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M.A./M.Sc. Part – II Supply – 2020 & Annual – 2021

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Subject: Statistics   Paper: I (Statistical Inference)   Time: 3 Hrs.   Marks: 100

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q.1 a) What happens to an estimate if an estimator is biased? **(06)**

b) Let $X_1, X_2, \ldots X_n$ be a random sample of size n from the Bernoulli distribution, and if $T=\sum_{i=1}^{n} X_i$,show that i) T/n is an unbiased estimator of θ **(12)**

ii) $\frac{T(T-1)}{n(n-1)}$ is an unbiased estimator of θ^2 and iii) $\frac{T(T-1)(T-2)}{n(n-1)(n-2)}$ is an unbiased estimator of θ^3

c) Let $Y_1<Y_2<Y_3$ be a set of ordered statistics corresponding to a random sample of size 3 from a Uniform distribution with parameters 'θ' and '2θ' for being $\theta>0$. Then show that $(4/5)Y_1$ and $(4/7)Y_3$ are unbiased estimators for θ. **(07)**

Q.2 a) Define and discuss i) mean squared error consistency ii) weak and strong consistency. **(3+5)**

b) With reference to Cramer-Rao inequality and with usual notations, prove that **(10)**

i) $E[(\partial \ln L)/(\partial\theta)]=0$,   ii) $E[(\partial \ln L)/\partial\theta]^2=-E[(\partial^2 \ln L)/\partial\theta^2]$.

c) Consider a random sampling from the distribution $f(x)=(1/\theta)\exp(-x/\theta)$, $0<x<\infty$. Show that sample mean is MVB estimator of θ, with variance θ^2/n. **(07)**

Q.3 a) Find the MLE of λ *and* λ^2 from the distribution $f(x)=(2x/\lambda^2)\exp(-x^2/\lambda^2)$, $0<x<\infty$, how you interpret the two MLEs? **(13)**

b) Let $Y_1<Y_2<Y_3\ldots<Y_n$ be the order statistic of a random sample of size 'n' from the Uniform distribution of continuous type over the closed interval $[\theta-p, \theta+p]$. Find the MLE of θ *and* p. **(12)**

Q.4 a) Find the most general form of the distribution for which arithmetic mean (grouped data) is the MLE of its parameter. **(10)**

b) Find the moment estimator of the parameter of U(0,a), that is uniform distribution. **(06)**

c) Find the asymptotic variance of the MLE of σ from the distribution $N(0,\sigma^2)$. **(09)**

Q.5 a) Compare the classical and Bayesian inferences. Explain, why the definition of loss function is needed while calculating risk in inferential statistics. **(6+4)**

b) What is the role of prior density in the Bayesian inference? **(04)**

c) Let $X_1, X_2, \ldots X_n$ be a random sample from $U(0,\theta)$ and if prior is also uniform i.e $g(\theta)=1, 0<\theta<1$ then derive the Bayes estimator of θ with respect to loss function is $(t-\theta)^2/\theta^2$. **(11)**

Q.6 a) Let $x_1, x_2, \ldots x_n$, $y_1, y_2, \ldots y_n$ and $z_1, z_2, \ldots z_n$ been observations with same unknown variance, respective means are given as $E(x_i)=0.1\theta_1+0.2\theta_2+0.3\theta_3$, $E(y_i)=0.2\theta_1+0.3\theta_2+0.1\theta_3$, $E(z_i)=0.3\theta_1+0.1\theta_2+0.2\theta_3$ where $\theta_1,\theta_2,\theta_3$ are unknown parameters. Apply least square method to estimate the contrasts $(\theta_1-\theta_2)$ and $(\theta_2-\theta_3)$ by using the condition $\theta_1+\theta_2+\theta 3=0$. Also compute the variance of contrasts above. **(12)**

b) In connection with the sequential probability ratio test, show that $A=(1-\beta)/\alpha$ and $B=\beta/(1-\alpha)$, where α and β, respectively be the error sizes for testing $H_0;\theta=\theta_0, H_1;\theta=\theta_1$. **(08)**

c) Write the importance of sequential sampling in statistical inference. **(05)**

Q.7 a) Let $X_1, X_2, \ldots, X_n$ denote a random sample from a distribution which has p.d.f. $f(x_i)$ that is positive on only non negative integers. It is desired to test the simple hypothesis $H_o: f(x)=e^{-1}/x!$, $x=0,1,2,\ldots\ldots.$ against alternative simple hypothesis $H_1: f(x)=(1/2)^{x+1}$ $x=0,1,2,\ldots\ldots$ Derive the expression for BCR (Best critical region). Consider the case of n=1 and k=1, k being any positive integer in the expression $(L(\theta', x_1, x_2, \ldots\ldots, x_n)/L(\theta'', x_1, x_2, \ldots\ldots, x_n)) \le k$ where $H_o: \theta=\theta'$, $H_1=\theta=\theta''$. Find the power of the test for this combination of n and k when H_o is true. **(11)**

b) What is the length of confidence interval ~~? How~~ it can be minimized? **(08)**

c) What do you mean by BCR(Best Critical Region) and how it can be obtain? **(06)**

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Subject: Statistics | **Paper: II (Regression Analysis and Econometrics)** | **Time: 3 Hrs.** | **Marks: 100**

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q.1.a) Define Econometrics? Under what reasons an error term is introduced in econometric models? Explain. **(10)**

b) Consider $\underline{Y} = X\underline{\beta} + \underline{\in}$, such that $\underline{\in} \sim N(\underline{0}, \sigma^2 I)$, develop the test statistic and explain the procedure involved in testing the statistical significance of all regression coefficient. **(15)**

Q.2.a) Differentiate between **(6)**

i) Distributed lagmodel and Autoregressive model.
ii) ANOVA and ANOCOV models.

b) Consider $\underline{Y} = X\underline{\beta} + \underline{\in}$, such that $\underline{\in} \sim N(\underline{0}, \sigma^2 I)$ and elements of $\underline{\beta}$ obey the relations $C\underline{\beta} = \underline{\gamma}$ Obtain restricted L.S. estimator of $\underline{\beta}$ and its variance covariance matrix. **(19)**

Q.3.a) A data matrix of full column rank is portioned as $X = [X_1\, X_2]$ X_1 *is* $n \times k_1$ and X_2 *is* $n \times k_2$. **(10)**

Show that the upper left-hand block in $(X'X)^{-1}$ may be expressed as $(X_1' M_2 X_1)^{-1}$ where $M_2 = 1 - X_2(X_2' X_2)^{-1} X_2'$. Give a least-squares interpretation of $M_2 X_1$ and hence $X_1' M_2 X_1$.

b) State and prove Aitken Theorem **(15)**

Q.4. The following estimated equation was obtained by OLS regression using quarterly data for 1958 to 1976 inclusive: **(25)**

$$\underset{(3.4)}{y_t = 2.20} + \underset{(0.005)}{0.104x_{t1}} - \underset{(2.2)}{3.48x_{t2}} + \underset{(0.15)}{0.34x_{t3}}$$

Standard errors are in parentheses, the explained sum of squares was 109.6, and the residual sum of squares 18.48.

a) Test the significance of each of the slop coefficients.

b) Calculate the coefficient of determination R^2.

c) When three seasonal dummy variables were added and the equation was reestimated, the explained sum of squares rose to 114.8. Test for the presence of seasonality.

d) Two further regressions, based on the original specification, were computed for the subperiods 1958, quarter 1, to 1968, quarter 4; and 1969, quarter1, to 1976, quarter 4, yielding residual sums of squares of 9.32 and 7.46, respectively. Test the following hypotheses:

i) The error variances are identical in the two subperiods.
ii) The coefficients are identical in the two super periods.

Q.5.a) What is the rationale of using ridge regression? Also obtain the mean and variance of its estimators. **(15)**

b) Define orthogonal polynomials and discuss their use in regression analysis. **(10)**

Q.6.a) What understanding do you have about heteroskedasticity? How it is removed from the system? **(13)**

b) Define Autocorrelation. How autocorrelation is detected by using Durbin Watson test? Discuss. **(12)**

Q.7. Consider the following model $y_2 = \beta y_{2t} + u_{1t}$ $\qquad y_{2t} = \alpha_1 y_{1t} + \alpha_2 x_{1t} + \alpha_3 x_{2t} + u_{2t}$ **(25)**

i) Show that OLS estimate of β is inconsistent estimate.
ii) Obtain consistent estimates of the structural parameters β and $\alpha's$, where possible, by appropriate method using the following calculations

$$\sum x_1^2 = 1, \sum x_2^2 = 20, \sum x_1 x_2 = 0, \quad \sum x_1 y_1 = 5, \quad \sum x_2 y_1 = 40, \quad \sum x_1 y_2 = 10 \quad \sum x_2 y_2 = 20$$

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Subject: Statistics

Paper: III (Part-A) (Data Processing and Computer Programming)

Time: 3 Hrs. Marks: 75

NOTE: Attempt any FOUR questions.

Q.1. **(a)** Describe the functions of the following components of a digital computer:

(i) Compiler (ii) Types of Storage devices

(iii) Input Devices (iv) Hardware

(b) Describe the usage of following functions of

i) XCOPY ii) DEL iii) DIR iv) MD

(c) Differentiate between source program and object program.

(8+4+7)

Q.2. **(a)** Write an algorithm and code in FORTRAN to find the area of a triangle given the length of its three sides are given.

(b) Write the following mathematical expression into FORTRAN expressions.

(i) $\dfrac{e^{x+y}}{x+y}$ (ii) $\sqrt[3]{|y|} - \dfrac{e^{\frac{1}{2}(x^2)}}{x+y+z}$

(iii) $\dfrac{-x}{y} + (x+y)^{3/4}$ (iv) $\dfrac{\sin x}{|y| + \cos Z}$

(v) $\dfrac{1}{2\sqrt{\pi}}\dfrac{x^5 y}{abc} + a^x$

(c) Determine the output of the following programs.

(i)
```
I=4
K = 6
L= K + 2 *I
I=2*L+1/2
K=K/4
L=I+K+L
WRITE(*,*)I,K,L
STOP
END
```

(ii)
```
A=6
B=600
W=20
Z=A+B*W
WRITE(*,10)A,B,W
STOP
END
```

Q.3. **(a)** Write a FORTRAN program that calculate nCr, nPr.

(b) Write a FORTRAN program which calculates the sum of first 50 terms of following series:

$$1 - \frac{2^3}{3^3} + \frac{4^3}{5^3} - \frac{6^3}{7^3} + \dots\dots\dots + \frac{(2N)^3}{(2N+1)^3}$$

(c) Write a FORTRAN program, which reads and compute employee's salary after paying health premium according to the following plan

Premium = 1000 if single

Premium = 2500 if married without children

Premium = 5000 if married with children **(6+7+6)**

Q.4. **(a)** Define the functions of following FORTRAN statements. Give two examples in each case.

i. DO statement (ii) END and STOP statements

b) Write a FORTRAN program that calculates and print the values of $y = 8x^3 - 6x^2 + 2x$, for values of x from -5 to 5 steps of 0.5.

(c) Write a program for the Fibonacci series up to 20 terms

1, 1, 2, 3, 5, 8, 13.....

(6+6+7)

Q.5. **(a)** What is an Array? What are the advantages of using Arrays?

(b) A mega store gives a discount on the total sale of items as follows:

Discount	If
5%	Total sale < Rs. 5000
7.5 %	Rs.5000 ≤ Total sale < Rs.10000
10%	Rs.10000 ≤ Total sale < Rs.15000
12.5%	Rs.15000 ≤ Total sale < Rs.20000
15%	Total sale ≥ Rs.20000

Write a FORTRAN program that reads the number items sold and their prices, then prints the total discounted price.

(c) Write a FORTRAN program which calculates the multiplication of two matrices A(M*N) and B(N*L).

(5+7+7)

Q.6. **(a)** Write a program to calculate overtime rate of 10 employees using do while ( ). Overtime is paid at the rate of Rs.80 per hour for over 40 working hours.

(b) Distinguish between switch() statement and else-if() statement.

(c) Write a program using switch statement to make a four function calculator

(4+7+8)

Q.7. **(a)** Write commonly used functions for looping in C++. Describe two forms of looping.

(b) Write and run C++ program which inputs 'amount' as opening balance in your saving account, calculates the balance at the end of 1 year. The interest 7.5% can provide quarterly. Print interest earned and balance at the end of each quarter.

(c) Write and run a C++ program using 'functions' that print the sum of following series

1/2, 3/4, 7/8,...........up to 50 values

(5+8+6)

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Subject: Statistics **Paper: VI (i) [Statistical Quality Control]** **Time: 3 Hrs. Marks: 100**

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q#1 (a)	Explain in brief three important postulates concerning the laws basic to control.	15
(b)	Differentiate Warning, Natural Tolerance and Action Limits?	10
Q#2 (a)	In order to meet Government regulations, the contained weight of a product must at least be equal to the labeled weight 98% of the time. Control charts for $\bar{x}$ *and* σ are maintained on the weight in ounces of the contents, using a subgroup size of 10, after 20 subgroups, $\sum\bar{x} = 731.4$ and $\sum\sigma = 9.16$ Compute 3-sigma control limits for $\bar{x}$ *and* σ and estimate the value of σ' assuming that the process is in statistical control. If the labeled weight is 36 oz, and assuming the process generator a normal distribution, does it meet federal requirements?	25
(b)	Suppose $\bar{x}$ chart is used with usual 3-sigma limits. The sample size is 5. Find the probability of detecting a shift to $\mu_1 = \mu_0 \pm 2\sigma$ on the first sample following the shift?	
Q#3 (a)	The following table gives the number of missing rivets noted at aircraft final inspection:	15

Air plane No.	*No. of missing rivets*	*Air plane No.*	*No. of missing rivets*	*Air plane No.*	*No. of missing rivets*
1	11	10	12	19	8
2	9	11	23	20	16
3	10	12	16	21	14
4	22	13	9	22	19
5	7	14	25	23	11
6	28	15	15	24	15
7	9	16	9	25	8
8	9	17	11		
9	14	18	21		

a. Find $\bar{c}$ and compute the control limits.
b. Plot control chart and make a decision about rejected lots.
c. What value of c'_0 would you suggest for the subsequent period?
d. Make revised control limits if necessary.

(b)	Discuss some situations in which p-chart is most applicable.	10
Q#4(a)	Differentiate between Single Sampling Plan and Double Sampling Plan.	10

P.T.O

(b)	Draw type-B OC curve for the single sampling plan $n = 100,\ c = 1$.	15
Q#5(a)	Take a sampling plan with $n_1 = 50,\ c_1 = 0,\ n_1 + n_2 = 100,\ c_2 = 3$ If the incoming lots have fraction nonconforming $p = 0.05$ then what is the probability of final acceptance? Calculate the probability of rejection on the first sample?	15
(b)	Use the following data to set up short run $\bar{x}$ *and* R charts using the DNOM approach. The nominal dimensions for each part are $N_A = 50,\ N_B = 25$	10

Sample No.	Part No.	M_1	M_2	M_3
1	A	49	51	52
2	A	48	50	51
3	A	49	49	52
4	A	50	53	51
5	B	24	27	26
6	B	25	27	24
7	B	27	26	23
8	B	25	24	23
9	B	24	25	25
10	B	26	24	25

Q#6 (a)	State some modern definitions of reliability and life testing.	10
(b)	In a plan, 10 items were tested for 500 hours with replacement and an acceptance number of 1. Construct an OC-curve showing probability of acceptance as a function of mean life.	15
Q#7	Write a short note on any **Five** of the following: i. Sequential Sampling Plan ii. OC-Curve iii. Modified Control Chart iv. Rectifying Inspection v. Average Run Length (ARL) vi. Dodge-Romig Sampling Plans	5 each

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Subject: Statistics   Paper: VII (ii) (Multivariate Analysis)   Time: 3 Hrs.   Marks: 100

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q1. Consider the data below: (5+4+16)

X_1	3	4	2	6	8	2	5
X_2	5	5.5	4	7	10	5	7.5

a) Write down data matrix. How many variables and observations are there? What is the order of the data matrix?

b) What is the 4^{th} observation on first variable? And how would you denote it?

c) Write down the sample mean vector, sample covariance matrix and sample correlation matrix for the data above.

Q2. Write down the spectral decomposition of the matrix below. (25)

$$A = \begin{bmatrix} 8 & 5 \\ 5 & 3 \end{bmatrix}$$

Q3. Derive conditional distribution for multivariate normal distribution. (25)

Q4. ~~Explain~~ the difference between central and non-central Wishart distribution. Derive the additive property of Wishart matrices. (12+13)

Q5. a) Data for two variables give the summary: (10+15)

$$n = 4, \bar{X} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}, S = \begin{bmatrix} 8 & \\ -3.33 & 2 \end{bmatrix}$$

Obtain T^2 simultaneous confidence intervals for the components of $\boldsymbol{\mu}$

b) Let x follows $N_3(\mu, \Sigma)$. Find the distribution of $\begin{bmatrix} X_1 - X_2 \\ X_2 - X_3 \end{bmatrix}$.

Q6. Let X_1 and X_2 be two random variables with covariance matrix: (25)

$$\Sigma = \begin{bmatrix} 9 & \sqrt{6} \\ \sqrt{6} & 4 \end{bmatrix}$$

Carryout principal component analysis for the matrix above.

Q7. Consider the data below from two bivariate normal populations $\mathbf{P_1}$ and $\mathbf{P_2}$ with common covariance matrices (25)

$$X_1 = \begin{bmatrix} 2 & 12 \\ 4 & 10 \\ 3 & 8 \end{bmatrix}, \quad X_1 = \begin{bmatrix} 5 & 7 \\ 3 & 9 \\ 4 & 5 \end{bmatrix}$$

Obtain linear discriminant function and allocate the new observation $\mathbf{x_o} = \begin{bmatrix} 1 \\ 4.4 \end{bmatrix}$ to one of the two populations.

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Subject: Statistics Paper: VI (iii) [Operations Research] Time: 3 Hrs. Marks: 100

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q.1 a) Explain the phases of OR.
b) What is big M technique?
c) Write down the advantages of Linear Programming 25

Q.2a) What is degenerate solution and alternative optima? Discuss the types of degeneracy.
b) What is Dual Simplex Method?
c) Solve the following LP-model by Dual Simplex Method.
Min $X_o = 2X_1 + X_2$ Subject to $3X_1 + X_2 \geq 3$; $4X_1 + 3X_2 \geq 6$; $X_1 + 2X_2 \leq 3$; $X_1, X_2, \geq 0$ 8+5+12

Q.3.a) Explain transportation model and its components
b) Find optimal solution of the following transportation modal?

	1	2	3	4	Supply
1	10	0	20	11	15
2	12	7	9	20	25
3	0	14	16	18	5
Demand	5	15	15	10	45

10+15

Q.4 a) Explain graphical solution of 2×N games and factors of queueing model
b) Solve the following payoff matrix.?

Firm A

Firm B	1	2	3
1	12	10	8
2	14	14	10
3	16	12	15

12+13

Q.5.a) What is generalized inventory system? Explain its main components.
b) A manufacturer has to supply his customer with 24000 units of his product per year. This demand is fixed and known. Since the unit is used by the customer is an assembly line operation and the customer has no storage space for the units, the manufacturer must ship a day's supply each day. If the manufacturer fails to supply the required units, he will lose the account and probably his business. Hence the cost of shortage is assumed to be infinite, and consequently, none will be tolerated. The inventory holding cost amounts to .59 per unit per month, and setup cost per run is Rs. 350. Find the optimum lot size and the length of optimum production run. 10+15

Q.6.a) What do you understand by Network Analysis? Write its objectives.
b) Distinguish between the CPM Modal and PERT modals.
c) The Following time-cost table (time in week and cost in rupees) applied to a project. Use it to arrive at the network associated with completing the project in minimum time with minimum cost.

Activity	Normal		Crash	
	Time	Cost	Time	Cost
1-2	2	800	1	1400
1-3	5	1000	2	2000
1-4	5	1000	3	1800
2-4	1	500	1	500
2-5	5	1500	3	2100
3-4	4	2000	3	3000
3-5	6	1200	4	1600
4-5	5	900	3	1600

5+6+14

Q 7 Write note on the following:
(i) Optimality and feasibility condition of dual simplex method
(ii) The Simplex method
(iii) Unbounded solution
(iv) Infeasible solution
(v) Dominance property method 25

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Subject: Statistics Paper: VI (iv) (Part-A-Survey and Report Writing) Time: 3 Hrs. Marks: 50

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q.1. Describe various types of errors in surveys. Also discuss the available methods to control these errors.

Q.2. Discuss the advantages and disadvantages of sample survey. What factors should be considered to make a survey successful?

Q.3. What are the different types of data? Explain sources of primary and secondary data.

Q.4. Describe and compare the face-to-face survey and drop-off survey with reference to their advantages and disadvantages.

Q.5. What are major sections of a survey report? Explain.

Q.6. Discuss and give examples to explain under what kind of situation you would use the following sampling schemes.

a) Cluster Sampling
b) Simple Random Sampling
c) Stratified Random Sampling

Q.7. Define and explain validity and its various types.