University of the Punjab

Examination: BSc 2-Years Program
Subject: Mathematics A-course (MODEL PAPER)
Time: 3 Hours

Class: III year (2016)
Paper-A
Max Marks: 100

Note: Attempt six questions by selecting two questions from Section I, two questions from Section II, one question from Section III and one question from Section IV

SECTION-I

Q. 1 **(a)** (i) Prove that $\left||a|-|b|\right| \le |a-b|$ for every $a,b \in R$ 4+4

(ii)Solve the inequality $\dfrac{x^2-2}{1-2x} > 1$

(b) (i) Evaluate the Bracket function $\lim\limits_{x\to 1} x^3\left[\dfrac{1}{x}\right]$ 4+5

(ii) Discuss the continuity of the function $f(x)=\begin{cases} x\sin\dfrac{1}{x}, & \text{if } x\ne 0 \\ 0, & \text{if } x=0 \end{cases}$ at $x=0$

Q. 2 **(a)** Differentiate $\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$ with respect to $\arccos x^2$ 8

(b) If $y=e^{m\arcsin x}$, Show that 9

$(1-x^2)y^{(n+2)}-(2n+1)xy^{(n+1)}-(n^2+m^2)y^{(n)}=0$

Find the values of $y^{(n)}$ at $x=0$

Q. 3 **(a)** Use Differential to find approximate value of $\cos 61°$. 9

(b) Prove that $\dfrac{x}{x+1} < \ln(x+1) < x$ 8

Q. 4 **(a)** Evaluate the Given Limits 4+4

(i) $\lim\limits_{x\to 0}\log_{\tan x}(\tan 2x)$

(ii) $\lim\limits_{x\to 0}\dfrac{\sqrt{x}-\sqrt{sinx}}{x^{3\backslash 2}}$

(b) State and Prove Cauchy Mean Value Theorem. 9

SECTION-II

Q. 5 **(a)** Evaluate $\int\left[\pi^{\sin x}+(sinx)^{\pi}\right]\cos x dx$ 8

(b) Evaluate $\int_0^{\pi/2}\ln(\sin x)dx$ 9

Q. 6 **(a)** Evaluate $\int\dfrac{dx}{a+b\cos x}$ 8

(b) Analyze and graph the conics represented $\sqrt{x}+\sqrt{y}=1$ 9

Q. 7 **(a)** Calculate the values of $\int_0^{2a} x^n\sqrt{2ax-x^2}\,dx$, n being a positive integer. Hence or otherwise calculate the values of $\int_0^{2a} x^4\sqrt{2ax-x^2}\,dx$ — 9

(b) Find the pedal equation of curve $x = a\cos^3\theta,\ y = a\sin^3\theta$. — 8

Q. 8 **(a)** Show that the curves $r^m = a^m\cos m\theta$ and $r^m = a^m \sin m\theta$ cut each other orthogonally. — 8

(b) If a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, with centre C, meets the major and minor axes in T and t, prove that $\frac{a^2}{CT^2}+\frac{b^2}{Ct^2}=1$. — 9

SECTION-III

Q. 9 **(a)** Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6r\sqrt{3}$. — 8

(b) Find the area of the region that lies outside the cardioid $r = 1+\cos\theta$ and inside the circle $r = 3\cos\theta$. — 8

Q. 10 **(a)** Prove that the intrinsic equation of the cycloid $x = a(\theta+\sin\theta)$, $y = a(1-\cos\theta)$ is $s = 4a\sin\theta$. — 8

(b) Find the envelope of the family of straight lines joining the extremities of a pair of conjugate diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. — 8

SECTION-IV

Q. 11 **(a)** A straight line makes angles of measure $\alpha,\beta,\gamma,\delta$ with the diagonals of a cube. Prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$. — 8

(b) Find an equation of the plane passing through the point $(2,-3,1)$ and containing the lines $x-3=2y=3z-1$. — 8

Q. 12 **(a)** Find equation of sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y-4z-8=0$ is a great circle. — 8

(b) Find the Direction of Qibla of Badshahi Mosque, Lahore, latitude $=31^\circ 35.4' N$ and longitude $=74^\circ 18.4' E$. — 8

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Examination: BSc 2-Years Program **Class: III year (2016)**
Subject: Mathematics B Course (MODEL PAPER) **Paper-A**
Time: 3 Hours **Max Marks: 100**

NOTE: Attempt SIX questions by selecting **ONE** question from **Section-I**, **TWO** questions from **Section-II**, **TWO** questions from **Section-III** and **ONE** question from **Section-IV**.

SECTION-I

Q.1(a) If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then show that $\vec{a}$ and $\vec{b}$ are perpendicular to each other. 8

(b) Define reciprocal system of vectors and find a set of vectors reciprocal to the set $-\hat{i}+\hat{j}+\hat{k}, \hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+2\hat{k}$. 8

Q.2(a) Define the gradient of a scalar point function and evaluate $\nabla(r+\ln r+\tan 3r+\sqrt{r})$. 8

(b) Show that $\text{curl}\left(\frac{\vec{a}\times\vec{r}}{r^3}\right)=\frac{3\vec{a}.\vec{r}}{r^5}\vec{r}-\frac{\vec{a}}{r^3}$, where $\vec{a}$ is a constant vector. 8

SECTION-II

Q.3(a) Two forces $\vec{P}+\vec{Q}$ and $\vec{P}-\vec{Q}$ make an angle 2α with one another and their resultant makes an angle θ with the bisector of the angle between them. Show that $P\tan\theta = Q\tan\alpha$. 9

(b) Prove that the necessary and sufficient condition for the equilibrium of a system of coplanar couples is that the algebraic sum of their moments is zero. 8

Q.4(a) *AB* and *AC* are similar uniform rods, of length *a*, smoothly joined at *A*. *BD* is a weightless bar, of length *b*, smoothly joined at *B*, and fastened at *D* to a smooth ring sliding on *AC*. The system is hung on a small smooth pin at *A*. show that the rod *AC* makes with the vertical an angle

$$\tan^{-1}\frac{b}{a+\sqrt{a^2-b^2}}$$ 9

(b) Two uniform solid spheres composed of the same material and whose diameters are 6 in. and 12 in. respectively, are firmly united. Find the centre of mass of the combined body. 8

Q.5(a) Find the position of the centroid of a quadrant of an elliptic lamina. 9

(b) Two bodies, weights W_1, W_2, are placed on an inclined plane and are connected by a light string which coincides with a line of greatest slope of the plane. If the coefficients of friction between the bodies and the plane be respectively μ_1 and μ_2. Find the inclination of the plane to the horizon when both bodies arc on the point of motion, it being assumed that the smoother body is below the other. 8

Q.6(a) A regular octahedron form of twelve equal rods each of weight W freely jointed together is suspended from the one corner. Show that the thrust with each horizontal rod is $\frac{3}{\sqrt{2}}W$. 9

(b) A uniform hemispherical shell rest on a rough inclined plane whose angle of friction λ. Show that inclination of the plane base to the horizontal cannot be greater than $\sin^{-1}(2\sin\lambda)$. 8

SECTION-III

Q.7(a) Find the radial and transverse components of the velocity of a particle moving along the curve $ax^2 + by^2 = 1$ at any time t if the polar angle $\theta = ct^2$. 9

(b) A particle is projected vertically upwards. After a time *t*, another particle is sent up from the same point with the same velocity and meets the first at height *h* during the downward flight of the first. Find the velocity of projection. 8

Q.8(a) A particle moves in a straight line with an acceleration kv^3. If its initial velocity is *u*, find the velocity and the time spent when the particle has travelled a distance *x*. 9

(b) A particle of mass m moves along x-axis under the influences of a conservative field of force having potential $V(x)$. If a particle is located at positions x_1 and x_2 at time t_1 and t_2 respectively. Prove that if E is the total energy. Then $t_2 - t_1 = \sqrt{\frac{m}{2}} \int_{x_1}^{x_2} \frac{dx}{\sqrt{E - V(x)}}$ 8

Q.9(a) The range of a rifle bullet is 1200 yards when α is the angle of elevation of projection. Show that if the rifle bullet is fired with the same elevation from a car travelling at 10 miles per hour towards the target, the range will be increased by $220\sqrt{\tan\alpha}$.

9

(b) A ball is dropped from the top of a tower of height h. At the same moment, another ball is thrown from a point of the ground at a distance k from the foot of tower so as to strike the first ball at a depth d. Show that the initial speed and the direction of projection of the second ball are respectively

$\sqrt{\frac{g(h^2+k^2)}{2d}}$ and $\tan^{-1}\left(\frac{h}{k}\right)$ 8

Q.10(a) A shell of mass M is moving with speed V. An internal explosion generates an amount of energy E and breaks the shell into two portions whose masses are in the ratio $m_1 : m_2$. The fragments continue to move in the original line of motion of the shell. Show that their speeds are

$V + \sqrt{\frac{2m_2E}{m_1M}}$ and $V - \sqrt{\frac{2m_1E}{m_2M}}$ 9

(b) An un-damped oscillator is subject to the restorative force $-m\omega_0^2 x$ and an applied force $mF_0 \sin\omega t$. Find the motion assuming that initially when $t = 0, x = 0$ and $\dot{x} = 0$. 8

SECTION-IV

Q.11 (a) State and prove Kepler's First Law. 8

(b) A smooth sphere impinges on another one at rest. After the collision their directions of motion are at right angle. Show that, if they are elastic, their masses must be equal. Where 2*a* the major axis, *e* the eccentricity, and *T* is the periodic time. 8

Q.12 (a) Two elastic spheres of masses *m* and *m'* moving with velocity *u* and *u'* impinge directly. If e be the coefficient of restitution, find their velocities after impact. 8

(b) A planet is describing an ellipse about the sun as focus. Show that its velocity away from the sun is greatest when the radius vector to the planet is at right angles to the major axis of the path, and that it then is $\frac{2\pi ac}{T(1-e^2)}$. 8

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Examination: BSc 2-Years Program | **Class: III year (2016)**
Subject: General Mathematics (MODEL PAPER) | **Paper-A**
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(b) (i) Evaluate the Bracket function $\lim_{x\to 1} x^3\left[\dfrac{1}{x}\right]$ 4+5

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(i) $\lim_{x\to 0}\log_{\tan x}(\tan 2x)$

(ii) $\lim_{x\to 0}\dfrac{\sqrt{x}-\sqrt{sinx}}{x^{3\backslash 2}}$

(b) Find the Maclaurin series of $f(x)=\ln(1+x)$. 9

SECTION-II

Q. 5 (a) Evaluate $\int\left[\pi^{\sin x}+\left(sinx\right)^{x}\right]\cos x dx$ 8

(b) Evaluate $\int_0^{\pi/2}\ln(\sin x)dx$ 9

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(b) Find the envelope of the family of straight lines joining the extremities of a pair of conjugate diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, 8

SECTION-IV

Q. 11 (a) Let $f(x,y)=\begin{cases}\frac{xy(x^2-y^2)}{x^2+y^2} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$ 8

Show that $f_{xy}(0,0)\neq f_{yx}(0,0)$

(b) Show that the volume of the largest parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{b^2}=1$ is $\frac{8abc}{3\sqrt{3}}$ 8

Q. 12 (a) Find the volume of solid bounded by the paraboloid $z=4-x^2-y^2$ and the xy – plane. 8

(b) Use cylindrical coordinates to evaluate $I=\iiint_s z\sqrt{x^2+y^2}\,dV$, where S is the hemisphere $x^2+y^2+z^2\le 4,\quad z\ge 0.$ 8