UNIVERSITY OF THE PUNJAB

M.A./M.Sc. Part – I Annual Exam – 2019

Roll No. ....

Subject: Physics **Paper: I (Mathematical Methods of Physics)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt FIVE questions in all, selecting at least TWO questions from each section.

SECTION-I

Q1: Discuss cylindrical polar coordinate system and derive expressions for scale factors of this system.[20]

Q2 (a): Show that the rotation of the coordinate axes through an angle θ counterclockwise about the x_3- axis is a proper orthogonal transformation.[10]

Q2 (b): Define addition and subtraction of a Cartesian tensor. Elaborate your answer by giving two examples of each.[5 + 5]

Q3(a): Find the Fourier series of $f(x)$ given by if: $f(x) = \begin{matrix} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{matrix}$ [10]

Q3(b) Find the Fourier transform of the function defined as: $f(t) = \begin{matrix} 1; & -a \le t \le a \\ 0, & otherwise \end{matrix}$ [10]

Q4(a): State and prove Cauchy's Residue theorem. Q4(b): Evaluate $\int_0^{2\pi} \frac{4d\theta}{5+4sin\theta}$. [10 +10]

Q5.Show that the Cauchy Riemann equations are satisfied for the function $(z) = \exp[-z^{-4}]$, [20]

P.T.O.

SECTION-II

Q6(a):Prove that the Eigen values of Strum-Liouville's problem are real.[10]

Q6(b): Find the Eigen functions and the Eigen values of Strum-Liouville's problem

$y'' + \lambda y = 0$ with boundary conditions $y(0)' = y(c)' = 0$.[10]

Q7(a): For the Bessel's functions, show that: $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$[10]

Q7(b): Prove that $J_n'(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$[10]

Q8(a): Prove that the polynomials (i) $P_1(x) = x$ (ii) $P_2(x) = \frac{1}{2}(3x^2 - 1)$ (iii) $P_3(x) = \frac{1}{2}(5x^3 - 3x)$ (iv) $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$ (v) $P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$ are the solutions of Legendre's differential equation $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ [10].

Q8(b): Sketch the graphs of the Legendre's polynomials (i) $P_0(x) = 1$ (ii) $P_1(x) = x$

(iii) $P_2(x) = \frac{1}{2}(3x^2 - 1)$ (iv) $P_3(x) = \frac{1}{2}(5x^3 - 3x)$ (v) $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$ [10]

Q9(a):Express $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ in spherical polar coordinates.[10]

Q9(b): If there is any common region in which $W_1 = u(x,y) + iv(x,y)$ and $W_2 = u(x,y) - iv(x,y)$ are both analytic, prove that $u(x,y)$ and $v(x,y)$ are constants. [10]

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Roll No. ....

Subject: Physics **Paper: II (Classical Mechanics)** **Time: 3 Hrs. Marks: 50**

NOTE: Attempt FOUR questions, selecting at least ONE questions from each section. All Questions carry equal marks.

SECTION-I

1. (a) An ant moves freely on the surface of a sphere. Write equation of constraint and find most convenient generalized coordinates.

   (b) A bead of mass m slides freely on a frictionless circular wire of radius r that rotates in a horizontal plane about a point on the circular wire with constant angular velocity ω. Derive the Lagrange equation of motion.

   (c) Define holonomic and non-holonomic constraints. Give at least two examples of each case, to illustrate your definition. Also distinguish between rheonomic and scleronomic constraints by giving suitable examples.(4+4+4.5)

2. (a) Show explicitly that

$$\frac{\partial \mathbf{x}}{\partial q_i} = \frac{\partial \dot{\mathbf{x}}}{\partial \dot{q}_i}$$

   where $\mathbf{x} = \mathbf{x}(q_1, q_2, \cdots, q_n)$.

   (b) State D' Alembert's principle and use it to derive

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0.$$

   (c) (a) Obtain the Lagrange equation of the second kind

$$\frac{\partial L}{\partial t} - \frac{d}{dt}\left(L - \dot{q}_i \frac{\partial L}{\partial \dot{q}_i}\right) = 0$$

   Also derive Beltrami's identity. (4+4+4.5)

3. (a) Consider the motion of a particle in a central force field

$$V(r) = -\frac{k}{r}.$$

   Write down the Lagrangian in polar coordinates and integrate the equation of motion to derive

$$\theta(r) = \frac{l\, dr}{r^2\sqrt{2\mu\left(E + \frac{k}{r} - \frac{l^2}{2\mu\, r^2}\right)}} + \text{constant},$$

   where E is the total energy and l is the angular momentum. Now change variables as $u = \frac{l}{r}$ to derive the equation of a conic section

$$\frac{\alpha}{r} = 1 + \varepsilon \cos\theta.$$

   (b) Use the above expressions to derive Kepler's third law of planetry motion. (7.5+5).

4. (a) A bead of mass m slides freely on a smooth circular wire of radius b that rotates in horizontal plan about a point on the circle with a constant angular velocity ω. Determine the Lagrangian function and Lagrange's equations of motion.
   (b) Also find the reaction of the wire.
   (c) Obtain the equation of motion for a body of mass m moving under central force with Lagrangian

$$L = \frac{1}{2}m(\dot{r}^2 + r\dot{\theta}) - V(r)$$

   also calculate the generalized momentum corresponding to the cyclic coordinate and discuss the result. (4+4+4.5)

P.T.O.

5. (a) A mass point glides without friction on a cycloid which is given by

$$x = a(\theta - sin\theta)$$

$$y = a(1 + cos\theta)$$

Obtain the Lagrangian, generalized momentum and equation of motion

(b) Show that the motion is just like the vibration of an ordinary pendulum of length $l = 4a$.

(c) Obtain the Lagrange equation of motion for a system of two particles of unequal masses connected by an inextensible string passing over a small smooth pulley. (Atwood's machine). (4+4.5+4)

SECTION-II

6. (a) The Lagrangian for a particle of mass m and charge e moving in the general electromagnetic field $(\mathbf{E}, \mathbf{B})$ is given by

$$L(\mathbf{x}, \dot{\mathbf{x}}, t) = \frac{1}{2} m\dot{\mathbf{x}}.\dot{\mathbf{x}} - e\phi + e\dot{\mathbf{x}}.\mathbf{A} \ ,$$

where $\mathbf{x} = (x, y, z)$ and $(\phi, \mathbf{A})$ are the electromagnetic potentials of fields $(\mathbf{E}, \mathbf{B})$. Show that the corresponding Hamiltonian is given by

$$H(\mathbf{x}, \mathbf{p}, t) = \frac{(\mathbf{p} - e\mathbf{A}).(\mathbf{p} - e\mathbf{A})}{2m} + e\phi \ ,$$

where $\mathbf{p} = (p_x, p_y, p_z)$.

(b) Consider a one parameter family of transformations

$$q_i(t) \to Q_i(s, t) \qquad s \in \mathbb{R}$$

such that $Q_i(0, t) = q_i(t)$. Show that if the Lagrangian is invariant under this transformation, then there exists a conserved quantity.

(c) Show that if the Lagrangian of a closed system is invariant under rotations, then the total angular momentum of the system is a constant vector in time (isotropy of space). (4+4.5+4) .

7. (a) State Hamilton's principle of least action and use it to derive Hamilton's equations of a dynamical system.

(b) Show that the Hamiltonian for a simple harmonic oscillator

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2$$

can be written in the form

$$H = \omega a^* a,$$

where

$$a = \sqrt{\frac{m\omega}{2}} \left(x + \frac{ip}{m\omega} \right),$$
$$a^* = \sqrt{\frac{m\omega}{2}} \left(x - \frac{ip}{m\omega} \right).$$

Show that

$$\{a, a^*\} = -i, \quad \{a, H\} = -i\omega a \ \text{ and } \ \{a^*, H\} = i\omega a^*$$

(c) Show that the path followed by a particle in sliding from one point to another in the absence of friction in the shotest time is a cycloid. (4+4+4.5)

8. (a) Show that the total time rate of change of any dynamical variable $A(q_i, p_i, t)$ is given by

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + \{A, H\}$$

(b) Define canonical transformations and derive the expression of a general generationg function of these transformations. Classify canonical transformation with repect to four types of different generating functions. Derive expressions of four types of canonical transformations.

(3+9.5)

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M.A./M.Sc. Part – I Annual Exam – 2019

Subject: Physics **Paper: III (Quantum Mechanics)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt FIVE questions, selecting at least ONE questions from each section. All Questions carry equal marks.

Section I

1. (a) State any three postulates of Quantum mechanics
   (b) What are Hermitian operators. Show that the eigenfunctions of same Hermitian operators belonging to distinct eigenvalues are always orthogonal.

(9+3+8)

2. State and prove Ehrenfest equations.

(20)

3. Consider the states $|\Psi\rangle = 9i\,|\phi_1\rangle + 2\,|\phi_2\rangle$ and $|\chi\rangle = -\frac{i}{\sqrt{2}}\,|\phi_1\rangle + \frac{1}{\sqrt{2}}\,|\phi_2\rangle$, where $|\phi_1\rangle$ and $|\phi_2\rangle$ are orthonormal. Calculate

   (a) $|\Psi\rangle\langle\chi|$ and $|\chi\rangle\langle\Psi|$. Are they equal?
   (b) $|\Psi\rangle\langle\Psi|$ and $|\chi\rangle\langle\chi|$
   (c) Find the scalar product $\langle\Psi|\,\chi\rangle$ and $\langle\chi|\,\Psi\rangle$
   (d) Are $|\Psi\rangle\langle\Psi|$ and $|\chi\rangle\langle\chi|$ projection operators.

(5+5+5+5)

Section II

4. Solve time independent Schrodinger wave equation for a potential well for the case $E > V_0$,where

$$V(x) = \begin{cases} V_0, & \text{for } -\infty < x < 0 \\ 0, & \text{for } 0 < x < a \\ V_0, & \text{for } a < x < \infty \end{cases}$$

Find the values of R and T(reflection and transmission coefficients)

(20)

5. (a) Define angular momentum in Quantum Mechanics. Write down the expressions for $\hat{L}_x, \hat{L}_y, \hat{L}_z$ and $\hat{L}^2$. Derive their expressions in spherical polar coordinates.
   (b) Obtain the eigenvalue spectrum of $\hat{L}^2$.

(10+10)

6. (a) What are identical particles. What is the difference between fermions and bosons. Write down the expressions for the wavefunctions of three identical fermions and three identical bosons.
   (b) Define exchange operator. Show that exchange operator commutes with Hamiltonian.

(10+10)

Section III

7. What is perturbation theory? Obtain the first order correction in energy and wavefunction of a system using time independant perturbation (nondegenerate case)

(20)

8. What is variational method. Obtain the ground state energy of simple harmonic oscillator by using variational method. (Use: $\Psi = Ae^{-\alpha x}$)

(20)

9. (a) Define Dirac/interaction picture. Derive equation of motion for the wavefunction and operators in Interaction picture.
   (b) Use generalized uncertainty relation to obtain the uncertainty between $\hat{L}^2$ and $\hat{L}_z$.

(12+8)

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Roll No. ....

Subject: Physics **Paper: IV (Solid State Physics-1)** **Time: 3 Hrs.** **Marks: 50**

NOTE: Attempt FOUR questions, selecting at least ONE question from each section.
All Questions carry equal marks.

Section 1

Q.1 (a) Define the packing fraction of a crystal, find its value for hcp structure. (8.5)

(b). Explain the Miller Indices of planes, sketch planes $(2\bar{1}0)$, $(\bar{2}10)$ in the unit cell of cubic crystal. (4)

Q.2. (a) Explain the crystal structure of Diamond in detail. (6.5)

(b) Show that the volume of first Brillouin zone is $(2\pi/V)^3$ where V is the volume of crystal primitive cell. (6).

Q.3 (a) Derive the Bragg law in reciprocal lattice and then explain the Ewald construction in the reciprocal lattice. (8)

(b) Find the reciprocal lattice and first Brillion zone of the simple cubic lattice. (4.5)

Q.4. Considering solid as a continuous medium, derive the dispersion relation for the longitudinal elastic waves in [110] direction in cubic crystal. (12.5)

P.T.O.

Section 11

Q.5. Derive the dispersion relation for acoustic phonons and sketch it in the first Brillion Zone. Also explain and sketch this acoustic mode. (12.5)

Q.6. (a) Derive the expression for the heat capacity of solid in the Einstein model. (9.5)

(b) Define and differentiate between the Edge and Screw dislocation. (3)

Q.7. Write short notes on the following. (6.5, 6)

i) Einstein model of heat capacity in solids

ii) Ficks law of diffusion in solids

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Subject: Physics **PAPER: V (Electronics)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt FIVE questions, selecting at least ONE question from each section.

Q. No.	Questions	Marks
	Section I	
Q.1	**(a)** Draw the circuit of full wave rectifier with center tapped transformer and explain its operation. Derive the expression for V_{dc} in it.	**10**
	(b) A full wave rectifier circuit with capacitor filter, operating from a 50 Hz line, supplies a load with 100 mA and 25 V dc at full load. What value of C is needed to limit the ripple factor to 0.01(1%)?	**10**
Q.2	**(a)** Discuss the performance of a common collector transistor amplifier in terms of A_{VC}. A_{iC}, R_{ic} and R_{oC}	**10**
	(b) Determine A_{VC}, A_{iC}, R_{iC} and R_{oc} for a load of 2 KΩ. in a C-C circuit with a transistor having h_{ie} = 900 Ω, h_{fe} = 25, h_{re} = negligible and h_{oe} = 16 x 10^{-6} mho.	**10**
Q.3	**(a)** Draw the voltage feedback bias circuit and find the relation for collector current and stabilizing ratio.	**10**
	(b) Design voltage feedback bias circuit and find out the value of stabilizing ratio when I_C = 3 mA, V_{BE} = 0.5V, h_{fe} = 75, V_{CC} = 15V, V_{CE} = 0.5V_{CC} and V_E = 0.1V_{CC}.	**10**
	Section II	
Q.4	**(a)** Discuss the low frequency response of RC coupled amplifier and find the expression for $A_{v(low)}$.	**10**
	(b) Derive the following relation for C E circuit having unbypassed emitter Resistance; $A'_v = A_{v(mid)} / (1 + g_m R_E)$	**8**
	(c) What is Miller Effect?	**2**

P.T.O.

Q.5	**(a)** Explain the construction, draw the symbol and describe the operation of n-channel MOSFET in depletion mode with the help of its characteristics.	**10**
	(b) Describe the construction of JFET and explain how pinch-off is obtained in an n-channel JFET.	**8**
	(c) Why do we limit FET operation to small signals?	**2**
Q.6	**(a)** How the bandwidth of an RC coupled amplifier is modified by the use of negative feedback?	**10**
	(b) An amplifier has $A_{Vmid)} = 200$ and $f_2 = 50$ KHz. If negative feedback is added with $\beta = 0.10$, what will be the mid frequency gain? What will be the high frequency band limit?	**8**
	(c) Which form of feedback increases the output resistance of an amplifier?	**2**
	Section III	
Q.7	**(a)** Draw the circuit and explain the working of practical Hartley oscillator.	**10**
	(b) In a Transistor Colpitts oscillator $C_1 = C_2 = 310$ pF and $L = 37$ µH. Find the frequency of oscillation.	**6**
	(c) What is the purpose of RFC and C_E in practical Colpitts and Hartley oscillators?	**4**
Q.8	**(a)** What are the types of power amplifiers in terms of operating conditions?	**4**
	(b) Explain class A power amplifier and find out expression for its efficiency.	**10**
	(c) An amplifier has only second harmonic distortion. Find out the second harmonic distortion D_2, when $i_{max} = 250$ mA, $i_{min} = 5$ mA and $I_C = 100$ mA.	**6**
Q.9	Write a note on any **two** of the following.	
	(i) The Differential Amplifier.	**10**
	(ii) Astable multivibrator.	**10**
	(iii) Logic gates.	