Each product is processed on two machines, X and Y. Type A requires 2 minutes of processing time on X and 3 minutes of processing time on Y. And type B requires 2 minutes of processing time on X and 2 minutes of processing time on Y. The machine, X is available for not more than 5 hours and 30 minutes while machine Y is available for 8 hours.

(a) Formulate the problem as a Linear Programming Model. (10)

(b) Solve the problem using the Graphical Method. (10)

Q.2 (a) Write algorithm for Big-M method (M-technique). (8)

(b) By means of M-technique, show that the following problem has no feasible solution. (12)

Maximize: $Z = 2x_1 + 5x_2$
Subject to
$3x_1 + 2x_2 \geq 6$
$2x_1 + x_2 \leq 2$
$x_1, x_2 \geq 0$

Q.3 (a) Apply Simplex Method to solve the following problem (10)

Minimize: $Z = x_1 - 3x_2 + 2x_3$
Subject to
$3x_1 - x_2 + 3x_3 \leq 7$
$-2x_1 + 4x_2 \leq 12$
$-4x_1 + 3x_2 + 8x_3 \leq 10$
$x_1, x_2, x_3 \geq 0$

(b) Apply Dual Simplex Algorithm to solve (10)

Minimize: $Z = x_1 + x_2$
Subject to
$2x_1 + x_2 \geq 2$
$-x_1 - x_2 \geq 1$
$x_1, x_2 \geq 0$

Q.4 (a) Write algorithm for Least Cost Method. (8)

(b) Apply method of multiplier to solve the following model using initial basic feasible solution by Vogel's Approximation Method (VAM). (12)

				Supply
	1	2	6	7
	0	4	2	12
	3	1	5	11
Demand	10	10	10	

P.T.O.

Q.5 **(a)** In the following transportation problem, the total demand exceeds total supply. Suppose that the penalty cost per unit of unsatisfied demands is 5, 3 and 2 for destinations 1, 2 and 3 respectively. Solve the problem using method of multiplier with VAM as starting solution. **(12)**

				Supply
	5	1	7	10
	6	4	6	80
	3	2	5	15
Demand	75	20	50	

(b) Find all possible assignments using the Assignment Model. (8)

5	5	--	2
7	4	2	3
9	3	5	--
7	2	6	7

SECTION-II

Q.6 **(a)** Consider the following LP-Model **(8)**

Maximize: $Z = x_1 + 4x_2 + 7x_3 + 5x_4$

Subject to

$2x_1 + x_2 + 2x_3 + 4x_4 = 10$

$3x_1 - x_2 - 2x_3 + 6x_4 = 5$

$x_1, x_2, x_3, x_4 \geq 0$

Generate the simplex tableau associated with the basis $B = (P_3, P_4)$.

(b) Use the Revised Simplex Method to Solve **(12)**

Maximize: $Z = x_1 + x_2$

Subject to

$3x_1 + 2x_2 \leq 6$

$x_1 + 4x_2 \leq 4$

$x_1, x_2 \geq 0$

Q.7 Solve the following LPP using the Bounded Variable Algorithm. (20)

Maximize: $Z = 3x_1 + 5x_2 + 2x_3$

Subject to

$x_1 + x_2 + 2x_3 \leq 14$

$2x_1 + 4x_2 + 3x_3 \leq 43$

$0 \leq x_1 \leq 4,\ 7 \leq x_2 \leq 10,\ 0 \leq x_3 \leq 3$

Q.8 Solve the following problem by Mixed Integer al Algorithm. (20)

Maximize: $Z = 7x_1 + 9x_2$

Subject to

$-x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 35$

where x_1 is an integer and $x_2 \geq 0$.

Q.9 **(a)** Use Dynamic Programming to solve **(12)**

Minimize: $Z = u_1^2 + u_2^2 + u_3^2$

Subject to

$u_1 + u_2 + u_3 \geq 15$

$u_1, u_2, u_3 \geq 0$

(b) Write a brief note on (i) Dynamic Programming (ii) Parametric linear Programming (8)

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Part-II A/2016
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt.xi) [Theory of Approximation & Splines]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions, select at least TWO questions from each section.

Section1

Q.1

a) Determine the image of line $y = 2x + 3$ under the reflection in line passing through origin making an angle $\frac{\pi}{4}$ in anticlockwise direction with X-axis.

b) Prove that rotation is distance preserving transformation. (10+10)

Q.2

a) Show that Euclidean congruence is an equivalence relation.

b) If t_1 is an Euclidean transformation defined as:

$$t_1(\vec{x}) = A\vec{x}+\vec{a}, \quad \forall\, x \in R^2 \text{ and some } a \in R^2$$

t_2 is defined in R^2 as:

$$t_2(\vec{x}) = A^{-1}\vec{x} - A^{-1}\vec{a}$$

then prove that t_2 is also an Euclidean transformation and is inverse of t_1. (10+10)

Q.3

a) Show that composition of two reflections is a rotation. (10+10)

b) Find the least squares line $y = Ax + B$ for all data and calculate root mean square error.

x_i	-2	-1	0	1	2
y_i	1	2	3	3	4
$f(x_i)$	1.2	1.9	2.6	3.3	4.0

Q.4

a) Prove that translation is an isometry.

b) Find the power fit $y = Ax^M$ where $M = 1$, which is a line through the orgin, for the data $(1, 1.6), (2, 2.8), (3, 4.7), (4, 6.4)$ and $(5, 8.0)$. Also calculate $E_2(f)$.

(10+10)

Section2

Q.5

a) Prove that Bernstein Bezier rational quadratic form represents conic section.

b) Define Clamped conditions, Periodic conditions, 2nd derivative conditions, Not a knot conditions

(10+10)

Q.6 Define cubic spline. Derive cubic spline interpolation function with natural end conditions. (20)

Q.7

a) Calculate the control points corresponding to the subintervals $\left[0, \frac{1}{3}\right], \left[\frac{1}{3}, 1\right]$ for $\vec{P}(\theta) = \sum_{i=0}^{3} B_i^3(\theta)\vec{b_i}, \theta \in [0,1]$.

b) Derive Bernstein Bezier (B.B.) tensor product surface quadratic by quadratic patch. (10+10)

Q.8 Explain degree raising algorithm. Consider the following B. B. cubic form (20)

$$\vec{P}(\theta) = \sum_{i=0}^{3} B_i^3(\theta)\vec{b_i}, \theta \in [0,1].$$

Determine the new control points for B. B. quartic form by degree raising algorithm.

Q.9 Define uniform B-spline. Let the knot vector be $t_0 = 0, t_1 = 1, t_2 = 2, \cdots\ t_m = m$. (20)
Determine the basis functions for the uniform B-spline of degree 2 on this knot vector.

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Part-II A/2016
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: IV-VI (opt. xiii) [Solid Mechanics] **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions in all, selecting at least TWO questions from each section.

SECTION I

Q1(a) Discuss the components of strain in an elastic body. [10]

(b) Show that greatest shearing strain for the strain components ϵ_x, ϵ_y, γ_{xy} is given by

$$\gamma_{\theta \max} = \epsilon_1 - \epsilon_2$$

where ϵ_1 and ϵ_2 are the algebraically greatest and least values of ϵ_θ as a function θ. [10]

Q2(a) State Saint-Venant's principle. Write in detail about its side effects. [10]

(b) Show that $\left(Ae^{\alpha y} + Be^{-\alpha y} + Cye^{\alpha y} + Dye^{-\alpha y}\right)\sin \alpha x$ is a stress function. [10]

Q3(a) Explain how membrane analogy is very useful in enabling us to visualize the stress distribution over the cross section of twisted bar. [10+10]

(b) Derive the differential equation of equilibrium of a small rectangular block of edges h, k and unity.

Q4(a) State and prove generalized Hook's law. [10]

(b) Find the stress function of the type $a_4 r^4 \cos 4\theta + b_2 r^4 \cos 2\theta$ that satisfied the conditions

$$\sigma_\theta = 0, \tau_{r\theta} = sr^2 \text{ on } \theta = \alpha$$

$$\sigma_\theta = 0, \tau_{r\theta} = -sr^2 \text{ on } \theta = -\alpha$$

s being a constant. Sketch the loading for positive s. [10]

SECTION II

Q5(a) Differentiate between the primary and secondary waves. [10]

(b) Work out the existence of Rayleigh waves for two dimensional case of plane waves propagating in x-direction. [10]

Q6(a) Determine the reflection coefficient for a transverse incident wave (with oscillations in the plane of incidence). [10]

(b) Work out the dispersion relation of elastic waves in a crystal of the hexagonal system. [10]

Q7(a) Show that the ratio of the mechanical impedances determines the nature of reflection and transmission at the interface. [10]

(b) Consider beams of incident, reflected and refracted waves and examine the averaged transmission of energy, in particular for the case $\frac{c_T^B}{c_T} \sin \theta_0 > 1$. [10]

Q8(a) Work out the spherically symmetric waves in the infinite medium. Also, discuss the outgoing and ingoing waves related with the problem of explosion and implosion. [10]

(b) Show that the love waves are dispersive. Also, extract the constraints for which the phase velocity decreases. [10]

Q9(a) Discuss the dispersion relation of elastic waves in a cubic crystal which are propagated (a) in the crystal plane, that of a cube face, (b) in the crystal direction, that of a cube diagonal. [10]

(b) Consider an infinite string with an elastic spring located at x = 0 and let a step plus be incident on this discontinuity. Determine the reflected and transmitted wave systems. [10]

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Part-II A/2016
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: IV-VI (opt.iii) [Group Theory] **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions by selecting at least TWO from each section.

	SECTION 1	
Q1.	a) Let G be direct product of groups A and B i.e. $G = A \otimes B$ Then show that the sets $\overline{A} = \{(a,e'): a \in A\}$ and $\overline{B} = \{(e,b): b \in B\}$ are normal subgroups of G. Also $\overline{A} \cong A$, $\overline{B} \cong B$, $\overline{A} \cap \overline{B} = \{(e,e')\}$ b) Prove that a finite group whose order is divisible by a prime number p contains a sylow p-subgoup.	10 10
Q2	a) Prove that there are two types of groups of order p^2, one a cyclic group of order p^2 and the other a direct product of two cyclic groups both of order p. b) Prove that every proper subgroup of p-group is properly contained in its normalizer.	10 10
Q3	a) Define characteristic subgroups. Show that center of a group is always characteristic. Give an example of normal subgroup which is not characteristic. b) Let G be a group with normal subgroup A. then prove that the following statements are equivalent i. G is normal product of A by B ii. $G = AB$ and $A \cap B = \{e\}$ iii. Each element of G can be uniquely written as $g = ab$.	10 10
Q4	a) Discuss the simplicity of A_n for $n \leq 4$. b) State and prove Orbit - Stabalizer Theorem. Hence use it to find the conjugacy classes of S_3.	10 10
	SECTION II	
Q5	a) Prove that a group G has a composition series if and only if all of its ascending and descending normal chains break off. b)Define the normal series of a group G. what is a subnormal subgroup of a group? Give an example of a group in which a subnormal subgroup is not necessarily a normal subgroup of the group?	10 10
Q6	a) Let N be normal subgroup of G. Then show that G is solvable if and only if N and G/N are solvable. b) Prove that $\langle a,b : a^2 = b^2, a^4 = 1, bab = a^{-1} \rangle$ is metabelian by using derived series. c) Prove that group of order 39 is solvable.	8 6 6
Q7	a) Prove that a finite group G is nilpotent if and only if it is direct product of all of its Sylow p-subgroups. b) Let $G = G_0 \rhd G_1 \rhd \ldots \rhd G_k = E$ be a central series for G. Then Show that *i)* $G_i \supseteq \gamma_i(G)$, $0 \leq i \leq k$ *ii)* $G_{k-i} \supseteq \xi_i(G)$, $0 \leq i \leq k$ Where $\gamma_i(G)$ and $\xi_i(G)$ are terms of lower central series and upper central series of G, respectively.	10 10
Q8	a) Define partial complement of a subgroup and prove that a normal subgroup H of G is contained in Frattini subgroup of G if and only if H has no partial complement in G. b) Define a Frattini subgroup of a group. Write the Frattini subgroups of A_4 and of the group Q of quaternions.	10 10
Q9	a) Define general linear group $GL(n,q)$, special linear group $SL(n,q)$ and representation of group and prove that $\lvert SL(n,q) \rvert = (q^n - 1)(q^n - q)......(q^n - q^{n-1})/(q-1)$ b) Let G be an extension of a group N by H. Then define section of G through H and sectional factor set and elaborate these with the help of examples.	12 8