UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part-II
Annual Exam - 2017

Roll No. ....

Subject: Mathematics A Course-II **TIME ALLOWED: 3 hrs.**
PAPER: (Linear Algebra and Differential Equations) **MAX. MARKS: 100**

Note: Attempt Six Questions by selecting Two Questions from Section-I, One Question from Section-II, One Question from Section-III and Two Questions from Section-IV.

Section I

Q-1 (a): For a nonsingular matrix A, show that $\overline{(A^T)}^{-1} = \overline{(A^{-1})}^T$ (9, 8)

(b): If A and B are 3×3 matrices such that $\det(A^2B^2)=108$ and $\det(A^3B^2) = 72$, *Find* $\det(2A)$ *and* $\det(B^{-1})$

Q-2(a): Solve the system of equations by Gauss elimination method.

$$x_1 + 5x_2 + 2x_3 = 9$$
$$x_1 + x_2 + 7x_3 = 6$$
$$-3x_2 + 4x_3 = -2$$

(b): Without expanding show that: (9, 8)

$$\begin{vmatrix} (a^m + a^{-m})^2 & (a^m - a^{-m})^2 & abc \\ (b^n + b^{-n})^2 & (b^n - b^{-n})^2 & abc \\ (c^p + c^{-p})^2 & (c^p - c^{-p})^2 & abc \end{vmatrix} = 0$$

Q-3(a) Find an equation defining the subspace W of R^3 spanned by $V_1 = (1,-3,2) V_2 = (-2,1,2), V_3 = (-3,1,6)$ by expressing an arbitrary elements $(x, y, z) \in R^3$ as a linear combination of V_1, V_2, V_3. (9, 8)

(b) Find a basis and dimension of the subspace W of R^4 *spanned by* $(1,4,-1,3), (2,1,-3,-1)$ *and* $(0,2,1,-5)$

Q-4(a): Find the rank of the matrix $A=\begin{bmatrix} 1 & -2 & 1 & 0 & 5 \\ -1 & 0 & 1 & -2 & 2 \\ 1 & -6 & 3 & -2 & 12 \\ 2 & -3 & 0 & 2 & 3 \end{bmatrix}$ Also write an echelon matrix row equivalent to A.

(b) The matrix $\begin{bmatrix} 3 & 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & -1 & 1 & 1 & 1 \end{bmatrix}$ of a linear transformation $T: R^n \to R^m$. (9, 8)

Determine m, n and express T in term of Coordinates.

Section II

Q-5(a): Let u,v be elements of an inner product space V over R then prove that

$|\langle u, v\rangle| \leq \|u\| \, \|v\|$. (8, 8)

(b): Find an orthogonal matrix whose first row is $(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$

Q- 6(a): Find the eigenvalue and eigenvectors of the matrix $A=\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$ (8, 8)

(b): Prove that eigenvalues of a symmetric matrix are all real.

P.T.O

Section III

Q-7(a): Solve the D.E (2x+y+1)dx +(4x+2y-1)dy =0 (8, 8)

(b): Solve the initial value problem $(2xcosy + 3x^2y)dx + (x^3 - x^2 siny - y)dy = 0, \quad y(0) = 2$

Q-8(a): Solve D.E. $dx + (\frac{x}{y} - siny)dy = 0$ (8, 8)

(b): solve the initial value problem $\frac{dr}{d\theta} + r\tan\theta = \cos^2\theta, \quad r\left(\frac{\pi}{4}\right) = 1$

Section IV

Q-9(a): Solve D.E. $(D^3 + 4)y = 4sin^2x$ (9, 8)

(b): Solve D.E. $x^2y'' - 2xy' + 2y = x\, lnx$, y (1)=1 , y′(1)= 0

Q-10(a): Solve D.E. $x^2\frac{d^2y}{dx^2} - (x^2 + 2x)\frac{dy}{dx} + (x+2)y = x^3e^x$ (9, 8)

(b): Find the general solution of $x^2\frac{d^2y}{dx^2} - x(x+2)\frac{dy}{dx} + (x+2)y = x^3$

Q-11 (a): Evaluate (4, 4)

i. $\angle\{\frac{sint}{t}\}$
ii. $\angle\{e^{3t+5}\}$

(b): Evaluate

i. $\angle^{-1}\{\frac{1}{s^2(s^2-a^2)}\}$ (4+5)

ii. $\angle^{-1}\{ln\frac{s^2+1}{(s-1)^2}\}$

Q-12(a): Using the Laplace transformation to solve the D.E. (9, 8)

$$\frac{dx}{dt} - 4x - 5y = e^{-4t} \qquad x(0) = 0$$

$$\frac{dy}{dt} + 4x + 4y = e^{4t} \qquad y(0) = 0$$

(b): Apply the power series method to solve D.E. $y' = y(1 + \frac{1}{x})$

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part-II
Annual Exam - 2017

Roll No. ....

Subject: Mathematics A Course-II
PAPER: (Linear Algebra and Differential Equations)

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

Note: Attempt Six Questions by selecting Two Questions from Section-I, One Question from Section-II, One Question from Section-III and Two Questions from Section-IV.

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$$\begin{vmatrix} (a^m + a^{-m})^2 & (a^m - a^{-m})^2 & abc \\ (b^n + b^{-n})^2 & (b^n - b^{-n})^2 & abc \\ (c^p + c^{-p})^2 & (c^p - c^{-p})^2 & abc \end{vmatrix} = 0$$

Q-3(a) Find an equation defining the subspace W of R^3 spanned by $V_1 = (1,-3,2)V_2 = (-2,1,2), V_3 = (-3,1,6)$ by expressing an arbitrary elements $(x, y, z) \in R^3$ as a linear combination of V_1, V_2, V_3. (9, 8)

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Section II

Q-5(a): Let u,v be elements of an inner product space V over R then prove that

$$|\langle u, v\rangle| \leq ||u||\,||v||.$$ (8, 8)

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P.T.O

Section III

Q-7(a): Solve the D.E $(2x+y+1)dx + (4x+2y-1)dy = 0$ (8, 8)

(b): Solve the initial value problem $(2x\cos y + 3x^2y)dx + (x^3 - x^2\sin y - y)dy = 0, \quad y(0) = 2$

Q-8(a): Solve D.E. $dx + (\frac{x}{y} - \sin y)dy = 0$ (8, 8)

(b): solve the initial value problem $\frac{dr}{d\theta} + r\tan\theta = \cos^2\theta, \quad r\left(\frac{\pi}{4}\right) = 1$

Section IV

Q-9(a): Solve D.E. $(D^3 + 4)y = 4\sin^2 x$ (9, 8)

(b): Solve D.E. $x^2y'' - 2xy' + 2y = x\ln x, \; y(1) = 1, \; y'(1) = 0$

Q-10(a): Solve D.E. $x^2\frac{d^2y}{dx^2} - (x^2 + 2x)\frac{dy}{dx} + (x+2)y = x^3e^x$ (9, 8)

(b): Find the general solution of $x^2\frac{d^2y}{dx^2} - x(x+2)\frac{dy}{dx} + (x+2)y = x^3$

Q-11 (a): Evaluate (4, 4)

i. $\angle\{\frac{\sin t}{t}\}$

ii. $\angle\{e^{3t+5}\}$

(b): Evaluate (4+5)

i. $\angle^{-1}\{\frac{1}{s^2(s^2-a^2)}\}$

ii. $\angle^{-1}\{\ln\frac{s^2+1}{(s-1)^2}\}$

Q-12(a): Using the Laplace transformation to solve the D.E. (9, 8)

$$\frac{dx}{dt} - 4x - 5y = e^{-4t} \qquad x(0) = 0$$

$$\frac{dy}{dt} + 4x + 4y = e^{4t} \qquad y(0) = 0$$

(b): Apply the power series method to solve D.E. $y' = y(1 + \frac{1}{x})$

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part-II
Annual Exam - 2017

Roll No. ....

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

Subject: Mathematic General-II
PAPER:(Mathematical Methods (Geomt, Series, Compl No.LA, DE)

Note: Attempt Six Questions by selecting two questions from Section-I Two Questions from Section-II, One Questions from Section-III and One Question from Section-IV.

Section - I

Q.1 (a) If $x = \cos\theta + i\sin\theta$, show that $x^n + \frac{1}{x^n} = 2\cos n\theta$ and $x^n - \frac{1}{x^n} = 2i\sin\theta$. 9

(b) Show that $2 + i = \sqrt{5}\, e^{i\tan^{-1}(1/2)}$. 8

Q.2 (a) If $\log\sin(x + iy) = u + iv$, show that $\cos h2y = \cos 2x + 2e^{2u}$. 9

(b) Evaluate the sum of the series, $1 + \frac{1}{2}\cos 2\theta - \frac{1}{2.4}\cos 4\theta + \frac{1.3}{2.4.6}\cos 6\theta + \ldots$ 8

Q.3 (a) Determine whether the sequence $\frac{2^n}{(2n)!}$ converges or diverges. If it converges find its limit. 9

(b) Apply any appropriate test to determine the convergence or divergence of series $1 + \frac{1.2}{1.3} + \frac{1.2.3}{1.3.5} + \frac{1.2.3.4}{1.3.5.7} + \ldots$ 8

Q.4 (a) Determine whether the series $\sum_{1}^{\infty} (-1)^{n-1} \frac{n+2}{n(n+3)}$ converges absolutely, converges conditionally or diverges. 9

(b) Show that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ 8

Section - II

Q.5 (a) Use scalar products to prove that the triangle with vertices A (1, 0, 1), B (1, 1, 1) and C (1, 1, 0) is a right isosceles triangle. 9

(b) Whether the vector $-6\hat{i} + 3\hat{j} + 2\hat{k}$, $3\hat{i} - 2\hat{j} + 4\hat{k}$, $6\hat{i} + 7\hat{j} + 3\hat{k}$ and $-13\hat{i} + 17\hat{j} - \hat{k}$ are coplanar? 8

Q.6 (a) If $a = 2x\bar{i} - 3yz\bar{j} + x^2z\bar{k}$ and $\phi = 2z - x^3y$, then find $a.\nabla\phi$ at point (1, −1, 1). 9

(b) If $\phi = \ln|\bar{r}|$, then find $\nabla\phi$. 8

Q.7 (a) Find distance of the point P (2, −1, 4) to the line with equation $\bar{r} = \bar{a} + t\bar{d}$ where $\bar{a} = [3, -1, -1]$ and $\bar{d} = [1, 2, 1]$. 9

(b) Find an equation of the plane that passes through the points (3, 2, −1) and (1, −3, 4) and contains a line parallel to $2\bar{i} - 4\bar{j} + 3\bar{k}$. 8

Q.8 (a) Find an equation of the sphere which passes through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ at the point (1, 2, 3). 9

(b) Find the direction of Qibla of Badshahi Mosque, Lahore lattitude = 31° 35.4′ N and longitude = 74° 18.7′ E. 8

Section - III

Q.9 (a) Find inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 4 & 1 \\ 1 & 3 & 0 \end{bmatrix}$. 8

(b) Solve the equation $\begin{vmatrix} 1 & 2+x & 3 \\ 2 & 1 & 3+x \\ 3 & 2+x & 1 \end{vmatrix} = 0$ for x. 8

Q.10 (a) Prove that the determinant $\begin{vmatrix} bc & ca & ab \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ a^2 & b^2 & c^2 \end{vmatrix}$ vanishes. 8

(b) Determine whether the vectors (1, −2, 4, 1), (2, 1, 0, −3) and (1, −6, 1, 4) in R^4 are linearly independent or linearly dependent. 8

Section - IV

Q.11 (a) Solve $(1 + 2y^2)\, dy = y\cos x\, dx$, $y(0) = 1$. 8

(b) Find the orthogonal trajectories of the curve $x^2y^2 = c$. 8

Q.12 (a) Find the general solution of $(D^2 - 2D - 3)\, y = 2e^x - 10\sin x$. 8

(b) Solve $(x^2D^2 + 7xD + 5)\, y = x^5$. 8

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part-II
Annual Exam - 2017

Roll No. ....

Subject: Mathematics
PAPER: Optional

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

Note: Attempt any Five Questions in all, selecting at least Two questions from each section.

SECTION-I

Q.1	(a)	Solve $x^2 - 2x + 2 > 0$	**(10)**
	(b)	Evaluate $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ when n tends to infinity through positive integral values only.	**(10)**
Q.2	(a)	Let $f(x)=\begin{cases} x^2 sin\left(\frac{1}{x}\right) & if\ x\neq 0 \\ 0 & if\ x=0 \end{cases}$ Discuss the continuity of f at $x=0$	**(10)**
	(b)	Find the Maclaurin's series of $f(x)=\tan x$	**(10)**
Q.3	(a)	Find $\lim_{x\to\frac{\pi}{2}}(1-\sin x)^{\cos x}$	**(10)**
	(b)	Evaluate $\int\sqrt{x}\cos\sqrt{x}dx$.	**(10)**
Q.4	(a)	Solve $\int\frac{dx}{x\sqrt{a^2+x^2}}$	**(10)**
	(b)	Solve $\int e^x\left(\frac{1+x\ln x}{x}\right)dx$	**(10)**

SECTION-II

Q.5	(a)	If possible, find the inverse of the matrix $\begin{bmatrix}1 & 2 & -3\\ 1 & -2 & 1\\ 5 & -2 & -3\end{bmatrix}$	**(10)**
	(b)	Determine whether the vectors are linearly independent or not? $v_1=(1,1,1), v_2=(0,1,1), v_3=(0,1,-1)$.	**(10)**
Q.6	(a)	Solve the system of linear equations $2x+z=1,\quad 2x+4y-z=-2,\quad x-8y-3z=2$	**(10)**
	(b)	Find the reduced echelon form of the matrix $\begin{bmatrix}1 & 3 & -1 & 2\\ 0 & 11 & -5 & 3\\ 2 & -5 & 3 & 1\\ 4 & 1 & 1 & 5\end{bmatrix}$	**(10)**
Q.7	(a)	Show that $\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^2 & b^2 & c^2\end{vmatrix}=(a-b)(b-c)(c-a)$.	**(10)**
	(b)	Prove that $\begin{vmatrix}(b+c)^2 & a^2 & a^2\\ b^2 & (c+a)^2 & b^2\\ c^2 & c^2 & (a+b)^2\end{vmatrix}=2abc(a+b+c)^3$	**(10)**
Q.8	(a)	Show that $\begin{vmatrix}a & 1 & 1 & 1\\ 1 & a & 1 & 1\\ 1 & 1 & a & 1\\ 1 & 1 & 1 & a\end{vmatrix}=(a-1)^3(a+3)$	**(10)**
	(b)	If A is any non-singular matrix, then show that $(A^{-1})^{-1}=A$	**(10)**

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part-II
Annual Exam - 2017

Roll No. ....

Subject: Mathematics B Course-II
PAPER: (Mathematical Methods, Group Theory & Matrix Space)

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

Note: Attempt SIX quuestion in all by selecting TWO questions from Section-I, TWO questions from Section-II, ONE question from Section-III and ONE questions from Section-IV.

SECTION I

Q.1 (a) Separate into real and imaginary part$(\alpha + i\beta)^{(P+iq)}$ 9,8

(b) Prove that $cos4\theta = 8cos^4\theta - cos^2\theta + 1$

Q.2 (a) Prove that $tan^{-1}z = \frac{1}{2i}log\left(\frac{1+iz}{1-iz}\right)$ 9,8

(b) Examine $f(x,y) = 2x^2 - 4x + xy^2 - 1$ for relative extrema.

Q.3 (a) A rectangular plate expands in such a way that its length changes from 10 to 10.03 and its breadth changes from 8 to 8.02 .Find approximate value for the change in its area. 9,8

(b) Let $f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2+y^2} & if\ (x,y) \neq (0,0) \\ 0 & if\ (x,y) = (0,0) \end{cases}$ Examine the continuity at (0,0). Do $f_x\,(0,0)$ *and* $f_y(0,0)$ exist.

Q.4 (a) Find the area bounded by the parabola $y = x^2$ and the straight line $y = 2x + 3$ 9,8

(b) Use the cylindrical coordinate to evaluate $I = \iiint_S z\sqrt{x^2+y^2}\ dv$, S is the hemisphere $x^2 + y^2 + z^2 \leq 4, z \geq 0$

SECTION II

Q5(a)Test the series for absolute convergence ,conditional convergence or divergence $\sum_1^\infty \frac{sin\sqrt{n}}{\sqrt{n^3+1}}$.

(b) Test the series $\sum_1^\infty \frac{1}{(2n-1)^{1/3}}$ 9,8

Q6(a) If a,b ∈ Z,where a,b are not both zero and (a,b)=d then show that $\left(\frac{a}{d},\frac{b}{d}\right) = 1$ 9,8

(b) Find the solution set of the equation $23x - 49y = 179$.

Q7(a) Find the remainder when 7^{23} is divided by 8. 9,8

(b) Prove that if $\sum_1^\infty a_n$ converges then $\lim_{n\to\infty} a_n = 0$.

Q8(a) Using Integral Test show that Harmonic series $\sum_1^\infty \frac{1}{n}$ is divergent. 9,8

(b) Define a Prime Divisor and prove that every Integer n>1 has a prime divisor.

SECTION III

Q9(a)Let G be a group and H is a subgroup of G.Then the set $aHa^{-1} = \{aha^{-1}: h \in H\}$ is a subgroup of G.

(b) Determine whether the Permutation is even or odd $\begin{pmatrix}1&2&3&4&5&6\\3&5&2&1&6&4\end{pmatrix}$. 8,8

Q10(a) State and prove Lagrange Theorem. 8,8

(b) Let H.K be subgroups of an Abelian group G. Then show that the set $HK = \{hk!h \in H\ , k \in\ K\}$ Is a subgroup of G.

SECTION IV

Q11(a) Let X be a metric space and Let $\{x_0\}$ be a single ton subset of X. Then show that X- $\{x_o\}$ is open.

(b) Show that $d: R \times R \to R$ defined by d(x,y)= $\left[\frac{1}{x} - \frac{1}{y}\right]$ for all x,y $\in R - \{0\}$ is metric. 8,8

Q12(a) If A and B are two subsets of a metric space X.Then A $\subseteq B$ Implies that $A^d \subseteq B^d$. 8,8

(b) If A and B are two subsets of a metric space X. Then $\overline{AUB} = \bar{A}U\bar{B}$