UNIVERSITY OF THE PUNJAB

Part-II : Supplementary Examination 2018

Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **MAX. TIME: 3 Hrs.**

PAPER: II (Methods of Mathematical Physics) **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

	Section-I	
1(a)	Define eigensolutions and eigenvalues of an S-L-system. Show that the S-L operator associated with a regular S-L-system is self-adjoint.	10
1(b)	Find the integral surface of the PDE $(y^2 - z^2)z_x - xyz_y = xz$ containing the curves $x = y = z,\ x > 0$.	10
2(a)	Show that the generating function for Legendre polynomials $P_n(x)$ is $(1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)t^n$.	10
2(b)	Find the general solution of the inhomogeneous linear second order PDE $(D^2 + 3DD' + 2D'^2)z = \sin(x + y)$.	10
3(a)	Show that eigenvalues and eigensolutions of the problem $\frac{d}{dx}\left(e^x \frac{dy}{dx}\right) + e^x\lambda^2 y = 0, \quad y(0) = 0, \quad y(a) = 0, \quad 0 < x < a,$ are ${\lambda_n}^2 = \left(\frac{n\pi}{a}\right)^2 + \frac{1}{4}$ and $y_n(x) = e^{-\pi/2} \sin\frac{n\pi x}{a}$.	10
3(b)	Solve the non-homogenous boundary value problem $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + x^2, \quad 0 \le x \le 1, \quad t > 0,$ $u(x,0) = f(x), \quad u_t(x,0) = 0, \quad 0 \le x \le 1,$ $u(0,t) = 0 = u(1,t), \quad t > 0.$	10
4(a)	Obtain solution of the boundary value problem defined by the following equations for a circular region with radius a $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 0 \le \rho \le a,\ 0 \le \varphi < 2\pi, \quad u(a,\varphi) = f(\varphi),$ where $f(\varphi)$ is an arbitrary function that is continuous inside and on the boundary of the circle of radius a.	10
4(b)	Use the method of Frobenius to obtain two linearly independent solutions about the regular singular point $x_0 = 0$ of the differential equation $x\frac{d^2y}{dx^2} + (x - 6)\frac{dy}{dx} - 3y = 0.$	10
5(a)	Define transient temperature distribution. Discuss the uniqueness of the solution of heat flow problem.	10

P.T.O.

5(b)	Find the value of $J_{3/2}(x)$.	**10**
	Section-II	
6(a)	A uniform cable is fixed at its ends at the same level in space and is allowed to hang under gravity. Find the final shape of the cable.	**10**
6(b)	Find the Green's function associated with the periodic S-L system $u'' + \lambda u = 0, \quad u(0) = u(1), \qquad u'(0) = u'(1).$	**10**
7(a)	Find the Fourier transform of $g(x) = \frac{1}{(x^2 + a^2)^2}.$	**10**
7(b)	Find the eigenvalues and eigenfunctions of the functional $I[y] = \int_0^3 [(2x+3)^2 y'^2 - y^2] dx$ subject to endpoint conditions $y(0) = 0 = y(3)$ and the side condition $\int_0^3 y^2(x) = 1.$	**10**
8(a)	Use Laplace transforms method to obtain a unique solution of the initial value problem $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \qquad 0 < x < 1, \quad 0 \le t < \infty,$ $u(0.t) = 1, \qquad u(1,t) = 1, \qquad t > 0,$ $u(x,0) = 1 - \sin \pi x, \qquad 0 < x < 1.$	**10**
8(b)	Write and explain the properties of Green's function associated with the regular S-L system.	**10**
9(a)	Verify that $L^{-1}\{\tan^{-1}(a/s)\} = \frac{1}{t}\sin at.$	**10**
9(b)	Define Dirac delta function. State and prove second shifting property of Laplace transform.	**10**

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Part-II : Supplementary Examination 2018

Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics

PAPER: IV-VI (Opt. i) [Mathematical Statistics]

MAX. TIME: 3 Hrs.

MAX. MARKS: 100

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

		SECTION-I	Marks
Q.1	(a)	(i) If A, B and C are three independent events in a sample space S. Show that $A \cup B$ and C are also independent. (ii) If A and B are any two events in a sample space S with $P(B) \neq 1$. Prove that, $$P(A/\bar{B}) = \frac{P(A) - P(A \cap B)}{1 - P(B)}$$	(10)
	(b)	In a college 4 % of boys and 1% of girls are wearing glasses. Also, 35% of students are girls. If a student is selected at random and is wearing glasses. What is the probability that the student is a girl?	(10)
Q.2	(a)	Find the Mean and Variance of the Hypergeometric distribution?	(10)
	(b)	In a college 40 % of students are boys and remaining are girls. If we select four students at random. a) What is the probability that fourth student is the third boy. b) What is the probability that there are three boys when we select five students at random.	(10)
Q.3	(a)	Prove that if Y and Z are independent Gamma variables, with parameters m and n respectively, then $\frac{Y}{Z}$ is a $\beta_2(m, n)$ variable.	(10)
	(b)	An urn contains 2 black, 3 red and 4 green balls. If 3 balls are selected at random with X is the number of black balls and Y is the number of red balls, find (i) the joint probability function $f(x, y)$ (ii) $P(X + Y > 1)$.	(10)
Q.4	(a)	Prove that the normal distribution is symmetrical. That is, mean, mode and median are equal.	(10)
	(b)	The completion time of writing a research paper is normally distributed with a mean of 47.6 weeks and a standard deviation of 16.2 weeks. a) What proportion of the research will take less than 50 weeks? b) What is the probability that the research time will be between 40 and 60 weeks? c) How many months will it take for 47% of the research to be completed?	(10)
		SECTION-II	
Q.5	(a)	If the multiple regression equation of X_1 on X_2 and X_3 is given by $x_1 = b_{12.3}x_2 + b_{13.2}x_3$, then show that $b_{13.2} = \frac{S_1(r_{13} - r_{12}r_{23})}{S_3(1 - r_{23}{}^2)}$ where r_{ij} are the linear correlation coefficients and S_k are the standard deviations.	(10)

P.T.O.

	(b)	Write the six partial regression coefficients and show that the partial correlation coefficients can be expressed as a geometric mean of some two of the above six partial regression coefficients. Justify your answer.	(10)
Q.6	**(a)**	If the joint probability density of X_1 and X_2 is given by $f(x_1, x_2) = \begin{cases} e^{-x_1 - x_2}, & for\ 0 < x_1, x_2 \\ 0 & elsewhere \end{cases}$ Using Distribution Technique, find the probability density function of $Z = \frac{X_1 + X_2}{2}$	(10)
	(b)	(i) Using The Moment Generating Function Technique, find the probability distribution finction for n independent Poisson random variates. (ii) In a Poisson distribution, $P(X = 2) = P(X = 3)$, find $P(X > 1)$	(10)
Q.7	**(a)**	The joint probability density function of random variables X and Y is given by $g(x, y) = \begin{cases} \alpha(x + 2y) & for\ \ 0 < x, y < 1 \\ 0 & elsewhere \end{cases}$ Find the followings: (i) the constant α (ii) $\mu_{Y/x}$ (iii) $\mu_{X/y}$.	(10)
	(b)	Find the probability distributions for each of the random variable X whose moment generating functions are given by: (i) $M(t) = (\frac{3}{7}e^t + \frac{4}{7})^8$ (ii) $M(t) = \frac{15e^t}{30 - 15e^t}$ (iii) $M(t) = e^{t(5+32t)}$	(10)
Q.8	**(a)**	Define Student's t statistic. Prove that all odd order moments about the mean of the t-distribution are zero.	(10)
	(b)	Find the mean and the mode of the F-distribution.	(10)
Q.9	**(a)**	Show that χ^2-distribution tends to normal distribution when the number of degrees of freedom is large.	(10)
	(b)	Discuss skewness and kurtosis for the χ^2-distribution.	(10)

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Part-II : Supplementary Examination 2018

Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: III (Numerical Analysis)

MAX. TIME: 3 Hrs.
MAX. MARKS: 100

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

Section I

Q1.

a) Define differential equation. Given that $\frac{dy}{dt} = t^2 + y^2, y(0) = 0, h = 0.5$, find $y(1.5)$ by Taylor's series algorithm of order 3.

b) Define forward difference operator. Prove that forward difference operator is a linear operator.

[14+06]

Q2.

Define approximate value. Apply Runge Kutta method of order four to find an approximation value of y when $x = 0.6$, given that

$$\frac{dy}{dx} = x + y^{0.5}, \qquad y(0.0) = 1.0, \qquad h = 0.2.$$

[20]

Q3.

a) Write an algorithm for Secant Method to find an approximate root of the non-linear equation $f(x) = 0$.

b) Define dominant eigenvalue and eigenvector. Compute the eigenvalue of matrix A corresponding to the given eigenvector $\boldsymbol{x}$:

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}, \ \boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

[10+10]

Q4.

a) Find a root of $2x^3 + 4x^2 - 2x - 5 = 0$, near to 1.0 correct to three decimal places by an iterative method.

b) Solve by Gauss-Seidel method, the following system of equations:

$$\begin{bmatrix} 28 & 4 & -1 \\ 1 & 3 & 10 \\ 2 & 17 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 32 \\ 24 \\ 35 \end{bmatrix}$$

[10+10]

P.T.O.

Q5.

a) Find a root of $xe^x - 3 = 0$ by Regula-Falsi method correct to three decimal places.

b) Solve by triangular factorization method the following linear system:

$$3x + 2y + z = 10$$
$$x + 4y - z = 6$$
$$x + 2y + 5z = 20$$

[10+10]

Section II

Q6.

Derive four points Newton divided difference interpolation formula. Find a polynomial of degree 3 or less, such that $f(0) = 1, f(1) = -1,\ f(4) = 1,$ $f(6) = -1.$

[20]

Q7.

a) Write an algorithm for Weddle's Rule to approximate the integral of $f(x)$ over the interval $[a, b]$ using n subintervals.

b) Interpolate the missing values in the following table:

x	-3	-2	-1	0	0.5	1	2	2.5	3
f(x)	-28	-9	-2	-1		0	7		26

[10+10]

Q8.

a) Apply seven points Weddle's Rule to evaluate $\int_0^{\pi} \frac{\sin x}{1+x} dx$

b) The population of a certain town is shown in the following table:

Year	1977	1987	1997	2007	2017
Population in Thousands	19.96	36.65	58.81	77.21	94.21

Find the rate of growth of the population in the year 2018.

[10+10]

Q9.

Define difference equation. Solve the following difference equations:

$$y_{k+2} + y_{k+1} + y_k = k^2 + k + 1$$
$$y_{t+2} - 9y_{t+1} + 20y_t = 4^t(t^2 + 1)$$

[10+10]

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Part-II : Supplementary Examination 2018

Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **MAX. TIME: 3 Hrs.**

PAPER: IV-VI (Opt. ii) [Computer Applications] **MAX. MARKS: 50**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

Section 1

1. [5+5 marks] What are the outputs of the following programs?

a)
```
Program First
Integer :: a, b=10
Real:: c=2.5, d=4.9
Do a = 1,3,1
c = c + mod(b,a)
b = d - c
End DO
Print*, b
End
```

b)
```
Program Second
Real:: N(3)
Integer:: k
Do k = 1,3
If(k<=2)
N(k)=k*k
Else
N(k)=0
Print*,N(k)
End If
End Do
End
```

2. (a) [5 marks] What are different forms of Do loop structure? Give examples.
   (b) [5 marks] What is the structure of select case statement? Give examples.

3. (a) [5 marks] Write a Fortran 90 program to swap values of two variables using subroutine.
   (b) [5 marks] Write a Fortran 90 program to find whether a point (x, y) lies in the first, second, third or fourth quadrant.

4. [10 marks] Write a Fortran 90 program to find roots of a quadratic equation using case statement.

Section 2

5. [10 marks] Write a Fortran 90 program to find Lagrange interpolating polynomial for the the following data $(0,2),(1,3),(2,6),(3,11)$.

6. [10 marks] Write a Fortran 90 program to implement two step Adams Bashforth (AB2) formula to solve the initial value problem $y' = \frac{x+2y}{y-x}$ $y(0) = 1$.

7. [10 marks] Write a Fortran 90 program to find the roots of $x^3 - e^x = 0$ using bisection method.

8. [10 marks] Write a Fortran 90 program to implement Simpson's $\frac{3}{8}$ formula to evaluate the integral

$$\int_1^2 \frac{1}{xln(x)}dx$$

9. Write the MATHEMATICA statements for the following.
   (a) [2 marks] Solve $x - y = 5$, $2x + y = 10$.
   (b) [2 marks] Draw graph of $e^x sin(x/2)$, $-3 \le x \le 3$. Draw frame and grid lines.
   (c) [2 marks] Numerically integrate $\int_2^3 sin(cos(y))dy$.
   (d) [2 marks] Find the highest power of y in the expression $y^3 - y^2 + y$.
   (e) [2 marks] Evaluate $\frac{\partial^2}{\partial x \partial y}(e^x - tan(y))$.

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Part-II : Supplementary Examination 2018

Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **MAX. TIME: 3 Hrs.**

PAPER: IV-VI (Opt. iii) [Group Theory] **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

SECTION 1

Q1.	a) Define Sylow p-subgroups of a finite group. Using Sylow's theorem check the simplicity of group of order 40.	10
	b) If A and B are two cyclic groups of order n and m, respectively then show that direct product of A and B is cyclic group of order nm if and only if gcd(n,m)=1.	10
Q2	a) By using Sylow Theory, prove that a group G of order pq is not simple. Also if in particular p > q and $p > q, \& p \not\equiv 1(mod\ q)$then show that $G \cong C_{pq}$.	10
	b) Prove that a p-sub group of a finite group G is contained in some Sylow p-sub group of G .	10
Q3	a) Define characteristic subgroup. Show that derived subgroup of a group is fully invariant. Give an example of normal subgroup which is not fully invariant.	10
	b) Let G be normal product of A and B. Then show that $A' = \{(a, e'): a \in A\}$ is normal subgroup of G and $\frac{G}{A'} \cong B$	10
Q4	c) Show that for $n \geq 5$, if a normal subgroup N of A_n contains a 3- cycle, then show that $N = A_n$.	10
	d) Define group action, G-set, orbit and stabilizer with examples. Also show that if x and y are in same orbits then stabilizer subgroups G_x and G_y are conjugate.	10

SECTION II

Q5	a) State and prove Zassenhaus Butterfly Lemma..	10
	b) Define chief series. What is the difference between chief series and composition series. Also give examples of normal series i. Which is composition but not chief series ii. Which is a chief series but not composition iii. Which cannot be refined to chief series	10

P.T.O.

Q6	a) Prove that a finite group G is solvable if and only if the factor groups in a composition series are cyclic of prime order. b) Discuss the solvability of S_n for all n by using derived series. c) Determine whether the group $G = < a, b : a^8 = 1, a^4 = b^2, bab = a^{-1} >$ is solvable or not?	8 8 4
Q7	a) Prove that every subgroup and factor group of nilpotent group is nilpotent. Does converse hold? Justify your answer. b) Prove that a finite group G is nilpotent if and only if all Sylow p- subgrops of G are normal c) Let $G = G_0 \triangleright G_1 \triangleright \ldots \triangleright G_k = E$ be a central series for G. Then Show that $G_i \supseteq \gamma_i(G), 0 \leq i \leq k$, where $\gamma_i(G)$ are terms of lower central series G, respectively.	8 8 4
Q8	a) Define partial complement of a subgroup and prove that a normal subgroup H of G is contained in Frattini subgroup of G if and only if H has no partial complement in G. b) If K is a normal subgroup and H is any subgroup of G such that $K \subseteq \phi(H)$ then show that $K \subseteq \phi(G)$.	10 10
Q9	a) Define special linear group and projective linear groups. Also prove that general linear group is not simple. b) Let G be an extension of a group N by H. Then define section of G through H $\{s(h) : h \in H\}$ and sectional factor set $\{f(h_1, h_2),\ h_1, h_2 \in H\}$. Also show that iii) For all h in H, $f(1, h) = 1 = f(h, 1)$ iv) For all $h_1, h_2, h_3 \in H$ $f(h_1, h_2) f(h_1 h_2, h_3) = f(h_2, h_3)^{s(h_1)} f(h_1, h_2 h_3)$	10 10

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Part-II : Supplementary Examination 2018

Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics
PAPER: (IV-VI) (Opt. iv) [Rings & Modules]

MAX. TIME: 3 Hrs.
MAX. MARKS: 100

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

Section I

Q. 1. a)Let R be an integral domain such that $R[x]$ is a principal ideal domain, then show that R is a field.

b)Let R be an integral domain, R is Unique Factorization Domain if and only if R is a factorization domain and every irreducible element is prime. 10+10

Q. 2. a) Let R be an integral domain, $p \in R\backslash\{0\}$. Then p is prime if and only if $R/_{pR}$ is an integral domain.

b) The polynomial $x - c$ is a factor of a polynomial$p(x) \in K[x]$if and only if $x = c$ is a root of $p(x) = 0$. 10+10

Q. 3.a) If R is an integral domain, show that

(i) s|tif and only if $tR \subseteq sR$

(ii) u is a unit of R if and only if $uR = R$

(iii) the set of all units of R is an abelian group.

b) Let F be a field. Then show that the polynomial ring $F[x]$ is a principal ideal domain. 10+10

Q. 4. a) Let K be a field, an element $a \in K$ is algebraic over F if and only if $[F(a): F]$ is finite. 10+10

b) Define extension of a field. Find the smallest extension of $\mathbb{Q}$ having a root of $x^3 - 2 \in \mathbb{Q}[x]$.

Q. 5. a) Polynomial $x^2 + 1$ over $\mathbb{R}$ and $\mathbb{R}$ is not splitting field for $x^2 + 1$over $\mathbb{Q}$. 10+10

b) Show that if L is a finite extension of F and K is a subfield of L which contains F, then $[K: F]$is a divisor of $[L: F]$.

Section II

Q. 6. a) Prove that an R- module which is finitely generated has a submodule which is not finitely generated. 10+10

b) Let A and B be submodules of an R- module M. Then prove that $^{(A+B)}/_{B} \cong {^A}/_{A \cap B}$.

Q. 7. a) Every FG – R-module is homomorphic image of a free module. 10+10

b) Let M be a module over an integral domain R and let T denote the set of torsion elements of M. Show that $T$is a submoduleof M, the quotient module $^M/_T$ is torsion free.

Q.8a)Show that in $\mathbb{Z}[\sqrt{-5}]$ the elements 6 and $2(1 + i\sqrt{5})$ do not have a gcd. 10+10

b) Prove if R –module $M = M_1 \oplus M_2$ and M can be generated by "s" elements. Then M_1 can be generated by "s" elements.

Q. 9 Let M be a module over PIDR and suppose that M is freely generated by a finite set of n elements. Prove that every basis of M contains exactly n elements. 20

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Part-II : Supplementary Examination 2018

Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics — **MAX. TIME: 3 Hrs.**

PAPER: IV-VI (Opt. v) [Number Theory] — **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

SECTION-I

Marks

Q.1 **(a)** Let a and b be any two integers at least one of which is non-zero, then there exist integers x,y such that $gcd(a,b)=ax+by$. (10)

(b) State and prove the Division Algorithm for integers. (10)

Q.2 **(a)** Let m be a positive integer, a and b any integers. Prove that the linear congruence $ax \equiv b \pmod{m}$ is solvable if and only if $d/b,\ d=(a,m)$. In the solvable case, prove that the given congruence has d mutually incongruent solutions. (7+7)

(b) By means of Hensel's lemma, solve the polynomial congruences $x^2+x+7\equiv 0 \pmod{27}$. (6)

Q.3 (a) Define Complete Residue system. If k integers $a_1, a_2, \ldots, a_k$ form a CRS *(mod m)* then show that $k=m$. (7)

(b) (i) There are an infinite number of primes, prove or disprove. (5+8)

(ii) Show that $\tau(n)$ is odd if and only if n is a perfect square.

Q.4 **(a)** State Chines Remainder Theorem and hence apply to solve the following system of linear congruences (3+7)

$$x \equiv 3 \pmod{5}$$
$$x \equiv 1 \pmod{7}$$
$$x \equiv 5 \pmod{11}$$

(b) (i) For each integer $m \geq 0$, show that $F_{m+1} = F_0F_1F_2\ldots F_m + 2$, where F_n is a Fermat number. Deduce that $(F_m, F_n)=1,\ m\neq n.$ (5+5)

(ii) Let $(a,57)=1$. Prove that $a^{18}\equiv 1 \pmod{57}$.

P.T.O.

Q.5 (a) Discuss all integers which can (or cannot) serve as a primitive root of $2^n, n \geq 1$. (10)

(b) Let p be an odd prime and a be a primitive root modulo p^2, then prove that a is a a primitive root modulo $p^k, k \geq 2$. (10)

SECTION II

Q.6 (a) Let p be an odd prime and a an integer co-prime to p. Let m denote the number of integers that leave negative least residues in the set $\{a, 2a, ..., \frac{p-1}{2}a\}$ then prove that $\left(\frac{a}{p}\right) = (-1)^m$. (10)

(b) Show that $x^2 \equiv a \pmod{2^n}$ has a solution if and only if $n \equiv 1 \pmod 8$. (10)

Q.7 (a) Let θ be a real algebraic number of degree $n > 1$, over R. Then there is a positive number M such that $|\vartheta - \frac{p}{q}| \geq \frac{M}{q^n}$ for all rational numbers $\frac{p}{q}, q > 0$. Prove or Disprove. (10)

(b) Show that an irreducible polynomial of degree n over F has n distinct roots. (10)

Q.8 (a) Differentiate between transcendental and algebraic numbers. Determine whether the number $\sqrt{\sqrt{7} - 3}$ is algebraic? If so, find its defining polynomial. (4+6)

(b) Prove that every algebraic number has a unique minimal polynomial. Further, if θ is algebraic of degree n, then any $\alpha \in F[\vartheta]$ can be expressed uniquely as $\alpha = \sum_{i=0}^{n-1} a_i \vartheta^i$, $a_i \in F$. (5+5)

Q.9 (a) Show that an element $\alpha \in R(\theta)$ is a unit if and only if its norm is ± 1. (10)

(b) Define discriminant for Cyclotomic polynomial, and verify it for the Cyclotomic field generated by fifth roots of unity. (10)

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Part-II : Supplementary Examination 2018

Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **MAX. TIME: 3 Hrs.**

PAPER: IV-VI (opt.vi) [Fluid Mechanics] **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

Section I

Q.1(a)	What is a fluid? Why is fluid considered as a continuum? Explain.	(10)
(b)	A flat plate having dimensions of $2m \times 2m$ slides down an inclined plane at an angle of one radian to the horizontal at a speed of $6\ m/s$. The inclined plane is lubricated by a thin film of oil having a viscosity of $30 \times 10^{-3}\ Pa.s$. The plate has a uniform thickness of 20 mm and a density of $40{,}000\ kg/m^3$. Determine the thickness of lubricating oil film.	(10)
Q.2(a)	What are the general methods for describing the fluid motion and explain the method which is commonly used in fluid mechanics.	(10)
(b)	If every particle of fluid moves on the surface of a sphere, prove that the equation of continuity is $\frac{\partial \rho}{\partial t} \cos\theta' + \frac{\partial(\rho\omega' \cos\theta')}{\partial \theta'} + \frac{\partial(\rho\omega\cos\theta')}{\partial\varphi} = 0$, ρ being the density, θ', φ the latitude and longitude of any element, and ω', ω the angular velocities of the element in latitude and longitude respectively.	(10)
Q.3(a)	State and prove Kelvin's theorem on the constancy of circulation.	(10)
(b)	The velocity components for certain flow fields is $u = -y/(x^2+y^2)$, $v = x/(x^2+y^2)$. Is the flow irrotational? Calculate the volumetric flow rate through the square with corners at (3,0,2), (3,0,1), (3,1,2), (3,1,1).	(10)
Q.4(a)	Derive the Bernoulli's equation for unsteady, irrotational and inviscid flow under conservative forces.	(10)
(b)	Find the velocity potential, stream function, complex velocity, flownet for a uniform flow.	(10)
Q. 5	For the velocity components of a certain fluid $$v_r = -\left(1-\frac{a^2}{r^2}\right)\cos\theta, \qquad v_\theta = \left(1+\frac{a^2}{r^2}\right)\sin\theta,$$ Find (i) The complex velocity $\frac{dw}{dz}$ (ii) The speed V and the complex velocity potential $w(z)$ (iii) The velocity potential φ and stream function ψ (iv) The equipotential lines and streamlines.	(20)

P.T.O.

Section II

Q.6(a)	If a cylinder of an aerofoil shape is placed in a uniform stream of speed U, with circulation Γ around the cylinder, then the lift per unit length of the cylinder is of magnitude $\rho U\Gamma$ in the direction perpendicular to the direction of the stream.	(10)
(b)	Show that the superposition of a uniform stream over a doublet represents the streaming flow past a circular cylinder of radius a.	(10)
Q. 7	Derive the Navior Stokes Equations and write it in Cylindrical coordinates system.	(20)
Q.8(a)	What is simple Couette flow? Modelling the same flow, evaluate the velocity field, Maximum velocity, volumetric flow rate and shearing stress.	(10)
(b)	Discuss the pulsating flow of viscous fluid between two parallel plates when both the plates are rest and $\frac{\partial p}{\partial x} = P_x \cos \omega t$. Find the expression for the velocity field.	(10)
Q.9(a)	State and prove Blausius Theorem	(10)
(b)	Determine whether the following velocity field are steady, 2-dim , 3dim, and find the associated shear stress with $p = 185$, $\mu = 0.273$ and t=5 . a) $\vec{V} = ax\hat{i} - by\hat{j} + (t - cz)\hat{k}$ b) $\vec{V} = \frac{x}{y^2}\hat{i} + t(\frac{1}{y} - 1)\hat{j}$	(10)

UNIVERSITY OF THE PUNJAB

Part-II : Supplementary Examination 2018

Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **MAX. TIME: 3 Hrs.**

PAPER: IV-VI (Opt.viii)[Special Theory of Relativity and Analytical Dynamics] **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

SECTION I

1. (a) Using the result that the velocity four-vector transforms like a four-vector, find the equations of Lorentz transformation for the veloc ty components $\mathbf{u} = (u_x, u_y, u_z)$. (10 marks)

   (b) What should the wavelength of light be so that the scattered light has double the wavelength of the original light when scattered through angles of
   (i) $\pi/6$ (ii) $\pi/3$ (iii) $5\pi/6$. (10 marks)

2. (a) Write a comprehensive note on the null cone structure. (10 marks)

   (b) An observer sees a clock as showing 1 hour to be half an hour. If he sees an object lying at an angle of $\pi/4$ as having a length of 2 m, what is the rest length of the object. (10 marks)

3. (a) Prove one dimensional Lorentz transformation. What are the corresponding expressions for time dilation and length contraction. (10 marks)

   (b) Show that $ds^2 = g_{\alpha\beta}dx^{\alpha}dx^{\beta}$ is invariant under Lorentz transformation. (10 marks)

4. (a) What do you understand by Doppler effect in light? Work out the expression for the transverse and longitudinal Doppler effect in light. Also, find the expression in case of no Doppler shift. (10 marks)

   (b) Find the components of Maxwell field tensor and use it to show that

$$F^{\mu\nu}F_{\mu\nu} = -2\mu_0\left(\epsilon_0\mathbf{E}.\mathbf{E} - \frac{\mathbf{B}.\mathbf{B}}{\mu_0}\right).$$

(10 marks)

5. (a) Show that $F^{\alpha\beta}_{\ \ ,\beta} = \mu_0 J^{\alpha}$, $F_{[\alpha\beta,\delta]} = 0$ represents Maxwell's equations in four-vector formalism. (10 marks)

   (b) Prove that the relativistic resultant of three co-linear speeds u, v, w is given by

$$\frac{u + v + w + \frac{uvw}{c^2}}{1 + (uv + vw + wu)/c^2}.$$

   What will be the formula for n co-linear speeds when n is even and when n is odd. (10 marks)

SECTION II

6. (a) State and explain D'Alembert's principle. (10 marks)

   (b) A mass M_2 hangs at one end of a string which passes over a fixed frictionless non-rotating pulley. At the other end of this string there is a non-rotating pulley of mass M_1 over which there is a string carrying masses m_1 and m_2. Set up the Lagrangian of the system. Also, work out its equations of motion. (10 marks)

7. (a) State and explain the Euler-Lagrange differential equation in the calculus of variation. (10 marks)

   (b) A solid sphere of mass m and radius b rests on top of another foxed sphere of radius a. The upper sphere is slightly displaced and it begins to roll down without slipping. By Lagrange's method of undetermined multipliers, find the normal reaction on the upper sphere and the frictional force at the point of contact. (10 marks)

8. (a) Explain the Hamilton's principle of least action, bringing out clearly the type of variation involved. (10 marks)

   (b) Show that the transformation $p = m\omega q\cot Q$ and $P = \frac{m\omega q^2}{2\sin^2 Q}$ is canonical, and obtain the generator of the transformation. (10 marks)

9. (a) Work out the Hamiltonian for the one dimensional harmonic oscillator of mass m. Also, write down the corresponding Hamilton-Jacobi equation. (10 marks)

   (b) Find the equations of motion of a system with the following Lagrangian (10 marks)

$$L(x, \dot{x}, t) = e^{-x^2}[e^{-\dot{x}^2} + 2\dot{x}\int_0^{\infty} e^{-\alpha^2}d\alpha].$$

UNIVERSITY OF THE PUNJAB

Part-II : Supplementary Examination 2018

Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **MAX. TIME: 3 Hrs.**

PAPER: IV-VI (opt. ix) [Electromagnetic Theory] **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

SECTION I

1. (a) Prove that the divergence of vector potential $\vec{A}$ is zero for electrostatic field. (10 marks)

   (b) A solenoid has length 1.23 m and inner diameter 3.55 cm. It has 5 layers of windings and having 850 turns each carrying current $I_0 = 5.57$ Amp. Find the magnetic induction at the center. (10 marks)

2. (a) Work out the the electric field due to a uniform spherical charge distribution at an internal and external point. (10 marks)

   (b) A infinite line charge produces a field of $4.52 \times 10^4 N/C$ at a distance of 1.96 m. Calculate the linear charge density. (10 marks)

3. (a) Calculate the potential in the presence of conducting sphere in a uniform electric field. (10 marks)

   (b) Work out the Poisson equation for the vector potential $\vec{A}$. (10 marks)

4. (a) Define self-inductance and mutual inductance. Work out the self-inductance of a toroid. (10 marks)

   (b) Work out the relationship between capacitance and resistance, i.e., $C = \frac{\rho\epsilon}{R}$. (10 marks)

5. (a) Work out the electromotance induced in a loop by a pair of long parallel wires carrying a variable current. (10 marks)

   (b) What must be the magnitude of uniform electric field if it is to have the same energy density as that possessed by a 0.5 T magnetic field. (10 marks)

SECTION II

6. (a) Define magnetic susceptibility, permeability and relative permeability. Show that the electric and magnetic field vectors satisfy the wave equation in free space. (10 marks)

   (b) Discuss the propagation of plane electromagnetic waves in non-conductors. (10 marks)

7. (a) Show that the flux of poynting vector through any closed surface gives the energy flow due to a plane wave through an imaginary cylinder. (10 marks)

   (b) Discuss the Lienard-Wiechert potentials for a moving charge. (10 marks)

8. (a) Discuss the propagation of electromagnetic wave in a hollow conducting waveguide. (10 marks)

   (b) Work out the ratio's of the amplitudes of the incident, reflected and transmitted waves for the case when the incident wave is polarized with its $\vec{E}$ vector parallel to the plane of incident. (10 marks)

9. (a) Show that there must be conservation of energy by formulating the coefficient of reflection and transmission at an interface when the incident wave is polarized with its $\vec{E}$ vector normal to the plane of incident. (10 marks)

   (b) The earth receives about $1300 watts/m^2$ radiant energy from the sun. Assuming the energy in the form of plane monochromatic wave, and also assuming normal incidence, compute magnitudes of electric and magnetic fields vectors in the sun light. (10 marks)

UNIVERSITY OF THE PUNJAB

Part-II : Supplementary Examination 2018

Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics **MAX. TIME: 3 Hrs.**

PAPER: IV-VI (opt.xi) [Theory of Approximation & Splines] **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each section.

Note: Attempt any five questions. Selecting at least two questions from each section.

Section I

Q1. (a) Show that the set of Euclidean transformations of $\mathbb{R}^2$ forms a group under the operation of composition of functions. (10)

(b) Determine the image of circle $x^2 + y^2 = 9$ after shearing along x-axis by factor 2. (10)

Q2. (a) Find an affine transformation which maps the points $(1,2)$, $(2,1)$ and $(3,5)$ onto the points $(2,1)$, $(1,5)$ and $(0,6)$, respectively. (10)

(b) Determine the image of $\frac{x^2}{4} - \frac{y^2}{9} = 1$, after scaling by factor 4. (10)

Q3. (a) Consider the data points $\{(x_k, y_k)\}_{k=1}^{N}$ for the distinct values $\{x_k\}_{k=1}^{N}$ that are approximated by the least-squares power curve $y = f(x) = Ax^M$, where M is constant. Then show that A can be obtained as $A = \left(\sum_{k=1}^{N} x_k^M y_K\right) / \left(\sum_{k=1}^{N} x_k^{2M}\right)$. (10)

(b) Find the curve fit $y = 1/(Ax + B)$ using the change of variables $X = x,\ Y = 1/y$ linearize the data points. (10)

x_k	-1	0	1	2	3
y_k	6.62	3.94	2.17	1.35	0.89
$1/y_k$	0.1511	0.2538	0.4608	0.7407	1.1236

Q4. (a) Find the Chebyshev polynomial $P_3(x)$ that approximates the function $f(x) = e^x$ over $[-1, 1]$. (10)

(b) Find the Pade approximation $R_{1,1}(x)$ for $f(x) = ln(1+x)/x$ and using this result establish the approximation $ln(1+x) \approx R_{2,1}(x) = \frac{6x+x^2}{6+4x}$. (10)

P.T.O.

Section II

Q5. (a) Discuss the subdivision algorithm to compute the following B. B. curve $\underline{P}(\theta) = \sum_{i=0}^{n} B_i^n(\theta)\underline{b}_i, \quad 0 \leq \theta \leq 1$ **(10)**

For $\alpha,\ \beta \in [0,\ 1]$, write the relation that defines the new control points of segment of B. B. curve corresponding to $\alpha \leq \theta \leq \beta$.

Derive the relation for new control points for

(i) $\alpha = 0$,

(ii) $\beta = 1$

as well.

(b) Determine the function $\underline{f} : [0,\ 2] \to I\!R^2$

$s.t.\ \underline{f}(0) = (5,2),\ \underline{f}(1) = (6,7),\ \underline{f}(2) = (8,15)$. **(10)**

Moreover, $\underline{f}$ is linear over $[0,\ 1]$ and is quadratic over $[1,\ 2]$.

Also $\underline{f}$ and $\underline{f}'$ are continuous.

Q6. (a) Show that **(10)**

$B_i^j(\theta) = (1-\theta)B_i^{j-1}(\theta) + \theta B_{i-1}^{j-1}(\theta),\ i = 0,1,2,\ldots,j.$

(b) Discuss the Variation diminishing property of B.B form of degree 3. **(10)**

Q7. (a) Show that rational quadratic (vector form) represents conic sections. **(10)**

(b) Find the natural cubic spline that passes through $(0,\ 0)$, $(1,\ 0.5)$ and $(2,\ 2.0)$ with the boundary conditions : **(10)**

$S''(0) = 0$ and $S''(2) = 0$.

Q8. (a) Determine the quadratic B-Spline function. **(10)**

(b) Determine $N_0^4(t)$ using $N_0^3(t)$ defined as **(10)**

$$N_0^3(t) = \begin{cases} t^2/2, & t \in\ ]0,\ 1], \\ -t^2 + 3t - 3/2, & t \in\ ]1,\ 2], \\ t^2/2 - 3t + 9/2, & t \in\ ]2,\ 3] \\ 0, & Otherwise. \end{cases}$$

Q9. (a) Discuss the degree reduction algorithm for B.B general form of degree n. Calculate the new control points of B.B. cubic form in terms of control points of B.B. quartic form. **(10)**

(b) Express the following quadratic spline as truncated power function representation **(10)**

$$S(x) = \begin{cases} 3x^2 + 2, & x \in [-2,\ -1[\\ -6x - 1, & x \in [-1,\ 1[\\ 6x^2 - 18x + 5, & x \in [1,\ 2[\\ 3x^2 - 6x - 7, & x \in [2,\ 3] \end{cases}$$