UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – I
Annual Examination - 2018

Roll No. ....

Subject: Mathematics A Course-I | **TIME ALLOWED: 3 hrs.**
PAPER: Calculus and Analytical Geometry | **MAX. MARKS: 100**

NOTE: **Attempt SIX questions by selecting TWO questions from Section-I, TWO questions from Section-II, ONE questions from Section-III and ONE question from Section-IV.**

SECTION – I

Q.1: **(9,8)**

(a) Solve the inequality $\frac{2x}{x+2} \geq \frac{x}{x-2}$

(b) Let $f(x) = \begin{cases} x\cos\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ Discuss the continuity of f at x = 0.

Q.2: **(9,8)**

(a) Differentiate $\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$ with respect to arcos x^2.

(b) If $y = e^{m \arcsin x}$, show that $(1+x^2)y^{(n+2)} - (2n+1)xy^{(n+1)} - (n^2+m^2)y^{(n)} = 0$

Q.3: **(9,8)**

(a) Use differentials to find approximate value of tan $29°$.

(b) Find $\frac{dy}{dx}$ *if* $y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$.

Q.4: **(9,8)**

(a) Evaluate the limit $\lim\limits_{x \to 0} (\cot\ x)^{\sin 2x}$

(b) If $f(x) = 1 - x^{3/4}$ *on* $[-1,1]$.discuss the validity of Rolle's Theorem. Find c if possible such that $f'(c) = 0$.

SECTION – II

Q.5: **(9,8)**

(a) Integrate $\int \frac{1}{(x^2-2x+2)\sqrt{x-1}}dx$

(b) Show that $\int x^n \arctan x dx = \frac{x^{n+1}}{n+1} arc \tan x - \frac{1}{x+1}\int \frac{x^{n+1}}{1+x^2}dx$ Hence evaluate $\int x^3 arc \tan x dx$

Q.6: **(9,8)**

(a) Prove that $\int_0^{\pi} \frac{x\sin x}{1+\sin x}dx = \frac{\pi^2}{2} - \pi$

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(b) Obtain a reduction formula for $\int \frac{x^n}{\sqrt{1-x^2}}dx$ *and Hence evaluate* $\int \frac{x^3}{\sqrt{1-x^2}}dx$.

(9,8)

Q.7:

(a) Show that the pedal equation of the astroid $x = a\cos^3\theta, y = \sin^3$ is $r^2 = a^2 - 3p^2$.

(b) Show that tangent to cardioids $r = a(1+\cos\theta)$ at points $0 = \frac{\pi}{3}$ *and* $\theta = \frac{2\pi}{3}$ are respectively parallel and parallel and perpendicular to initial line.

(9,8)

Q.8:

(a) Show that the curve $r^m = a^m \cos m\theta$ *and* $r^m = a^m \sin m\theta$ cut each other orthogonally.

(b) If a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, with centre C, meets the major and minor axes in T and t, prove that $\frac{a^2}{CT^2} + \frac{b^2}{Ct^2} = 1$.

SECTION – III

(8,8)

Q.9:

(a) Find the asymptotes of the curve $x^2y + xy^2 + xt + y^2 + 3x = 0$

(b) Find the relative maxima and minima of $r = 1\bar{c}\cos\theta$

(8,8)

Q.10:

(a) Show that the intrinsic equation of the parabola $x^2 = 4ay$ is

$S = a\tan\alpha\sec\alpha + a\ln(\tan\alpha + \sec\alpha)$

(b) Prove that radius of curvature at point (2a, 2a) on the curve $x^2y = a(x^2 + y^2)$ *is* $2a$

SECTION – IV

(8,8)

Q.11:

(a) If the edges of a rectangular parallelepiped are a, b, c. show that the angle between the four diagonals are given by $arc\cos\left(\frac{\pm a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2}\right)$

(b) Find an equation of the plane through the point (3,–2, 5) and perpendicular to the line $x = 2+3t, y = 1-6t, z = -2+2t$

(8,8)

Q.12:

(c) Find an equation of the sphere passing through the point (0,–2,–4),(2,–1,–1) and having its center on the st-line $2x - 3y = 0 = 5y + 2z$

(d) Find the direction of Qibla of Shahi Mosque, Islamabad, Latitude = $33°40'N$ and longitude = $78°8'E$

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – I
Annual Examination - 2018

Roll No. ....

Subject: Mathematics B Course-I
PAPER: Vector Analysis and Mechanics

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt SIX questions in all. Selecting ONE question from SECTION – I, TWO questions from SECTION – II, TWO questions from SECTION – III and ONE question from SECTION – IV.

SECTION – I

1. a. Show that $\vec{a}\times(\vec{b}\times\vec{c})$, $\vec{b}\times(\vec{c}\times\vec{a})$ and $\vec{c}\times(\vec{a}\times\vec{b})$ are coplanar. 8

   b. If $\bar{U}(t)$ is a unit vector, show that $\bar{U}.\left(\bar{U}+\frac{d^2\bar{U}}{dt^2}\right)+\left(\frac{d\bar{U}}{dt}\right)^2=1.$ 8

2. a. Find the divergence and curl of the vector point function $\vec{F}=(x^2+yz)\hat{\imath}+(y^2+zx)\hat{\jmath}+(z^2+xy)\hat{k}.$ 8

   b. Show that $\text{div}\left(\frac{\vec{r}}{r^n}\right)=0.$ 8

SECTION – II

3. a. State and prove the *LAMY*'s theorem. 9

   b. A Three forces P, Q and R acting at a point are in equilibrium and the angle between P and Q is double of the angle between P and R. Prove that $R^2=Q(Q-P)$. 8

4. a. Three forces P, Q, R act along the sides BC, CA, AB respectively of a triangle ABC. Prove that if $P\cos A+Q\cos B+R\cos C=0$, then the line of action of the resultant pass through the circumcenter of the triangle. 9

   b. A uniform solid right circular cone is suspended by a light inextensible string which has one end tied to the vertex and the other to a point on the circumference of the base and passes through two small smooth rings fixed at a distance a apart in a horizontal line, the altitude of the base is h $(>a)$ and the radius of the base is r. Prove that if the cone hangs with its axis horizontal, the length of the string is $a+\frac{(h-a)\sqrt{h^2+4r^2}}{h}$. 8

5. a. Find A uniform lamina is bounded by the asteroid $x=a\cos^3\theta$, $y=b\sin^3\theta$. Find the centre of gravity of its portion that lies in the first quadrant. 9

   b. Find the position of the centre of gravity of an octant of a uniform solid sphere. 8

6. a. Find the force necessary to support a heavy particle on an inclined plane of inclination α, $(\alpha>\lambda)$. 9

   b. State and prove PRINCIPLE OF VIRTUAL WORK for a system of forces along a plane rigid body. 8

P.T.O.

SECTION – III

7. a. Find radial and transverse components of velocity and acceleration. 9

b. A particle moving in a straight line starts with a velocity u and has acceleration v^3, where v is the velocity of the particle at time t. Find the velocity and the time as functions of the distance travelled by the particle. 8

8. a. Prove that the field of force F determined by $\vec{F} = (y^2 - 2xyz^3)\,\hat{\imath} + (3 + 2xy - x^2z^2)\,\hat{\jmath} + (6z^3 - 3x^2yz^2)\,\hat{k}$ is conservative and find its potential. 9

b. A particle of mass m moves along the curve defined by $\vec{r} = a\cos wt\ \hat{\imath} + b\sin wt\ \hat{\jmath}$. Find the torque and angular momentum about the origin. 8

9. a. Obtain equation of parabola of safety. 9

b. Prove that the speed required to project a particle from a height h to fall a horizontal distance a from the point of projection is at least $\sqrt{g(\sqrt{a^2 + h^2} - h)}$. 8

10. a. A particle describes the curve $\frac{a}{r} = \cosh n\theta$ under central attraction force F, find the law of force. 9

b. A particle of mass m under the central force $mM\{3au^4 - 2(a^2 - b^2)u^5\}, a > b$ and is projected from an apse at a distance $a+b$ with velocity $\frac{\sqrt{M}}{a+b}$; show that the orbit is $r = a + b\cos\theta$. 8

SECTION – IV

11. a. A heavy elastic ball is dropped upon a horizontal floor from a height of *20 ft* and after rebounding twice, it is observed to attain a height of *10 ft*. Find the coefficient of restitution. 8

b. Three perfectly elastic balls of masses m, $2m$ and $3m$ are placed in a straight line. The first impinges directly on the second with a velocity u and then the second impinges on the third. Find the velocity of the third ball after impact. 8

12. a. Two equal, smooth and perfectly elastic spheres moving at right angles to one another impinge obliquely, show that after impact, they will still move at right angle to each other. 8

b. Prove that when two smooth spheres impinge obliquely, the K.E is always lost by impact, unless the elasticity is perfect. 8

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – I
Annual Examination - 2018

Roll No. ....

Subject: Mathematics General-I | **TIME ALLOWED: 3 hrs.**
PAPER: Calculus (Differential and Integral Calculus) | **MAX. MARKS: 100**

Attempt any **SIX** questions by selecting **TWO** questions from **Section-I**, **TWO** questions from **Section-II**, **ONE** question from **Section-III** and **ONE** question from **Section-IV**.

Section-I

Q. 1. (a) — 4+4

i. Solve the inequality: $|x^2 - x + 1| > 1$

ii. Evaluate $\lim_{x\to\infty} \frac{a^x-1}{x}$, $(a > 1)$

(b) — 4+5

i. Examine the continuity of $f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$ $at\ x = 0$

ii. Evaluate $\lim_{x\to\pi} \frac{\tan(\sin x)}{\sin x}$

Q. 2. (a) Find $L\,f'(2)$ and $R\,f'(2)$ for the function $f(x) = |x^2 - 4|$ — 8+9

(b) If $y = \arctan x$. Show that

$(1 + x^2)y'' + 2xy' = 0$. Hence find the value of $y^{(n)}$ at $x = 0$

Q. 3. (a) Find $\frac{dy}{dx}$ when $\arcsin(\ln xy) = x + y^2$ — 8+9

(b) Use the Mean Value Theorem to show that

$|\sin x - \sin y| \leq |x - y|$ for any real numbers x, y.

Q. 4. (a) Evaluate the given limits: — 4+4

i. $\lim_{x\to 0} \left[\frac{a}{x} - \cot\left(\frac{x}{a}\right)\right]$

ii. $\lim_{x\to 0} \left(\frac{\sin hx}{x}\right)^{1/x^2}$

(b) Find first four terms of the Maclaurin Series of — 9

$f(x) = e^{\sin x}$

Section-II

Q. 5. (a) Evaluate $\int \frac{x^2+1}{\sqrt{(x+1)^2}} e^x dx$ — 8+9

(b) Evaluate $\int \frac{\sec x}{1+\csc x} dx$

Q. 6. (a) Show that $\int_0^1 \frac{\ln(1+x)}{1+x^2} dx = \frac{\pi}{8} \ln 2$ 8+9

(b) Show that : $\int sec^{2n+1}x \, dx = \frac{sec^{2n-1}x \tan x}{2n} + \left(1 - \frac{1}{2n}\right) \int sec^{2n-1}x \, dx$

Q. 7. (a) Find the angle of intersection of the curves $y^2 = 4ax$ and $x^2 = 4ay$ at the point other than (0,0) 8+9

(b) Sketch the graph of the curve $r = a \sin 3\theta$, $a > 0$

Q. 8. (a) Show that the curves $r^m = a^m \cos m\theta$ and $r^m = a^m \sin m\theta$ cut each other orthogonally. 8+9

(b) Show that the normal at any point of the curve $x = a\cos\theta + a\theta \sin\theta$, $y = a \sin\theta - a\theta \cos\theta$ is at a constant distance from the origin.

Section-III

Q. 9. (a) Find equations of the asymptotes of the curve 8+8

$$y^3 + x^2y + 2xy^2 - y + 1 = 0$$

(b) Find the position and nature of the multiple points on the curve:

$$x^4 + y^3 - 2x^3 + 3y^2 = 0$$

Q. 10. (a) Find the area inside the circle $r = 2a\sin\theta$ and outside the circle $r = a$ 8+8

(b) Find the intrinsic equation of the parabola $x^2 = 4ay$

Section-IV

Q. 11. (a) If $f(x,y) = \frac{x^2+y^2}{x+y}$, prove that $(f_x - f_y)^2 = 4(1 - f_x - f_y)$ 8+8

(b) The radius of a circle increases from 10 cm to 10.1cm. Find the corresponding change in the area of the circle. Also find the percentage change in the area.

Q. 12. (a) Examine $f(x,y) = 2x^2 - 4x + xy^2 - 1$ for relative extrema. 8+8

(b) Find the volume of the solid in the first octant bounded by the coordinate planes and the graphs of the equations $z = x^2 + y^2 + 1$ and $2x + y = 2$.