UNIVERSITY OF THE PUNJAB

Part-II A/2017
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: I (Advanced Analysis) **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

SECTION-I

Q.1 **(a)** Define countable set and prove that countable union of countable sets is countable. (10)

(b) Prove that a set X is countable if and only if there exists an injective function from X to set of natural numbers. (10)

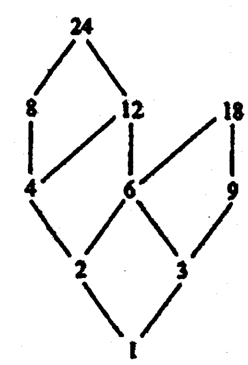
Q.2 **(a)** Define partial order, dual order and Quasi-order. Let τ be any collection of sets. Then show that the relation $\subseteq$ of set inclusion is a partial ordering on τ. (10)

(b) Let $P = \{1,2,3,...\}$ be the set of positive integers. Then show that $P \times P$ is denumerable. (10)

Q.3 **(a)** Let $X = \{1,2,3,4,6,8,9,12,18,24\}$ be ordered as given in the picture and let $A = \{4, 6, 9\}$. (10)

(i) Find maximal and minimal elements of X.

(ii) Find the *supremum* and *infimum* of A.



(b) Define ordinal numbers. Prove that if λ is any ordinal number then $\lambda + 1$ will be the immediate successor of λ. (10)

Q.4 **(a)** Let S be a subset of a well-ordered set A with the following properties (i) first element $a_0 \in S$ (ii) if $s(a) \subseteq S$, then $a \in S$. Then show that $S = A$. (10)

(b) Let R be a ring with unity 1. Use Zorn's Lemma to prove that every proper ideal J of R is contained in a maximal ideal. (10)

SECTION-II

Q.5 **(a)** Let $\mu^* : 2^R \to [0,\infty]$ be defined as $\mu^*(E) = \inf\left\{\sum_{i=1}^{\infty} l(I_n) : E \subseteq \bigcup_{i=1}^{\infty} I_n\right\}$ where I_n are open intervals. Show that μ^* is an outer measure. (10)

(b) Define measurable sets and prove that the interval $\{x \in \mathbb{R} : 7 < x < \infty\}$ is a measurable set. (10)

6 **(a)** Prove that Lebesgue outer measure is translation invariant. (10)

(b) (i) Prove that union and intersection of two measurable sets E_1, E_2 is a measurable set. (10)

(ii) If $m^*(A) = 0$, then $m^*(A \cup B) = m^*(B)$.

Q.7 **(a)** Let f be an extended real valued function defined on a measurable set D. Then show that the following are equivalent: (10)

PTO

(i) f is measurable

(ii) $\{x : f(x) \geq \alpha\}$ is a measurable set for all $\alpha \in R$

(iii) $\{x : f(x) < \alpha\}$ is a measurable set for all $\alpha \in R$

(iv) $\{x : f(x) \leq \alpha\}$ is a measurable set for all $\alpha \in R$.

(b) Write a short note on counting measure. (10)

Q.8 (a) (i) Evaluate the Lebesgue integral of the following function (10)

$$f(x) = \begin{cases} -4, & \text{if is rational number in } (-4,5) \\ 8, & \text{if is irrational number in } (-4,5) \end{cases}$$

(ii) Give an example of a function which is Lebesgue integrable but not Riemann integrable.

(b) Let f be a non-negative measurable function on E. Then $\int_E f = 0 \Leftrightarrow f = 0$ *a.e.* on E. (10)

Q.9 (a) Let $f_n : \mathbb{R} \to \mathbb{R}$ be a sequence of functions defined by $f_n(x) = \frac{1}{n}\chi_{[n,\infty)}$ $\forall n = 1,2,3,\ldots$ (10)

Then prove that $\underset{n}{Lim} \int_E f_n \neq \int_E \underset{n}{Lim}\ f_n$.

(b) Let $f(x) = \begin{cases} 1/x, & 0 < x \leq 1 \\ 1/(x-2), & 1 < x < 2 \end{cases}$ (10)

Then find the Lebesgue integrals $\int_{(0,2)} f^+$, $\int_{(0,2)} f^-$ and $\int_{(0,2)} f$.

UNIVERSITY OF THE PUNJAB

Part-II A/2017
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: II (Methods of Mathematical Physics)

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt FIVE questions in all, selecting at least TWO questions from each section.

Q. No		Marks
	Section-I	
1(a)	Find the integral surface of the quasi-linear PDE $xz_x - yz_y = z$ which contains the circle $x^2 + y^2 = 1, \quad z = 1$.	10
1(b)	Determine the general solution of the PDE $z_{xx} - \frac{1}{c^2} z_{yy} = 0$, where c is a constant.	10
2(a)	Show that if $Re(\beta) > 0$ and n is a nonnegative integer then $F\left(\frac{-1}{2}n, \frac{-1}{2}n + \frac{1}{2}; \beta + \frac{1}{2}; 1\right) = \frac{2^n (\beta)_n}{(2\beta)_n}$.	10
2(b)	Determine the eigenvalues and eigenfunctions of the system $u'' + \lambda u = 0, \quad u(0) = u(\pi), \quad u'(0) = 2u'(\pi)$.	10
3(a)	Use D' Alembert's solution of the one dimensional wave equation to solve the problem $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$, $u(0,t) = 0, \ u(a,t) = 0, \ u(x,0) = f(x), \ u_t(x,0) = g(x)$.	10
3(b)	Transform the Laplace equation $\nabla^2 u = \mathbf{0}$ from rectangular coordinates (x, y, z) to cylindrical polar coordinates(r, θ, z).	10
4(a)	Show that $\int_{-1}^{1} [P_k(x)]^2 dx = \frac{2}{2k+1}$.	10
4(b)	Use the method of Frobenius to find the power series solution of Bessel differential equation of the form $y(x) = c_1 J_v(x) + c_2 J_{-v}(x)$.	10
5(a)	Find the general solution of the differential equation $x^2 y'' + xy' + (x^2 - 1/4)y = 0$.	10
5(b)	Solve the PDE $(x - y)z_x - (x - y + z)z_y = z$, subject to the conditions $z = 1$ and $x^2 + y^2 = 1$.	10

PTO

	Section-II	
6(a)	Formulate and solve Dido's problem.	10
6(b)	Find the Green function associated with the boundary value problem $y'' + \frac{1}{4}y = f(x), \quad y(0) = 0, \quad y(\pi) = 0.$ Also, solve the problem when $f(x) = x/2$.	10
7(a)	Find the Fourier Transform of $g(x) = \frac{a}{x^2 + a^2}, \quad a > 0.$	10
7(b)	State the convolution theorem and use it to calculate the inverse Laplace transform of the function $s/(s^2+9)^2$.	10
8(a)	State and prove Parseval first and second theorems.	10
8(b)	Solve the problem using Laplace transform method $u_{xx} = u_{tt}, \quad 0 < x < 1, \quad t > 0,$ $u(0,t) = 0, \quad u(1,t) = 0,$ $u(x,0) = \sin \pi x, \quad u_t(x,0) = -\sin \pi x.$	10
9(a)	Find an extremal of the functional $I[y(x), z(x)] = \int_0^1 (y'^2 + z'^2 - 4xz' - 4z)dx$ with endpoint conditions $y(0) = 0, \; z(0) = 0, \; y(1) = 1, \; z(1) = 1,$ subject to the conditions $\int_0^1 (y'^2 - xy' - z'^2)dx = 2.$	10
9(b)	Show that the Euler-Lagrange equations for the functional $I = \int_a^b F(x, y, y', z, z')dx$ admit the following first integrals (i) $\frac{\partial F}{\partial y'} = c$ if F does not contain y. (ii) $F - y'\frac{\partial F}{\partial y'} - z'\frac{\partial F}{\partial z'} = c$ if F does not contain x.	10

UNIVERSITY OF THE PUNJAB

Part-II A/2017
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: III (Numerical Analysis) **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions, selecting at least TWO questions from each section.

Section I

Q1

a) Derive Heun's method and solve $\frac{dy}{dx} = 4e^{0.8x} - 0.5y$ from $x = 0$ to 4 taking $h = 1.0$ with an initial condition $y(0) = 2$.

(16)

b) Define over-determined and under-determined system of linear equations.

(04)

Q2.

a) Use a numerical technique to solve $4x - 2y + z = 3,\ 2x + 3y - z = 6,\ x + 3y + 7z = 11$.

(10)

b) Calculate five iterations of the power method with scaling to approximate a dominant eigenvector of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix}.$$

Use $\boldsymbol{x}_0 = (1,1,1)$ as initial approximation.

(10)

Q3.

Solve $\frac{dy}{dx} = x + y,\ y(0) = 1$ using Milne's method for $y(0.4)$. The solution for $x = 0.1$, 0.2, and 0.3 should be obtained by Runge Kutta method of fourth order. Also find the absolute error.

(20)

Q4.

Derive Newton Raphson method. Show that Newton Raphson method is quadratically convergent. Find the root between 0 and 1 of the equation $1 + cost - 4t = 0$ correct to four decimal places by Newton Raphson method.

(20)

Q5.

a) If $u = \frac{5xy^2}{z^3}$ and error in x, y, z be 0.001, 0.002 and 0.003, repectively, compute the relative error in u, where $x = y = z = 1$.

(03)

b) Show that $\mu = \frac{2+\Delta}{2\sqrt{1+\Delta}}$, $\mu\delta = \frac{\Delta+\nabla}{2}$, $\Delta = \frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$ (09)

c) Solve the system of equations $3x + y - z = 3, 2x - 8y + z = -5, x - 2y + 9z = 8$ using Gaussian elimination method.

(08)

P.T.O.

Section 2

Q6.

a) Use an interpolation formula of appropriate degree to obtain approximation to the value of function at $x = 1.16$ for the given data

x	1.0	1.1	1.2	1.3	1.4	1.5
$f(x)$	8.01	9.69	11.56	13.61	15.84	18.26

(10)

b) Write an algorithm (Boole's Rule) to approximate the integral of $f(x)$ over the interval $[a, b]$ using n subintervals.

(10)

Q7.

a) Apply five points Simpson's rule to evaluate $\int_{0.1}^{0.5} \frac{x+\ln x+\sin x+e^x}{x^4-1} dx$

(06)

b) Define Hermite interpolation. Derive Cardinal form of Cubic Hermite.

(14)

Q8.

a) Define two points Gaussain quadrature formula. Evaluate $\int_0^{\frac{\pi}{2}} \sin(x)\, dx$ using two points Gaussain quadrature formula.

(14)

b) Find the first and third derivatives at $x = 1.8$ if

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	2.25	1.73	0.97	-0.03	-1.27	-2.75

(06)

Q9.

Solve

$y_{t+2} - 9y_{t+1} + 20y_t = 3^t(t^2 + 1)$

$y_{t+2} + 3y_{t+1} + y_t = sint$

(10+10)

UNIVERSITY OF THE PUNJAB

Part-II A/2017
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt.i) [Mathematical Statistics]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section. All questions carry equal marks.

		SECTION-I	Marks
Q.1	(a)	If A and B are mutually exclusive events with $P(A\cup B)\neq 0$, then show that $P(A/A\cup B)=\frac{P(A)}{P(A)+p(B)}$	(10)
	(b)	Three drawing each of two balls are made from a box containing seven white and eight black balls. What are the probabilities that these drawings are made (i) in alternative colours (ii) in same colours When balls are not replaced before any draw.	(10)
Q.2	(a)	Prove that the recurrence formula for binomial distribution is given by $\mu_{r+1}=pq(nr\mu_{r-1}+\frac{\partial\mu_r}{\partial p})$, where n and p are the parameters of the binomial distribution.	(10)
	(b)	The number of bacteria observed in bottles of one cc water follows Poisson random variable with mean 4. (i) Find the probability that no bacteria are found in a given bottle of one cc water. (ii) Assume that the number of bacteria found in two different bottles are independent. Find the probability that at least two bacterias are observed in two different bottles having one cc water in each.	(5+5)
Q.3	(a)	Prove that if X and Y are independent Gamma variates, with parameters l and m respectively, then $\frac{X}{X+Y}$ is a $\beta_1(l,m)$ variate.	(10)
	(b)	Find the mean and variance of a Gamma distribution.	(10)
Q.4	(a)	Prove that the Normal distribution is a limiting case of the Binomial distribution.	(10)
	(b)	The Edwards's Theater chain has studied its movie customers to determine the how much money they spend on concessions. The study revealed that the spending distribution is approximately normally distributed with a mean of \$4.11 and a standard deviation of \$1.37. (i) What percentage of customers will spend less than \$3.00 on concessions? (ii) What spending amount corresponds to the top 87th percentile?	(10)
		SECTION-II	
Q.5	(a)	Prove that $$\frac{r_{12}-r_{13}r_{23}}{\sqrt{(1-r_{13}^2)(1-r_{23}^2)}}= r_{12.3}$$	(10)

(P.T.O.)

	(b)	If the multiple regression equation of X_3 on X_1 and X_2 is given by $x_3 = b_{13.2}x_1 + b_{23.1}x_2$, then show that $b_{23.1} = \frac{S_2(r_{23}-r_{13}r_{21})}{S_3(1-r_{13}^2)}$ where r_{ij} are the linear correlation coefficients and S_k are the standard deviations.	(10)
Q.6	**(a)**	Let X and Y be two random variables with zero mean, zero correlation and same variance and if $U = XCos\alpha + YSin\alpha$ and $V = XSin\alpha - YCos\alpha$, find the correlation between U and V.	(10)
	(b)	Find the probability distributions for each of the random variable X whose moment generating functions are given by: (i) $M(t) = (\frac{1}{5}e^t + \frac{4}{5})^{11}$ (ii) $M(t) = \frac{3e^t}{6-3e^t}$ (iii) $M(t) = e^{t(5+32t)}$	(10)
Q.7	**(a)**	If the joint density of X and Y is given by $f(x,y) = \begin{cases} e^{-x-y} & for\ x>0,\ y>0, \\ 0 & elsewhere \end{cases}$ Find **(i)** the joint density of $Z = \frac{X+Y}{3}$ **(ii)** the marginal densities of X and Y.	(10)
	(b)	Find moment ratios (β_1, β_2) and discuss Skewness and Kurtosis for Poisson distribution.	(10)
Q.8	**(a)**	Define F-statistic. Prove that all even order moments about origin of $t-$distribution with n degree of freedom is given by $\frac{n^r\Gamma(r+\frac{1}{2})\Gamma(\frac{n}{2}-r)}{\Gamma(\frac{1}{2})\Gamma(\frac{n}{2})}$	(10)
	(b)	If $F \sim F(\upsilon_1, \upsilon_2)$, then $(1+\frac{\upsilon_1}{\upsilon_2}F)^{-1} \sim \beta(\frac{\upsilon_1}{2}, \frac{\upsilon_2}{2})$	(10)
Q.9	**(a)**	Let $X_1, X_2, \ldots, X_n$ be a random sample of size n taken from a normal population with mean μ and variance σ^2. If $\bar{x}$ and s^2 represent the mean and unbiased variance of the sample chosen above. Then prove that $\frac{(n-1)s^2}{\sigma^2}$ is a Chi-square variate with $n-1$ degree of freedom.	(10)
	(b)	Derive the probability density function of the t - distribution.	(10)

UNIVERSITY OF THE PUNJAB

Part-II **A/2017**
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt.iii) [Group Theory]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions by selecting at least TWO from each section.

			Marks
		SECTION-I	
Q.1	**(a)**	Show that every group of order p^2 is abelian, p a prime number.	(10)
	(b)	Prove that there are two types of groups of order p^2, one a cyclic group of order p^2 and the other a direct product of two cyclic groups both of order p.	(10)
Q.2	**(a)**	Prove that a finite group whose order is divisible by a prime number p contains a sylow p-subgoup.	(10)
	(b)	For any prime divisor p of the order n of a goup G, G has a unique sylow p-subgroup H if and only if H is normal in G.	(10)
Q.3	**(a)**	Show that the center of a group G is a characteristic subgroup of G.	(10)
	(b)	Every fully invariant subgroup of a group is characteristic.	(10)
Q.4	**(a)**	Define the normal product of a group H by a group K. Give an example.	(10)
	(b)	What is meant by the holomorph of a group G? Find the holomorph of the group $G = \langle a, b: a^3 = b^2 = (ab)^2 = 1\rangle$. Write the order of the holomorph of G.	(10)
		SECTION-II	
Q.5		State and prove Zassenhaus Butterfly Lemma.	(20)
Q.6	**(a)**	Define the normal series of a group G. what is a subnormal subgroup of a group? Give an example of a group in which a subnormal subgroup is not necessarily a normal subgroup of the group?	(10)
	(b)	Define a composition series of a group G. Can a composition factor of a group have a proper normal subgroup? Elaborate by an example.	(10)
Q.7	**(a)**	Show that a group G is solvable if and only if it has a normal series with abelian factors.	(10)
	(b)	Define a nilpotent group and its nilpotency class. Illustrate by an example. Is every solvable group also nilpotent and vice versa?	(10)
Q.8	**(a)**	Define a Frattini subgroup of a group. Write the Frattini subgroups of A_4 and of the group Q of quaternions.	(10)
	(b)	Explain the concepts of (i)Extensions, (ii) central extensions with examples.	(10)
Q.9	**(a)**	Define a chief series of a group G. Show that every chief factor of a finite p-group G is of order p.	(10)
	(b)	Let Q and D_4 be the group of Quaternions and the dihedral group of order 8 respectively. Write their composition series and compare their compositon factors.	(10)

UNIVERSITY OF THE PUNJAB

Part-II A/2017
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt.vii) [Quantum Mechanics]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions in all, selecting at least TWO questions from each section.

SECTION I

1. (a) (i) Show that the de Broglie wavelength of an electron of kinetic energy $E(ev)$ is $\lambda_c = \frac{12.3\times10^{-8}}{\sqrt{E}}$ cm and that of proton is $\lambda_p = \frac{0.29\times10^{-8}}{\sqrt{E}}$cm. (10 marks)

   (ii) Define Hermitian operator and show that eigen values of hermitian operators are always real. (10 marks)

   (b) Define Compton effect. According to quantum theory, a monochromatic electromagnetic beam of frequency ν is regarded as a collection of particle like photons, each possessing an energy $E = h\nu$ and a momentum $p = h\nu/c$. The scattering of electromagnetic radiation becomes a problem of collision of a photon with a charged particle. suppose that a photon is scattered at an angle θ, and its frequency is changed. Find the increase in the photon's wavelength as a function of the scattering angle. Also find the angle after collision in different cases. (10 marks)

   (c) Show that the operators $\hat{A}$ and $\hat{B}$ commute with their commutator and show that (10 marks)

$$[\hat{A},\ \hat{B}^n] = n\hat{B}^{n-1}[\hat{A},\ \hat{B}].$$

2. (a) State the Heisenberg uncertainty principle. Describe the experiment to prove its validity. (10 marks)

   (b) An electron beam is incident on a barrier of heigh $10eV$. At $E = 10eV$, $T = 3.37\times10^{-3}$. What is the width of the barrier? (10 marks)

3. (a) Find the solution of the following Schrodinger equation in three dimensional rectangular box

$$-\frac{\hbar^2}{2m}\nabla^2\psi(x,y,z) = E\psi(x,y,z).$$

   Show that E_{211}, E_{121} and E_{112} are the corresponding degenerate eigen values of energies. (10 marks)

   (b) Prove

   (i) $e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + [\hat{A},\hat{B}] + \frac{1}{2!}[\hat{A},[\hat{A},\hat{B}]] + ...,$

   (ii) $[\hat{A}\hat{B},\hat{C}] = \hat{A}[\hat{B},\hat{C}] + [\hat{A},\hat{C}]\hat{B},$

   where $\hat{A}$, $\hat{B}$ and $\hat{C}$ are any three operators. (10 marks)

P.T.O.

SECTION II

4. (a) Apply the operators $L_+ \equiv L_x + \iota L_y$ and $L_- \equiv L_x - \iota L_y$ on the eigen states of L^2 and $L_z(|lm\rangle)$ and interpret the physical meaning of the results. Find the Hermitian conjugate of L_+. Calculate the norm of $L_+|lm\rangle$ and $L_-|lm\rangle$. (10 marks)

   (b) Particles are scattered from the potential $V(r) = g/r^2$, where g is a positive constant. Prove that the phase shifts are given by (10 marks)

$$\delta_1 = \frac{\pi}{2}\left[l + \frac{1}{2} - \sqrt{\left(l + \frac{1}{2}\right)^2 + \frac{2\mu g}{\hbar^2}}\right]$$

5. (a) Show that the expression

$$\langle J^2 \rangle = \hbar^2 j(j+1)$$

   is implied by the two assumptions:

   (i) The only possible values that the componenst of angular momentum can have on any axis are $\hbar(-j, \ , ..., +j)$.

   (ii) All these components are equally probable.

   (10 marks)

   (b) Compute the expression for ionization potential for hydrogen, helium and Lithium atoms. (10 marks)

6. (a) What is Schrödinger equation in momentum representation for a free particle moving in one dimension? What are the eigen functions $b(k)$ of this equation? (10 marks)

   (b) Show that eigrn states of $\hat{H}$ or equivalently $\hat{N} = \hat{a}^\dagger \hat{a}$ corresponding to eigen values $n < -1/2$ must vanish identically. Here, $\hat{a}^\dagger$ and $\hat{a}$ are promotion and demotion operators, respectively. (10 marks)

7. Define perturbation theory. Obtain first order correction in energy and eigen functions for time-independent non-degenerate perturbation theory. (20 marks)

8. (a) Prove the following relations for the angular momentum operator (10 marks) $[\hat{L}^2, \ \hat{L}_z] = 0$, $\mathbf{L} \times \mathbf{L} = \iota\hbar\mathbf{L}$.

   (b) Use transformation equations relating spherical coordinates $(r, \ \theta, \ \phi)$ with cartesian coordinates $(x, \ y, \ z)$ to obtain the expression $\hat{L}_z = -\iota\hbar\frac{\partial}{\partial\phi}$. (10 marks)

UNIVERSITY OF THE PUNJAB

Part-II A/2017
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: (IV-VI) (opt.iv) [Rings & Modules] **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section I

Q. 1. (a) Let $a, b \in F[x]$, $b \neq 0$ and F is a field. Then there exist $q, r \in F[x]$ such that $a = bq + r$ with $r = 0$ or $\deg(r) < \deg(b)$. Moreover q and r are unique. 12+8

(b) Find GCD of the Gaussian integers 7+3i and 5 – 8i.

Q. 2.(a) Prove or disprove that every Euclidean domain is principal ideal domain. 10+10

(b) Let R be an integral domain and let $p \in R\backslash\{0\}$. Then p is prime if and only if $R/_{pR}$ is an integral domain.

Q. 3. (a) The polynomial $x - c$ is a factor of a polynomial $p(x) \in K[x]$ if and only if $x = c$ is a root of $p(x) = 0$. 10+10

(b) Prove or disprove that every Euclidean domain is Principal Ideal Domain.

Q. 4. (a)Show that if L is a finite extension of F and K is a subfield of L which contains F, then [K: F] is a divisor of [L: F]. 10+10

(b) If L is finite extension of K and Kis a finite extension of F, then prove that Lis finite extension of F and [L: F] = [L: K][K: F].

Q. 5. (a) Define an algebraic extension of a field and prove that every finite extension of a field is algebraic. 10+10

(b) Let K be a field, an element a of K is algebraic over F if and only if [F(a): a] is finite.

Section II

Q. 6. (a)Every FG – R-module is homomorphic image of a free module. 10+10

(b) Let M be a module over an integral domain R and let T denote the set of torsion elements of M. Show that T is a submodule of M then quotient module $M/_T$ is torsion free.

Q. 7. (a) Let R be a ring with identity and M be irreducible R-module. Then M is cyclic. 10+10

(b) Let M and N be two R-modules, $f: M \to N$ and $g: N \to M$ be two module homomorphisms such that $gof = I_M$ (identity map on M). Show that $N = Kerg \oplus Imf$.

Q.8. (a) Show that any two cyclic R-modules are isomorphic if and only if they have same order ideal.

(b) Let M be an R- module, where R is a commutative ring with identity. Show that M is simple if and only if $\cong R/_I$, I is a maximal ideal of R. 10+10

Q. 9. Let M be an R-module and let $\{m_1, m_2, \dots\dots m_s\}$ be a finite subset of M. Prove that the following statements are equivalent: 20

i. $\{m_1, m_2, \dots\dots m_s\}$ generates M freely.
ii. $\{m_1, m_2, \dots\dots m_s\}$ is linearly independent and generates M.
iii. every element $m \in M$ is uniquely expressed in the form $m = \sum_{i=1}^{s} r_i m_i$.
iv. every m_i is torsion free and $M = Rm_1 \oplus Rm_2 \oplus \dots\dots \oplus Rm_s$.

UNIVERSITY OF THE PUNJAB

Part-II A/2017
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: IV-VI (opt.v) [Number Theory] **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

SECTION-I **Marks**

Q.1 **(a)** Let $k>0$ and a, b be any two positive integers, then prove that $\gcd(ka,kb)=k\gcd(a,b)$ (7)

(b) (i) Prove that every composite integer can be expressed as a product of primes in a unique way except the order of primes in which they appear. (7+6)

(ii) Find addition and multiplication of $(785)_9$ and $(518)_9$

Q.2 **(a)** State and prove Lagrange's Theorem for the solution of polynomial congruences modulo a prime number. (10)

(b) In modulo 15, discuss all solutions of the polynomial $f(x)=x^2-1\equiv 0$ (10)

Q.3 **(a)** Prove that $\sum_{d|n}\frac{\varphi(d)}{d}=\prod_{i=1}^{r}[1+k_i\frac{p_i-1}{p_i}]$, where φ is the Euler's phi function. (10)

(b) Define a Mobious pair. Prove that $\{n, \varphi(n)\}$ is a Mobious pair. (2+8)

Q.4 **(a)** State Fermat's last theorem and prove it for $n = 4$. (2+8)

(b) (i) For any integer n, the exponent e of any prime p, such that $p^e \mid n!$ is at most $\sum_{i=1}^{\infty}[\frac{n}{p^i}]$ where [ ], is the bracket function. (5+5)

(ii) Find $E_{100}(13)$. That is, exponent of 13 in 100!

Q.5 **(a)** Let a have exponent h modulo m. Then show that a^k has exponent h modulo m if and only if $\gcd(h,k)=1$. Find all primitive roots of 11. (8+2)

P.T.O.

(b) Prove the existence of primitive roots of any prime number. (10)

SECTION II

Q.6 (a) Define Legendre and Jacobi symbols. Evaluate (2+8)

(i) $\left(\frac{127}{179}\right)$ (ii) $\left(\frac{219}{383}\right)$

(b) Let p and q be distinct odd primes. Show that (10)

$$\left(\frac{p}{q}\right)=\begin{cases}-\left(\frac{q}{p}\right), & p,q\equiv 3(\text{mod } 4)\\ \left(\frac{q}{p}\right), & p \text{ or } q\equiv 1(\text{mod } 4)\end{cases}$$

Q.7 (a) Define a primitive polynomial. State and prove Gauss Lemma for primitive polynomials. (2+8)

(b) State any irreducibility criterion for polynomials. Apply your criterion to show that the cyclotomic polynomial of degree p-1 is irreducible where p is an odd prime. (2+8)

Q.8 (a) Define an algebraic number. Find the minimal polynomial for the algebraic number $\sqrt{2+\sqrt{3}}$ (2+5)

(b) Prove that, if ξ is algebraic number over a field F then all elements of the field $F(\xi)$ are algebraic over F. Determine whether $7^{\sqrt{7}}$ is an algebraic number? (10+3)

Q.9 (a) State and prove the existence of transcendental numbers. (10)

(b) Prove that any two integral bases have same discriminant. (10)

UNIVERSITY OF THE PUNJAB

Part-II A/2017
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt.vi) [Fluid Mechanics]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions in all, selecting at least TWO questions from each section.

Section I

Q.1(a)	Derive Newtonian law of viscosity.	(10)
(b)	Define the equation of continuity an drive it in Euler's methods of pecification.	(10)
Q.2(a)	Derive Bernoulli's equation for unsteady inviscid irrotational flow under conservative forces and discuss all its special cases	(10)
(b)	Define vortex line and derive equation for the vortex line.	(10)
Q.3(a)	Discuss the flow when a fluid is moving subject to external impulsive forces.	(10)
(b)	What is a steam function? Discuss the tangential and normal components of velocity in terms of stream function.	(10)
Q.4(a)	Derive Euler's equation of motion.	(10)
(b)	Define Circulation, Find the relationship between Circulation and vorticity.	(10)
Q. 5	Three sources each of strength m are placed at the points $(-a,0),(0,0),(a,0)$. Evaluate the following: (i) Velocity potential and Stream function (ii) Complex velocity potential (iii) Speed and Stagnation points (iv) Equipotential and Stream lines	(20)

Section II

Q.6(a)	An incompressible steady flow field has the velocity components $u=6y, v=7x^2z,\ w=x^3\sin z$ Evaluate the stress tensor σ_{ij}. Assuming that there is no hydrostatic pressure.	(10)
(b)	What is Poiseuille flow. Make a mathemamatical model and find the maxmimum velocity for it.	(10)
Q.7(a)	Define mean motion and fluctuations in a turbulent flow and prove that the mean value of a fluctuating quantity is always zero	(10)
(b)	Define uniform flow for two dimensional flow and draw flownet for it.	(10)
Q.8(a)	What is simple Couette flow? Modelling the same flow, evaluate the velocity field, Maximum velocity, volumetric flow rate and shearing stress.	(10)
(b)	Derive mathematical model for the steady, laminar flow through two cylinders. The cylinders are at rest body forces are negligible and flow is due to pressure difference along the common axis. Find the expression for the velocity field..	(10)
Q.9(a)	Solve first problem of Stokes . Write its mathematical formulation and solve it for the velocity field.	(10)
(b)	Show that the velocity field $u=0, v=0,\ w=\frac{3}{4\mu a}\left(-\frac{dp}{dz}\right)(x-a)\left(y^2-\frac{x^2}{3}\right)$ satisfies the Navier Stokes equations for and incompressible viscous steady flow.	(10)

UNIVERSITY OF THE PUNJAB

Part-II A/2017
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: IV-VI (opt.viii) [Special Theory of Relativity and Analytical Dynamics] **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions in all, selecting at least TWO questions from each section.

SECTION I

Q1(a) Describe Michelson Morley experiment in detail. What was the objective of this experiment? [10]

(b) An observer sees a clock as showing 1 hour to be half and hour. If he sees an object lying at an angle of $\frac{\pi}{4}$ as having a length of 2m. What is the rest length of the object? [10]

Q2(a) Describe particle scattering by considering two particles having four vector momenta in the laboratory frame, p_1^μ and p_2^μ with rest masses m_1 and m_2. Calculate the four momenta of these particles in the center of mass frame. [10]

(b) An electron is moved from rest by a photon which is deflected by $\frac{\pi}{3}$ and whose wavelength is doubled. Determine the resultant momentum of electron. [10]

Q3(a) Show that $P^2 - E^2/c^2$ is Lorentz invariant, where P, E are the momentum and energy of the particle and c is the speed of light. [10]

(b) A life-time of a π^+ mesons as measured in a laboratory (rest frame) is found to be $\tau = 2.5\times10^{-8}$ seconds. If a beam of π^+ mesons of velocity 0.8c is produced, find out the distance the beam will travel before its flux is reduced to $1/c^2$ times the initial flux. [10]

Q4(a) Consider a scalar electric potential and a vector magnetic potential to make a four vector electromagnetic field potential to define an electromagnetic field tensor. Find all the components of contravariant and covariant Maxwell field tensors. [10]

(b) Show that $F^{\alpha\beta}{}_{;\beta} = \mu_0 J^\alpha$, $F_{[\alpha\beta,\delta]}=0$ represent Maxwell's equations in 4-vector formalism. [10]

Q5(a) Determine a formula for the minimum kinetic energy required to produce a particle of mass m.

(b) A rod of length l makes angles $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with the x-axis and y-axis respectively. What does the length of the rod appear to be to an observer moving with velocity $\left(\frac{c}{\sqrt{3}},\frac{c}{\sqrt{6}},\frac{c}{\sqrt{3}}\right)$. [10+10]

SECTION II

Q6(a) What are the quasi coordinates? Derive Boltzmann-Hamilton equation of motion using these coordinates. [10]

(b) A particle of mass m is moving on the surface of sphere r in the gravitational field. Use Hamilton's principle to show that the equation of motion $\ddot{\theta}-\frac{P_\phi \cos\theta}{m^2r^2\sin^3\theta}=\frac{g}{r}\sin\theta$, where P_ϕ is the constant of integration. [10]

Q7(a) What are the generalized coordinates and generalized velocities? Prove D'Alembert principle of dynamical system. [10+10]

(b) Consider a particle moving in space using the spherical polar coordinates (r,θ,ϕ) as the generalized coordinates. Express the virtual displacement δ_x, δ_y and δ_z in terms of r, θ and ϕ.

Q8(a) State and prove Hamilton principle. [10]

(b) Obtain Hamilton's equations of a particle of mass m moving in a plane about a fixed point by an inverse square force $-\frac{k}{r^2}$. Hence (i) obtain the radial equation of motion, (ii) show that the angular momentum is constant. [10]

Q9(a) Show that Poisson bracket is invariant under canonical transformations. [10]

(b) Show that the transformation $p = m\omega q\cot Q$ and $P=\frac{m\omega q^2}{2\sin^2 Q}$ is canonical and obtain the generator of the transformation. [10]

UNIVERSITY OF THE PUNJAB

Part-II A/2017
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: IV-VI (opt.xi) [Theory of Approximation & Splines] **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions, select at least TWO questions from each section.

Section1

Q1.

a) Show that the concatenation of two rotations, the first through an angle θ about a point $P(x_0, y_0)$ and second about a point $Q(x_1, y_1)$ (distinct from P) through an angle $-\theta$, is equivalent to a translation. (10)

b) Prove that any unit quaternion $q = (s, v)$ has the form $q = (cos\theta, sin\theta\ \boldsymbol{I})$ for some angle θ and unit vector $\boldsymbol{I}$ (10)

Q2

a) Define Chebsheve polynomials and Pade approximations. (10)

b) Determine the quaternion q that represents a rotation about the axis $(-1,2,2)$ through an angle $\frac{\pi}{4}$. Apply the rotation to the point (5,6,7). (10)

Q3

a) Define Scalar and Vector Functions, Barcentric Coordinates, Convex Hull and Affine Maps. (10)

b) Define data linearization method. Use the data linearization method, find the exponential fit $y = ce^{Ax}$ for the data points (0, 1.5), (1, 2.5), (2, 3.5), (3, 5.0), (4, 7.5). (10)

Q4.

a) Demonstrate that if the coordinates of points are expressed by rational numbers, then a reflection in a line defined by reflection coefficients a, b, c can be computed using integer arithmetic. (10)

b) Calculate maximum, Average, Root mean squares errors for $y = 0.199769x^{\frac{3}{2}}$ to the data (58,88), (108,255), (150,365), (228,687). (10)

Section2

Q5.

a) Derive first, second and third derivatives of Bernstein basis function $B_{i,n}(t)$ of degree n. (10)

b) Write an algorithm to compute Bernstein Bezier form. (10)

Q6. a) Derive Ball cubic polynomial. (10)

b) Consider the Bernstein Bezier cubic form. Determine the new control points for Bernstein Bezier quartic form by degree raising algorithm. (10)

Q7. a) Prove that the intermediate control points defined in the de Casteljau algorithm satisfy

$$b_k^j = \sum_{i=0}^{j} B_{i,j}(t) b_{i+k}$$

(10)

b) Apply the de Casteljau algorithm to the quartic Bezier curve $\boldsymbol{B}(t)$ with control points $b_0(3,3)$, $b_1(4,2)$, $b_2(-1,0)$, $b_3(6,1)$, $b_4(8,5)$ and evaluate the point $\boldsymbol{B}(0.6)$. (10)

Q8. Define cubic spline. Derive cubic spline interpolation with periodic end conditions. (20)

Q9. Let $t_0 = 0$, $t_1 = 1$, $t_2 = 2$, $t_3 = 3$, $t_4 = 4$, $t_5 = 5$ and $t_6 = 6$ then determine the basis functions for a B-spline of degree 2. Discuss the properties of the B-spline basis functions (20)

UNIVERSITY OF THE PUNJAB

Part-II A/2017
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt. ix) [Electromagnetic Theory]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions in all selecting at least TWO questions from each section.

SECTION I

1. (a) Prove that the incident, reflected and transmitted waves are coplanar. (10 marks)

   (b) Two small identical conducting spheres have charges $2 \times 10^{-9}C$ and $-0.5 \times 10^{-9}C$, respectively, when they are placed 4cm apart. How much force they exert on each other. (10 marks)

2. (a) Show that there are effective charge distributions outside due to the polarized dielectric. (10 marks)

   (b) Find the potential at any point of the electric field in the presence of conducting sphere. (10 marks)

3. (a) Show that the free charge density decreases exponentially with time. (10 marks)

   (b) Deduce the divergence of the magnetic induction. (10 marks)

4. (a) Derive the Lorentz condition and use it to prove that the divergence of vector potential is zero for electrostatic field. (10 marks)

   (b) Calculate the electromotance induced in a loop by a pair of long parallel wires carrying a variable current. (10 marks)

5. (a) Find the potential difference between two points "a" and "b" (b>a) lying on spherical radial line (θ, ϕ=constant) from the origin. (10 marks)

   (b) In electric field given by

   $$E = \frac{1.5}{\epsilon_o}x^2y^2\hat{i} + \frac{1}{\epsilon_o}x^3y\hat{j}.$$

   How much charge lie within a cube, 4m of side, if its geometric center is at origin and its sides are parallel to the coordinate axis. (10 marks)

SECTION II

6. (a) Derive the electromagnetic wave equations for field vectors. (10 marks)

   (b) Prove that the electric and magnetic energy densities for a plane electromagnetic wave in free space are equal. (10 marks)

7. (a) Verify that the flux of Poynting vector through any closed surface gives the energy flow due to a plane wave through an imaginary cylinder. (10 marks)

   (b) Discuss the propagation of plane electromagnetic waves in non-conductors. (10 marks)

8. (a) Work out the coefficients of reflection and transmission at an interface using Fresnel's equations for case when $\vec{\mathbf{E}}$ is polarized normal to the plane of incidence. (10 marks)

   (b) Discuss the transverse electric waves in a hollow rectangular waveguide. (10 marks)

9. (a) State and prove Snell's law. (10 marks)

   (b) If $f_1(x,t) = A_1\cos(kx - \omega t + \theta_1)$ and $f_2 = A_2\cos(kx - \omega t + \theta_2)$ are two sinusoidal waves then show that their sum is also a sinusoidal wave. (10 marks)

UNIVERSITY OF THE PUNJAB

Part-II A/2017
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: IV-VI (opt. x) [Operations Research] **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions selecting atleast TWO questions from each section.

SECTION 1

Q1. The required daily batch of feed mix for a broiler diet is 100 pounds. The diet must contains

1) At least 0.8 % but not more than 1.2% calcium
2) At least 22 % protein
3) At least 5% fiber.

The main ingredients used include line-stone, corn and soya bean. The nutritive contents are

ingredients	calcium	protein	fibre	Cost/pounds
Line stone	0.380	0.00	0.00	0.0164
Corn	0.001	0.09	0.02	0.0645
Soya bean	0.002	0.50	0.08	0.1250

Formulate LP model and solve graphically for determining feed mix which can be prepared at least cost — 20

Q2
a) Write a note on Two Phase method. — 10
b) Solve the following by usin two phase method — 10

$$\text{Min: } z = 2x_1 + 3x_2 - 5x_3$$

Subject to

$$x_1 + x_2 + x_3 = 7$$
$$2x_1 - 5x_2 + x_3 \geq 10$$
$$x_1, x_2, x_3 \geq 0$$

Q3
a) Write a note on unbounded solution? — 8
b) Write the dual of the following primal LP model and determine the values of dual variables by solving primal LP model. — 12

$$\text{Max: } z = 15x_1 + 12x_2$$

Subject to

$$x_1 + 2x_2 \geq 3$$
$$2x_1 - 4x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

Q4
a) Explain dual simplex method. — 8
b) Solve the following by dual simplex method. — 12

$$\text{Max: } z = 5x_1 + 6x_2$$

Subject to

$$x_1 + x_2 \geq 2$$
$$4x_1 + 2x_2 \geq 4$$
$$x_1, x_2 \geq 0$$

(P.T.O.)

Q5

a) Write a note on Assignment model. **6**

b) The assignment cost of assigning any one operator to any one machine is given in the following table. Find the optimal assignment by Hungarian Method. **14**

Operators

	9	22	58	11	19	27
	43	78	72	50	63	48
Machine	41	28	91	37	45	33
	74	42	27	49	39	32
	36	11	57	22	25	18
	3	56	53	31	17	28

SECTION II

Q6

a) Write a note on revised simplex method. **6**

b) Solve the following LP model by revised simplex method **14**

$$\text{Max: } z = x_1 + 2x_2$$

Subject to

$$2x_1 + 5x_2 \geq 6$$
$$x_1 + x_2 \geq 2$$
$$x_1, x_2 \geq 0$$

Q7

Solve the following bounded LP model **20**

$$\text{Max: } z = 4x_1 + 2x_2 + 6x_3$$

Subject to

$$4x_1 + x_2 \leq 9$$
$$-x_1 + x_2 + 2x_3 \leq 8$$
$$-3x_1 + x_2 + 4x_3 \leq 12$$
$$0 \leq x_1 \leq 3,\ 0 \leq x_2 \leq 5,\ 0 \leq x_3 \leq 2$$

Q8

a) Write a note on Cutting plane algorithm. **8**

b) Solve the following LP model by cutting plane algorithm **12**

$$\text{Max: } z = 7x_1 + 10x_2$$

Subject to

$$-x_1 + 3x_2 \leq 6$$
$$7x_1 + x_2 \leq 35$$

where $x_1, x_2 \geq 0$ are non-negative integers

Q9

Solve the following parametric LP model **20**

$$\text{Max: } z = 3x_1 + 2x_2 + 5x_3$$

Subject to

$$x_1 + 2x_2 + x_3 \leq 40 - t$$
$$3x_1 + 2x_3 \leq 60 + 2t$$
$$x_1 + 4x_2 \leq 30 - 7t$$
$$x_1, x_2, x_3, t \geq 0$$

UNIVERSITY OF THE PUNJAB

Part-II **A/2017**
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: IV-VI (opt.ii) [Computer Applications]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 50

NOTE: Attempt any FIVE questions by selecting atleast TWO from each section.

Section 1

1. (a) [**6 marks**] Define the following
   i. Subroutines
   ii. Arrays
   iii. Select Case statement

   (b) [**4 marks**] What is the output of the following programs.

a)
```
Program looping
Real :: a
a = 10
Do p = 1, 5, 1
a = a - 2/p
End Do
Print * , a
End
```

b)
```
Program Multiplying
Character(LEN=2) :: p , q , r
p = 17609
q = 345
r = p * q
Print * , r
End
```

2. [**10 marks**] Given a set of n integers, write a Fortran 90 program which will find
   (a) The total number of even integers in the set
   (b) The total number of odd integers in the set
   (c) The sum of all even integers
   (d) The sum of all odd integers

   Structure the program using subroutines.

3. (a) [**5 marks**] Write a Fortran 90 program to calculate factorial of a given integer.
   (b) [**5 marks**] Give different forms of If-statements in Fortran 90 with one example each.

4. (a) [**5 marks**] Given a decimal number of arbitrary length, write a Fortran 90 program to find its octal equivalent.
   (b) [**5 marks**] Given a 4 digit number, representing a year, write a Fortran 90 program to find out whether it is a leap year.

(P.T.O.)

Section 2

5. **[10 marks]** Write a Fortran 90 program to implement Milne Predictor Corrector method for the solution of following initial value problem,

$$\frac{dy}{dx} = e^{x^2-y^2}, \qquad y(1) = 2 \qquad h = \frac{1}{10}.$$

6. **[10 marks]** Write a Fortran 90 program to implement Boole's rule to evaluate

$$\int_{-1}^{1} \frac{x^3}{1-x}\, dx$$

7. **[10 marks]** Write a Fortran 90 program to use Gauss Seidel method to solve

$$\begin{aligned} 2x + 3z &= 2 \\ x - 5y &= 3 \\ x - y - z &= 15 \end{aligned}$$

8. **[10 marks]** Write a Fortran 90 program to implement fixed point iteration method for finding root of $(x-1)^3 - 10x + 5 = 0$..

9. Write the **MATHEMATICA** statements for the following.

   (a) **[2 marks]** Find the sum of series, $\sum_{k=-\infty}^{-1} z^{1/k}$.

   (b) **[2 marks]** Find the intersection of two sets, $X = \{a, c, h, p\}$, $Y = \{h, m, n\}$.

   (c) **[2 marks]** Find the argument of a complex number $a + bi$.

   (d) **[2 marks]** Evaluate $\int_{-10}^{10} Sin(\frac{1}{y})\ dy$.

   (e) **[2 marks]** Plot the graph of e^x, $2 \le x \le 5$. Draw the frame as well.

UNIVERSITY OF THE PUNJAB

Part-II A/2017
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: IV-VI (opt. xiii) [Solid Mechanics] **MAX. MARKS: 100**

NOTE: Attempt any **FIVE** questions in all, selecting at least **TWO** questions from each section.

SECTION I

Q1(a) Write a detailed note on the components of stress and strain in the discussion of deformation of an elastic body. [10]

(b) Show that a stress vector cannot cross a free surface (one on which there is no external load). [10]

Q2(a) Investigate what problem is solved by $\phi = -\frac{F}{d^3}xy^2(3d-2y)$ applied to the region included in $y = 0$, $y=d$, $x = 0$ on the side x positive. [10]

(b) Show that $(Ae^{\alpha y} + Be^{-\alpha y} + Cye^{\alpha y} + Dye^{-\alpha y})\sin \alpha z$ is a stress function. [10]

Q3(a) Determine the value of constant C in the stress function $\phi = Cr^2(\cos 2\theta - \cos 2\alpha)$ required to satisfy the conditions

$$\sigma_\theta = 0, \tau_{r\theta} = s \text{ on } \theta = \alpha$$
$$\sigma_\theta = 0, \tau_{r\theta} = -s \text{ on } \theta = -\alpha,$$

corresponding to uniform shear loading on each edge of a wedge, directed away from the vertex. Verify that no concentrated force or couple acts on the vertex. [10]

(b) Find the general form of $f(r)$ in the stress function $\theta f(r)$, and find the expressions for the stress components σ_r, σ_θ, $\tau_{r\theta}$. Could such a stress function apply to a closed ring? [10]

Q4(a) Define the stress function S by $\tau_{ij} = S_{,ij} \equiv \frac{\partial^2 S}{\partial x_i \partial x_j}$ and consider the case of zero body force. Show that if Poisson's ration σ is assumed to vanish, then the equilibrium and compatibility equations given in the preceding problem reduce to $\nabla^2 S$ = constant. [10]

(b) State and prove generalized Hook's law. [10]

SECTION II

Q5(a) Work out the waves of dilatation and waves of distortion in isotropic elastic media. [10]

(b) Consider the reflection-transmission expressions $\frac{\partial^2 u_1}{\partial x_1^2} = \frac{1}{c_1^2}\frac{\partial^2 u_1}{\partial t^2}$, $\frac{\partial^2 u_2}{\partial x_2^2} = \frac{1}{c_2^2}\frac{\partial^2 u_2}{\partial t^2}$, where c_1 and c_2 are the bar velocities of the respective materials in the form $\sigma_t = K_t\sigma_i$, $\sigma_r = K_r\sigma_i$, where K_t and K_r approximately defined transmission and reflection coefficients. Plot K_t and K_r as a function of $\frac{Z_2}{Z_1}$ for various ratios of $\frac{A_1}{A_2}\left(\frac{1}{4}, \frac{1}{2}, 1, 2\right)$. [10]

Q6 Discuss the reflection and transmission of waves at plane boundaries in detail. [20]

Q7(a) Show that when a plane wave incident on a fixed boundary, a plane wave is reflected and an exponentially-attenuated wave is induced that propagates along the edge of the plate. [10]

(b) A rod of length l is fixed at one end and subjected to a compressive load P. The load is suddenly removed. Determine the resulting wave propagation in the wall. In particular, plot the displacement verses time for the end. [10]

Q8(a) Determine the displacement constants u_1 and u_2 due to two dimensional center of compression. [10]

(b) Show that in cylindrical coordinates the radial displacement due to a point load of magnitude $\rho g(t)$ acting in the axial direction may be expressed in the form

$$u_r = \frac{3zr}{4\pi R^5}\int_{R/C_L}^{R/C_T} s g(t-s)\,ds + \frac{zr}{4\pi R^3}\left[\frac{1}{C_L^2}g\left(t-\frac{R}{C_L}\right) - \frac{1}{C_T^2}g\left(t-\frac{R}{C_T}\right)\right].$$

where z and r are cylindrical coordinates and $R^2 = r^2 + z^2$. [10]

Q9(a) A thin layer of substance whose elastic constants are very small and can be neglected is adhered to the surface of an elastic half space. The mass density of the substance is ρ_s. A plane harmonic longitudinal wave is incident on the covered surface. Determine the influence of mass layer on the amplitude of the reflected waves. [10]

(b) Two semi-infinite elastic solids are in perfect contact. Draw a slowness diagram for the case $C_T^B < C_L^B < C_T < C_L$ and determine the critical angles for incident longitudinal waves. [10]