M.Sc. Programme

Two years M.Sc. Mathematics programme consists of two parts namely Part-I and Part II. The regulation, Syllabi and Courses of Reading for the M.Sc. (Mathematics) Part-I and Part-II (Regular Scheme) are given below.

Regulations

The following regulations will be observed by M.Sc. (Mathematics) regular students
   i. There are a total of 1200 marks for M.Sc. (Mathematics) for regular students as is the case with other M.Sc. subjects.
   ii. There are five papers in Part-I and six papers in Part-II. Each paper carries 100 marks.
   iii. There is a Viva Voce Examination at the end of M.Sc. Part II. The topics of Viva Voce Examination shall be from the following courses of M.Sc. Part-I (carrying 100 marks):
         a) Real Analysis
         b) Algebra
         c) Complex Analysis
         d) Mechanics
         e) Topology and Functional Analysis

M.Sc. Part-I

The following five papers shall be studied in M.Sc. Part-I:

<table>
<thead>
<tr>
<th>Paper</th>
<th>Course</th>
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</thead>
<tbody>
<tr>
<td>I</td>
<td>Real Analysis</td>
</tr>
<tr>
<td>II</td>
<td>Algebra</td>
</tr>
<tr>
<td>III</td>
<td>Complex Analysis and Differential Geometry</td>
</tr>
<tr>
<td>IV</td>
<td>Mechanics</td>
</tr>
<tr>
<td>V</td>
<td>Topology and Functional Analysis</td>
</tr>
</tbody>
</table>

Note: All the papers of M.Sc. Part-I given above are compulsory.

M.Sc. Part-II

In M.Sc. Part-II examinations, there are six written papers. The following three papers are compulsory. Each paper carries 100 marks.

<table>
<thead>
<tr>
<th>Paper</th>
<th>Course</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Advanced Analysis</td>
</tr>
<tr>
<td>II</td>
<td>Methods of Mathematical Physics</td>
</tr>
<tr>
<td>III</td>
<td>Numerical Analysis</td>
</tr>
</tbody>
</table>
Optional Papers

A student may select any three of the following optional courses:

- Paper IV-VI option (i) Mathematical Statistics
- Paper IV-VI option (ii) Computer Applications
- Paper IV-VI option (iii) Group Theory
- Paper IV-VI option (iv) Rings and Modules
- Paper IV-VI option (v) Number Theory
- Paper IV-VI option (vi) Fluid Mechanics
- Paper IV-VI option (vii) Quantum Mechanics
- Paper IV-VI option (viii) Special Theory of Relativity and Analytical Mechanics
- Paper IV-VI option (ix) Electromagnetic Theory
- Paper IV-VI option (x) Operations Research
- Paper IV-VI option (xi) Theory of Approximation and Splines
- Paper IV-VI option (xii) Advanced Functional Analysis
- Paper IV-VI option (xiii) Solid Mechanics
- Paper IV-VI option (xiv) Theory of Optimization

Note: The students who opt for Computer Applications paper shall have to pass in both the theory and practical parts of the examinations.
Detailed Outline of Courses

M.Sc. Part I Papers

Paper I: Real Analysis
NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

Real Number System
- Ordered sets, Fields, Completeness property of real numbers
- The extended real number system, Euclidean spaces

Sequences and Series
- Sequences, Subsequences, Convergent sequences, Cauchy sequences
- Monotone and bounded sequences, Bolzano Weierstrass theorem
- Series, Convergence of series, Series of non-negative terms, Cauchy condensation test
- Partial sums, The root and ratio tests, Integral test, Comparison test
- Absolute and conditional convergence

Limit and Continuity
- The limit of a function, Continuous functions, Types of discontinuity
- Uniform continuity, Monotone functions

Differentiation
- The derivative of a function
- Mean value theorem, Continuity of derivatives
- Properties of differentiable functions.

Functions of Several Variables
- Partial derivatives and differentiability, Derivatives and differentials of composite functions
- Change in the order of partial derivative, Implicit functions, Inverse functions, Jacobians
- Maxima and minima, Lagrange multipliers

Section-II (4/9)

The Riemann-Stieltjes Integrals
- Definition and existence of integrals, Properties of integrals
- Fundamental theorem of calculus and its applications
- Change of variable theorem
- Integration by parts
Functions of Bounded Variation
- Definition and examples
- Properties of functions of bounded variation

Improper Integrals
- Types of improper integrals
- Tests for convergence of improper integrals
- Beta and gamma functions
- Absolute and conditional convergence of improper integrals

Sequences and Series of Functions
- Definition of point-wise and uniform convergence
- Uniform convergence and continuity
- Uniform convergence and integration
- Uniform convergence and differentiation

Recommended Books


Paper II: Algebra (Group Theory and Linear Algebra)

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

Groups
- Definition and examples of groups
- Subgroups lattice, Lagrange’s theorem
- Cyclic groups
- Groups and symmetries, Cayley’s theorem

Complexes in Groups
- Complexes and coset decomposition of groups
- Centre of a group
- Normalizer in a group
- Centralizer in a group
- Conjugacy classes and congruence relation in a group

Normal Subgroups
- Normal subgroups
• Proper and improper normal subgroups
• Factor groups
• Isomorphism theorems
• Automorphism group of a group
• Commutator subgroups of a group

Permutation Groups
• Symmetric or permutation group
• Transpositions
• Generators of the symmetric and alternating group
• Cyclic permutations and orbits, The alternating group
• Generators of the symmetric and alternating groups

Sylow Theorems
• Double cosets
• Cauchy’s theorem for Abelian and non-Abelian group
• Sylow theorems (with proofs)
• Applications of Sylow theory
• Classification of groups with at most 7 elements

Section-II (4/9)

Ring Theory
• Definition and examples of rings
• Special classes of rings
• Fields
• Ideals and quotient rings
• Ring Homomorphisms
• Prime and maximal ideals
• Field of quotients

Linear Algebra
• Vector spaces, Subspaces
• Linear combinations, Linearly independent vectors
• Spanning set
• Bases and dimension of a vector space
• Homomorphism of vector spaces
• Quotient spaces

Linear Mappings
• Mappings, Linear mappings
• Rank and nullity
• Linear mappings and system of linear equations
• Algebra of linear operators
• Space L( X, Y) of all linear transformations

Matrices and Linear Operators
• Matrix representation of a linear operator
• Change of basis
• Similar matrices
• Matrix and linear transformations
• Orthogonal matrices and orthogonal transformations
• Orthonormal basis and Gram Schmidt process

**Eigen Values and Eigen Vectors**
• Polynomials of matrices and linear operators
• Characteristic polynomial
• Diagonalization of matrices

**Recommended Books**


**Paper III: Complex Analysis and Differential Geometry**

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

**Section-I (5/9)**

**The Concept of Analytic Functions**
• Complex numbers, Complex planes, Complex functions
• Analytic functions
• Entire functions
• Harmonic functions
• Elementary functions: Trigonometric, Complex exponential, Logarithmic and hyperbolic functions

**Infinite Series**
• Power series, Derived series, Radius of convergence
• Taylor series and Laurent series

**Conformal Representation**
• Transformation, conformal transformation
• Linear transformation
• Möbius transformations

**Complex Integration**
• Complex integrals
• Cauchy-Goursat theorem
• Cauchy’s integral formula and their consequences
• Liouville’s theorem
• Morera’s theorem
• Derivative of an analytic function

**Singularity and Poles**
• Review of Laurent series
• Zeros, Singularities
• Poles and residues
• Cauchy’s residue theorem
• Contour Integration

**Expansion of Functions and Analytic Continuation**
• Mittag-Leffler theorem
• Weierstrass’s factorization theorem
• Analytic continuation

**Section-II (4/9)**

**Theory of Space Curves**
• Introduction, Index notation and summation convention
• Space curves, Arc length, Tangent, Normal and binormal
• Osculating, Normal and rectifying planes
• Curvature and torsion
• The Frenet-Serret theorem
• Natural equation of a curve
• Involutes and evolutes, Helices
• Fundamental existence theorem of space curves

**Theory of Surfaces**
• Coordinate transformation
• Tangent plane and surface normal
• The first fundamental form and the metric tensor
• The second fundamental form
• Principal, Gaussian, Mean, Geodesic and normal curvatures
• Gauss and Weingarten equations
• Gauss and Codazzi equations

**Recommended Books**

**Paper IV: Mechanics**

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

**Section-I (5/9)**

**Vector Integration**
- Line integrals
- Surface area and surface integrals
- Volume integrals

**Integral Theorems**
- Green’s theorem
- Gauss divergence theorem
- Stoke’s theorem

**Curvilinear Coordinates**
- Orthogonal coordinates
- Unit vectors in curvilinear systems
- Arc length and volume elements
- The gradient, Divergence and curl
- Special orthogonal coordinate systems

**Tensor Analysis**
- Coordinate transformations
- Einstein summation convention
- Tensors of different ranks
- Contravariant, Covariant and mixed tensors
- Symmetric and skew symmetric tensors
- Addition, Subtraction, Inner and outer products of tensors
- Contraction theorem, Quotient law
- The line element and metric tensor
- Christoffel symbols

**Section-II (4/9)**

**Non Inertial Reference Systems**
- Accelerated coordinate systems and inertial forces
- Rotating coordinate systems
- Velocity and acceleration in moving system: Coriolis, Centripetal and transverse acceleration
- Dynamics of a particle in a rotating coordinate system

**Planar Motion of Rigid Bodies**
• Introduction to rigid and elastic bodies, Degrees of freedom, Translations, Rotations, instantaneous axis and center of rotation, Motion of the center of mass
• Euler’s theorem and Chasle’s theorem
• Rotation of a rigid body about a fixed axis: Moments and products of inertia of various bodies including hoop or cylindrical shell, circular cylinder, spherical shell
• Parallel and perpendicular axis theorem
• Radius of gyration of various bodies

**Motion of Rigid Bodies in Three Dimensions**
• General motion of rigid bodies in space: Moments and products of inertia, Inertia matrix
• The momental ellipsoid and equimomental systems
• Angular momentum vector and rotational kinetic energy
• Principal axes and principal moments of inertia
• Determination of principal axes by diagonalizing the inertia matrix

**Euler Equations of Motion of a Rigid Body**
• Force free motion
• Free rotation of a rigid body with an axis of symmetry
• Free rotation of a rigid body with three different principal moments
• Euler’s Equations
• The Eulerian angles, Angular velocity and kinetic energy in terms of Euler angles, Space cone
• Motion of a spinning top and gyroscopes- steady precession, Sleeping top

**Recommended Books**

**Paper V: Topology & Functional Analysis**

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

**Section-I (4/9)**

**Topology**
• Definition and examples
• Open and closed sets
• Subspaces
• Neighborhoods
• Limit points, Closure of a set
• Interior, Exterior and boundary of a set

Bases and Sub-bases
• Base and sub bases
• Neighborhood bases
• First and second axioms of countability
• Separable spaces, Lindelöf spaces
• Continuous functions and homeomorphism
• Weak topologies, Finite product spaces

Separation Axioms
• Separation axioms
• Regular spaces
• Completely regular spaces
• Normal spaces

Compact Spaces
• Compact topological spaces
• Countably compact spaces
• Sequentially compact spaces

Connectedness
• Connected spaces, Disconnected spaces
• Totally disconnected spaces
• Components of topological spaces

Section-II (5/9)
Metric Space
• Review of metric spaces
• Convergence in metric spaces
• Complete metric spaces
• Completeness proofs
• Dense sets and separable spaces
• No-where dense sets
• Baire category theorem

Normed Spaces
• Normed linear spaces
• Banach spaces
• Convex sets
• Quotient spaces
• Equivalent norms
• Linear operators
• Linear functionals
• Finite dimensional normed spaces
• Continuous or bounded linear operators
• Dual spaces

**Inner Product Spaces**
• Definition and examples
• Orthonormal sets and bases
• Annihilators, Projections
• Hilbert space
• Linear functionals on Hilbert spaces
• Reflexivity of Hilbert spaces

**Recommended Books**
M.Sc. Part II Papers

Paper I: Advanced Analysis
NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (4/9)

Advanced Set Theory
- Equivalent Sets
- Countable and Uncountable Sets
- The concept of a cardinal number
- The cardinals $\aleph_0$ and $c$
- Addition and multiplication of cardinals
- Cartesian product, Axiom of Choice, Multiplication of cardinal numbers
- Order relation and order types, Well ordered sets, Transfinite induction
- Addition and multiplication of ordinals
- Statements of Zorn’s lemma, Maximality principle and their simple implications

Section-II (5/9)

Measure Theory
- Outer measure, Lebesgue Measure, Measureable Sets and Lebesgue measure, Non measurable sets, Measureable functions

The Lebesgue Integral
- The Riemann Integral, The Lebesgue integral of a bounded function
- The general Lebesgue integral

General Measure and Integration
- Measure spaces, Measureable functions, Integration, General convergence theorems
- Signed measures, The $L^p$-spaces, Outer measure and measurability
- The extension theorem
- The Lebesgue Stieltjes integral, Product measures

Recommended Books
4. Royden, H. L. Real Analysis, (Prentice Hall, 1988)
6. Halmos, P. R. Naive Set Theory, (Springer, 1974)
7. Halmos, P. R. Measure Theory, (Springer, 1974)
Paper II: Methods of Mathematical Physics
NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.
Section-I (5/9)
Sturm Liouville Systems
- Some properties of Sturm-Liouville equations
- Regular, Periodic and singular Sturm-Liouville systems and its applications
Series Solutions of Second Order Linear Differential Equations
- Series solution near an ordinary point
- Series solution near regular singular points
Series Solution of Some Special Differential Equations
- Hypergeometric function $F(a, b, c; x)$ and its evaluation
- Series solution of Bessel equation
- Expression for $J_n(X)$ when $n$ is half odd integer, Recurrence formulas for $J_n(X)$
- Orthogonality of Bessel functions
- Series solution of Legendre equation
Introduction to PDEs
- Review of ordinary differential equation in more than one variables
- Linear partial differential equations (PDEs) of the first order
- Cauchy’s problem for quasi-linear first order PDEs
PDEs of Second Order
- PDEs of second order in two independent variables with variable coefficients
- Cauchy’s problem for second order PDEs in two independent variables
Boundary Value Problems
- Laplace equation and its solution in Cartesian, Cylindrical and spherical polar coordinates
- Dirichlet problem for a circle
- Poisson’s integral for a circle
- Wave equation
- Heat equation
Section-II (4/9)
Fourier Methods
- The Fourier transform
- Fourier analysis of generalized functions
- The Laplace transform
Green’s Functions and Transform Methods
- Expansion for Green’s functions
- Transform methods
- Closed form of Green’s functions
Variational Methods
- Euler-Lagrange equations
- Integrand involving one, two, three and n variables
- Necessary conditions for existence of an extremum of a functional
• Constrained maxima and minima

Recommended Books

Paper III: Numerical Analysis
NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.
Section-I (5/9)
Error Analysis
• Errors, Absolute errors, Rounding errors, Truncation errors
• Inherent Errors, Major and Minor approximations in numbers
The Solution of Linear Systems
• Gaussian elimination method with pivoting, LU Decomposition methods, Algorithm and convergence of Jacobi iterative Method, Algorithm and convergence of Gauss Seidel Method
• Eigenvalue and eigenvector, Power method
The Solution of Non-Linear Equation
• Bisection Method, Fixed point iterative method, Newton Raphson method, Secant method, Method of false position, Algorithms and convergence of these methods
Difference Operators
• Shift operators
• Forward difference operators
• Backward difference operators
• Average and central difference operators

Ordinary Differential Equations
• Euler’s, Improved Euler’s, Modified Euler’s methods with error analysis
• Runge-Kutta methods with error analysis
• Predictor-corrector methods for solving initial value problems
• Finite Difference, Collocation and variational methods for boundary value problems

Section-II (4/9)
Interpolation
• Lagrange’s interpolation
• Newton’s divided difference interpolation
• Newton’s forward and backward difference interpolation, Central difference interpolation
• Hermite interpolation
• Spline interpolation
• Errors and algorithms of these interpolations

Numerical Differentiation
• Newton’s Forward, Backward and central formulae for numerical differentiation

Numerical Integration
• Rectangular rule
• Trapezoidal rule
• Simpson rule
• Boole’s rule
• Weddle’s rule
• Gaussian quadrature formulae
• Errors in quadrature formulae
• Newton-Cotes formulae

Difference Equations
• Linear homogeneous and non-homogeneous difference equations with constant coefficients

Recommended Books
Paper (IV-VI) option (i): Mathematical Statistics

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.
Section-I (4/9)

Probability Distributions
- The postulates of probability
- Some elementary theorems
- Addition and multiplication rules
- Baye’s rule and future Baye’s theorem
- Random variables and probability functions

Discrete Probability Distributions
- Uniform, Bernoulli and binomial distribution
- Hypergeometric and geometric distribution
- Negative binomial and Poisson distribution

Continuous Probability Distributions
- Uniform and exponential distribution
- Gamma and beta distributions
- Normal distribution

Mathematical Expectations
- Moments and moment generating functions
- Moments of binomial, Hypergeometric, Poisson, Gamma, Beta and normal distributions

Section-II (5/9)

Functions of Random Variables
- Distribution function technique
- Transformation technique: One variable, Several variables
- Moment-generating function technique

Sampling Distributions
- The distribution of mean and variance
- The distribution of differences of means and variances
- The Chi-Square distribution
- The $t$ distribution
- The $F$ distribution

Regression and Correlation
- Linear regression
• The methods of least squares
• Normal regression analysis
• Normal correlation analysis
• Multiple linear regression (along with matrix notation)

Recommended Books


Paper (IV-VI) option (ii): Computer Applications

NOTE:
The evaluation of this paper will consist of two parts:
1. Written examination: 50 marks
2. Practical examination: 50 marks

The practical examination includes 10 marks for the note book containing details of work done in the computer laboratory and 10 marks for the oral examination. It will involve writing and running programs on computational projects. Practical examination will be of three hours duration in which one or more computational projects will be examined.

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (4/9)

Fortran Programming
• Constants
• Variables
• Implicit declaration
• Intrinsic functions
• Arithmetic operations
• Arithmetic expressions
• Assignment statements
• Relational operators
• Format statements
• The block if structure
• The block do loop structure
• Count controlled do loop structure
• Logical constants, variables and expressions
• The case statement
• Function subprograms
• Subroutines
• One dimensional and two dimensional Arrays
• Implementation of Fortran in terms of short programs

**Section-II (5/9)**

**Implementation of Fortran to the numerical methods**

**System of linear equations**
• Gaussian elimination method with pivoting
• LU Decomposition methods
• Jacobi’s iterative method
• Gauss-Seidel method

**Solutions of non-linear equations**
• Bisection method
• Newton-Raphson method
• Secant method
• Regula Falsi method

**Interpolation**
• Lagrange interpolation
• Newton’s divided and forward difference interpolation

**Numerical integration**
• Rectangular rule
• Trapezoidal rule
• Simpson’s rule
• Boole’s rule
• Weddle’s rule

**Differential equations**
• Euler’s methods
• Runge- Kutta methods
• Predictor-corrector methods

**Mathematica**
• Syntax of Fortran in Mathematica
• Symbolic representation
• Algebraic calculations
• Graphs

**Recommended Books**


**Paper (IV-VI) option (iii): Advanced Group Theory**

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

**Section-I (4/9)**

**The Orbit Stabilizer Theorem**
- Stabilizer, Orbit, A group with $p^2$ elements
- Simplicity of $A_n$, $n \geq 5$
- Classification of Groups with at most 8 elements

**Sylow Theorems**
- Sylow theorems (with proofs)
- Applications of Sylow Theory

**Products in Groups**
- Direct Products
- Classification of Finite Abelian Groups
- Characteristic and fully invariant subgroups
- Normal products of groups
- Holomorph of a group

**Section-II (5/9)**

**Series in Groups**
- Series in groups
- Zassenhaus lemma
- Normal series and their refinements
- Composition series
- The Jordan Holder Theorem

**Solvable Groups**
- Solvable groups, Definition and examples
- Theorems on solvable groups

**Nilpotent Groups**
• Characterisation of finite nilpotent groups
• Frattini subgroups

Extensions
• Central extensions
• Cyclic extensions
• Groups with at most 31 elements

Linear Groups
• Linear groups, types of linear groups
• Representation of linear groups
• The projective special linear groups

Recommended Books


Paper (IV-VI) option (iv): Rings and Modules
NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

Ring Theory
• Construction of new rings
• Direct sums, Polynomial rings
• Matrix rings
• Divisors, units and associates
• Unique factorisation domains
• Principal ideal domains and Euclidean domains

Field Extensions
• Algebraic and transcendental elements
• Degree of extension
• Algebraic extensions
• Reducible and irreducible polynomials
• Roots of polynomials

Section-II (4/9)
Modules
• Definition and examples
• Submodules
• Homomorphisms
• Quotient modules
• Direct sums of modules
• Finitely generated modules
• Torsion modules
• Free modules
• Basis, Rank and endomorphisms of free modules
• Matrices over rings and their connection with the basis of a free module
• A module as the direct sum of a free and a torsion module

Recommended Books


Paper (IV-VI) option (v): Number Theory

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section- I (5/9)
Congruences
• Elementary properties of prime numbers
• Residue classes and Euler’s function
• Linear congruences and congruences of higher degree
• Congruences with prime moduli
• The theorems of Fermat, Euler and Wilson

Number-Theoretic Functions
• Möbius function
• The function \([x]\), The symbols \(O\) and their basic properties

**Primitive roots and indices**
• Integers belonging to a given exponent \((\text{mod } p)\)
• Primitive roots and composite moduli
• Determination of integers having primitive roots
• Indices, Solutions of Higher Congruences by Indices

**Diophantine Equations**
• Equations and Fermat’s conjecture for \(n = 2, n = 4\)

**Section-II (4/9)**

**Quadratic Residues**
• Composite moduli, Legendre symbol
• Law of quadratic reciprocity
• The Jacobi symbol

**Algebraic Number Theory**
• Polynomials over a field
• Divisibility properties of polynomials
• Gauss’s lemma
• The Eisenstein’s irreducibility criterion
• Symmetric polynomials
• Extensions of a field
• Algebraic and transcendental numbers
• Bases and finite extensions, Properties of finite extensions
• Conjugates and discriminants
• Algebraic integers in a quadratic field, Integral bases
• Units and primes in a quadratic field
• Ideals, Arithmetic of ideals in an algebraic number field
• The norm of an ideal, Prime ideals

**Recommended Books**

Paper (IV-VI) option (vi): Fluid Mechanics

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)
Conservation of Matter

- Introduction
- Fields and continuum concepts
- Lagrangian and Eulerian specifications
- Local, Convective and total rates of change
- Conservation of mass
- Equation of continuity
- Boundary conditions

Nature of Forces and Fluid Flow

- Surface and body forces
- Stress at a point
- Viscosity and Newton’s viscosity law
- Viscous and inviscid flows
- Laminar and turbulent flows
- Compressible and incompressible flows

Irrotational Fluid Motion

- Velocity potential from an irrotational velocity field
- Streamlines
- Vortex lines and vortex sheets
- Kelvin’s minimum energy theorem
- Conservation of linear momentum
- Bernoulli’s theorem and its applications
- Circulation, Rate of change of circulation (Kelvin’s theorem)
- Aaxially symmetric motion
- Stokes’s stream function

Two-dimensional Motion

- Stream function
- Complex potential and complex velocity, Uniform flows
- Sources, Sinks and vortex flows
- Flow in a sector
- Flow around a sharp edge
- Flow due to a doublet

Section-II (4/9)

Two and Three-Dimensional Potential Flows

- Circular cylinder without circulation
- Circular cylinder with circulation
• Blasius theorem
• Kutta condition and the flat-plate airfoil
• Joukowski airfoil
• Vortex motion
• Karman’s vortex street
• Method of images
• Velocity potential
• Stoke’s stream function
• Solution of the Potential equation
• Uniform flow
• Source and sink
• Flow due to a doublet

**Viscous Flows of Incompressible Fluids**

• Constitutive equations
• Navier-Stokes equations and their exact solutions
• Steady unidirectional flow
• Poiseuille flow
• Couette flow
• Flow between rotating cylinders
• Stokes’ first problem
• Stokes’ second problem

**Approach to Fluid Flow Problems**

• Similarity from a differential equation
• Dimensional analysis
• One dimensional, Steady compressible flow

**Recommended Book**

Paper (IV-VI) option (vii): Quantum Mechanics

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (4/9)

Inadequacy of Classical Mechanics

- Black body radiation
- Photoelectric effect
- Compton effect
- Bohr’s theory of atomic structure
- Wave-particle duality
- The de Broglie postulate
- Heisenberg uncertainty principle

The Postulates of Quantum Mechanics: Operators, Eigenfunctions and Eigenvalues

- Observables and operators
- Measurement in quantum mechanics
- The state function and expectation values
- Time development of the state function (Schrödinger wave equation)
- Solution to the initial-value problem in quantum mechanics
- Parity operators

Preparatory Concepts: Function Spaces and Hermitian Operators

- Particle in a box
- Dirac notation
- Hilbert space
- Hermitian operators
- Properties of Hermitian operators

Additional One-Dimensional Problems: Bound and Unbound States

- General properties of the 1-dimensional Schrödinger equation
- Unbound states
- One-dimensional barrier problems
- The rectangular barrier: Tunneling

Section-II (5/9)

Harmonic Oscillator and Problems in Three-Dimensions

- The harmonic oscillator
- Eigenfunctions of the harmonic oscillator
- The harmonic oscillator in momentum space
- Motion in three dimensions
- Spherically symmetric potential and the hydrogen atom

Angular Momentum

- Basic properties
• Eigenvalues of the angular momentum operators
• Eigenfunctions of the orbital angular momentum operators $L^2$ and $L_z$
• Commutation relations between components of angular momentum and their representation in spherical polar coordinates

**Scattering Theory**

• The scattering cross-section
• Scattering amplitude
• Scattering equation
• Born approximation
• Partial wave analysis

**Perturbation Theory**

• Time independent perturbation of non-degenerate and degenerate cases
• Time-dependent perturbations

**Identical Particles**

• Symmetric and antisymmetric eigenfunctions
• The Pauli exclusion principle

**Recommended Book**


**Paper (IV-VI) optional (viii): Special Relativity and Analytical Dynamics**

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

**Section-I (5/9)**

**Derivation of Special Relativity**

• Fundamental concepts
• Einstein’s formulation of special relativity
• The Lorentz transformations
• Length contraction, Time dilation and simultaneity
• The velocity addition formulae
• Three dimensional Lorentz transformations

**The Four-Vector Formulation of Special Relativity**
• The four-vector formalism
• The Lorentz transformations in 4-vectors
• The Lorentz and Poincare groups
• The null cone structure
• Proper time

Applications of Special Relativity
• Relativistic kinematics
• The Doppler shift in relativity
• The Compton effect
• Particle scattering
• Binding energy, Particle production and particle decay

Electromagnetism in Special Relativity
• Review of electromagnetism
• The electric and magnetic field intensities
• The electric current
• Maxwell’s equations and electromagnetic waves
• The four-vector formulation of Maxwell’s equations

Section-II (4/9)

Lagrange’s Theory of Holonomic and Non-Holonomic Systems
• Generalized coordinates
• Holonomic and non-holonomic systems
• D’Alembert’s principle, D-delta rule
• Lagrange equations
• Generalization of Lagrange equations
• Quasi-coordinates
• Lagrange equations in quasi-coordinates
• First integrals of Lagrange equations of motion
• Energy integral
• Lagrange equations for non-holonomic systems with and without Lagrange multipliers
• Hamilton’s Principle for non-holonomic systems

Hamilton’s Theory
• Hamilton’s principle
• Generalized momenta and phase space
• Hamilton’s equations
• Ignorable coordinates, Routhian function
• Derivation of Hamilton’s equations from a variational principle
• The principle of least action

Canonical Transformations
• The equations of canonical transformations
Examples of canonical transformations
The Lagrange and Poisson brackets
Equations of motion, Infinitesimal canonical transformations and conservation theorems in the Poisson bracket formulation

Hamilton-Jacobi Theory
• The Hamilton-Jacobi equation for Hamilton’s principal function
• The harmonic oscillator problem as an example of the Hamilton-Jacobi method
• The Hamilton-Jacobi equation for Hamilton’s characteristic function
• Separation of variables in the Hamilton-Jacobi equation

Recommended Books


Paper (IV-VI) option (ix): Electromagnetic Theory
NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

**Electrostatic Fields**
• Coulomb’s law, The electric field intensity and potential
• Gauss’s law and deductions, Poisson and Laplace equations
• Conductors and condensers
• Dipoles, The linear quadrupole
• Potential energy of a charge distribution, Dielectrics
• The polarization and the displacement vectors

**Magnetostatic Fields**
• The Magnetostatic law of force
• The magnetic induction
• The Lorentz force on a point charge moving in a magnetic field
• The divergence of the magnetic field
• The vector potential
• The conservation of charge and the equation of continuity
• The Lorentz condition
• The curl of the magnetic field
• Ampere’s law and the scalar potential

**Steady and Slowly Varying Currents**

• Electric current, Linear conductors
• Conductivity, Resistance
• Kirchhoff’s laws
• Current density vector
• Magnetic field of straight and circular current
• Magnetic flux, Vector potential
• Forces on a circuit in magnetic field
• The Faraday induction law
• Induced electromotance in a moving system
• Inductance and induced electromotance
• Energy stored in a magnetic field

**Section-II (4/9)**

**The Equations of Electromagnetism**

• Maxwell’s equations in free space and material media
• Solution of Maxwell’s equations

**Electromagnetic Waves**

• Plane electromagnetic waves in homogeneous and isotropic media
• The Poynting vector in free space
• Propagation of plane electromagnetic waves in non-conductors
• Propagation of plane electromagnetic waves in conducting media
• Reflection and refraction of plane waves

**Guided Waves**

• Guided waves, Coaxial line, Hollow rectangular wave guide
• Radiation of electromagnetic waves
• Electromagnetic field of a moving charge

**Recommended Books**

Paper (IV-VI) option (x): Operations Research

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)
Linear Programming
- Linear programming, Formulations and graphical solution
- Simplex method
- M-Technique and two-phase technique
- Special cases
Duality and Sensitivity Analysis
- The dual problem, Primal-dual relationships
- Dual simplex method
- Sensitivity and postoptimal analysis
Transportation Models
- North-West corner
- Least-Cost and Vogel’s approximations methods
- The method of multipliers
- The assignment model
- The transhipment model
- Hungarian method

Section-II (4/9)
Net work Minimization and Integer Programming
- Network minimization
- Shortest-Route algorithms for acyclic networks
- Maximal-flow problem
- Matrix definition of LP problem
- Revised simplex method, Bounded variables
- Decomposition algorithm
- Parametric linear programming
- Applications of integer programming
- Cutting-plane algorithms
- Branch-and-bound method
- Elements of dynamic programming
- Programmes by dynamic programming

Recommended Books

**Paper (IV-VI) option (xi): Theory of Approximation and Splines**

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

**Section-I (4/9)**

**Euclidean Geometry**
- Basic concepts of Euclidean geometry
- Scalar and vector functions
- Barycentric coordinates
- Convex hull
- Affine maps: Translation, Rotation, Scaling, Reflection and shear

**Approximation using Polynomials**
- Curve Fitting: Least squares line fitting, Least squares power fit, Data linearization method for exponential functions, Nonlinear least-squares method for exponential functions, Transformations for data linearization, Linear least squares, Polynomial fitting
- Chebyshev polynomials, Padé approximations

**Section-II (5/9)**

**Parametric Curves (Scalar and Vector Case)**
- Cubic algebraic form
- Cubic Hermite form
- Cubic control point form
- Bernstein Bezier cubic form
- Bernstein Bezier general form
- Uniform B-Spline cubic form
- Matrix forms of parametric curves
- Rational quadratic form
- Rational cubic form
- Tensor product surface, Bernstein Bezier cubic patch, Quadratic by cubic Bernstein Bezier patch, Bernstein Bezier quartic patch
- Properties of Bernstein Bezier form: Convex hull property, Affine invariance property, Variation diminishing property
- Algorithms to compute Bernstein Bezier form
- Derivation of Uniform B-Spline form

**Spline Functions**
- Introduction to splines
- Cubic Hermite splines
- End conditions of cubic splines: Clamped conditions, Natural conditions, 2<sup>nd</sup> Derivative conditions, Periodic conditions, Not a knot conditions
• General Splines: Natural splines, Periodic splines
• Truncated power function, Representation of spline in terms of truncated power functions, examples

Recommended Books

Paper (IV-VI) option (xii): Advanced Functional Analysis
NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.
Section-I (4/9)
Compact Normed Spaces
• Completion of metric spaces
• Completion of normed spaces
• Compactification
• Nowhere and everywhere dense sets and category
• Generated subspaces and closed subspaces
• Factor Spaces
• Completeness in the factor spaces

Complete Orthonormal set
• Complete orthonormal sets
• Total orthonormal sets
• Parseval’s identity
• Bessel’s inequality

The Specific geometry of Hilbert Spaces
• Hilbert spaces
• Bases of Hilbert spaces
• Cardinality of Hilbert spaces
• Linear manifolds and subspaces
• Orthogonal subspaces of Hilbert spaces
- Polynomial bases in $L^2$ spaces

Section-II (5/9)
**Fundamental Theorems**
- Hahn Banach theorems
- Open mapping and closed graph theorems
- Banach Steinhass theorem

**Semi-norms**
- Semi norms, Locally convex spaces
- Quasi normed linear spaces
- Bounded linear functionals
- Hahn Banach theorem

**Dual or Conjugate spaces**
- First and second dual spaces
- Second conjugate space of $l_p$
- The Riesz representation theorem for linear functionals on a Hilbert spaces
- Conjugate space of $C[a,b]$
- A representation theorem for bounded linear functionals on $C[a,b]$

**Uniform Boundedness**
- Weak convergence
- The Principle of uniform boundedness
- Consequences of the principle of uniform boundedness

**Recommended Books**

Paper (IV-VI) option (xiii): Solid Mechanics
Five questions to be attempted, selecting at least two questions from each section.

Section-I (4/9)
**Elasticity**
- Analysis of stress and strain
- Generalized Hook’s law
- Differential equations of equilibrium in terms of stress and in terms of displacements
- Boundary conditions
- Compatibility equations
- Plane stress, Plane strain, Stress functions
- Two-dimensional problems in rectangular and polar co-ordinates
- Torsion problems

Section-II (5/9)
Elastodynamics
- Equations of wave propagation in elastic solids
- Primary and secondary waves
- Reflection and transmission at plane boundaries
- Surface wave: Love waves and Raleigh waves
- Dispersion relations
- Geophysical applications

Recommended Books

Paper (IV-VI) optional (xiv): Theory of Optimization
NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.
Section-I (5/9)
The Mathematical Programming Problem
- Formal statement of the problem
- Types of maxima, the Weierstrass Theorem and the Local-Global theorem
- Geometry of the problem

Classical Programming
- The unconstrained case
- The method of Lagrange multipliers
- The interpretation of the Lagrange multipliers

Non-linear Programming
- The case of no inequality constraints
- The Kuhn-Tucker conditions
- The Kuhn-Tucker theorem
- The interpretation of the Lagrange multipliers
• Solution algorithms

**Linear Programming**
• The Dual problems of linear programming
• The Lagrangian approach; Existence, Duality and complementary slackness theorems
• The interpretation of the dual
• The simplex algorithm

**Section-II (4/9)**

**The Control Problem**
• Formal statement of the problem
• Some special cases
• Types of Control
• The Control problem as one of programming in on infinite dimensional space; The generalized Weierstrass theorem

**Calculus of Variations**
• Euler equations
• Necessary conditions
• Transversality condition
• Constraints

**Dynamic Programming**
• The principle of optimality and Bellman’s equation
• Dynamic programming and the calculus of variations
• Dynamic programming solution of multistage optimization problems

**Maximum Principle**
• Co-state variables, The Hamiltonian and the maximum principle
• The interpretation of the co-state variables
• The maximum principle and the calculus of variations
• The maximum principle and dynamic programming
• Examples

**Recommended Books**