UNIVERSITY OF THE PUNJAB

First Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Elementary Mathematics-I (Algebra) **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-111 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

SHORT QUESTIONS

Q.2 Solve the following Short Questions: **$(2 \times 10 = 20)$**

(i) Find the $|z_1 - z_2|$, where $z_1 = 1 + i$ and $z_2 = 6 - i$.

(ii) If $A = \begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix}$ Find $A(\overline{A})'$.

(iii) Find the three cube roots of unity.

(iv) Show that the roots of equation $2x^2 + (mx-1)^2 = 3$ are equal if $3m^2 + 4 = 0$.

(v) Which term of the sequence $-6, -2, 2, \ldots$ is 70?

(vi) Find the term involving x^4 in the expansion of $(2x+3)^6$.

(vii) Find the nth term of the H.P. $\frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \ldots$

(viii) Prove that $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

(ix) Solve $\sqrt{x-1} + \sqrt{x+4} = 5$

(x) Simplify $(a+b)^5 + (a-b)^5$

LONG QUESTIONS **$(6 \times 5 = 30)$**

Q.3 Show that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.

Q.4 Solve the system of linear equations by Cramer's rule
$2x + y + z = 1$, $3x + y - 5z = 8$, $4x - y + z = 5$

Q.5 If α and β are the roots of $2x^2 + 7x + 5$ then find the equation whose roots are α^2, β^2.

Q.6 If the 5th term of A.P is 16 and 20th term of A.P is 46. Find the 12th term of A.P.

Q.7 Prove the identity $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

Q.8 If $x = a\sin\theta - b\cos\theta$ and $y = a\cos\theta + b\sin\theta$, then show that $x^2 + y^2 = a^2 + b^2$.

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Elementary Mathematics-I (Algebra) **TIME ALLOWED: 30 mins.**

Course Code: MATH-111 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Multiple Choice Questions

Q.1 Please Tick (√) the correct answer in the following MCQs.

(i) For any subset A of a universal set X, $A \cap X =$ ------------

(a) A' (b) A (c) ϕ (d) X

(ii) For any subsets A, B of a universal set X, $A \subset B$ implies that ------------

(a) $B \subset A$ (b) $A' \subset B'$ (c) $B' \subset A'$ (d) X

(iii) If $\begin{vmatrix} 2 & x \\ x & 2 \end{vmatrix} = 0$, then $x =$ ________

(a) ± 2 (b) 4 (c) $\pm\frac{1}{2}$ (d) $\pm\frac{1}{4}$

(iv) The Product of all three cube roots of unity is

(a) 1 (b) -1 (c) 3 (d) 9

(v) If $a_{n-1} = 3n - 1$, then 5th term is:

(a) 14 (b) 17 (c) 10 (d) 13

(vi) If $a = 3, r = 2$, then the nth term of the G.P. is

(a) 2.3^{n-1} (b) 3.2^{n} (c) 3.2^{n+1} (d) 3.2^{n-1}

(vii) The number of terms in the expansion of $(a+b)^9$ is:

(a) 10 (b) 11 (c) 12 (d) 13

(viii) The expansion of $(1-2x)^{\frac{2}{3}}$ is valid if

(a) $|x| < \frac{1}{2}$ (b) $|x| > \frac{1}{2}$ (c) $|x| < \frac{2}{3}$ (d) $|x| < 1$

(ix) $\cos^2\theta + \sin^2\theta =$

(a) -2 (b) -1 (c) 0 (d) 1

(x) If $\sin x > 0, \sec < 0$ then the terminal arm of the angle lies in _____quadrant.

(a) I st b) II nd c) III rd d) IV th

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First Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics A-I [Calculus(I)] **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-101 / MTH-11309/11010 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

SECTION – II

Q. 2	SHORT QUESTIONS	
(i)	Show that $\lim_{x\to 0}\frac{Tan^{-1}x^2}{x} = 0$.	(4)
(ii)	Evaluate $\int x^3\sqrt{x^2+16}\,dx$.	(4)
(iii)	If $x = \tan\theta$ and $y = \tan p\theta$, then prove that $(1+x^2)y' - p(1+y^2) = 0$.	(4)
(iv)	Find domain and range of the function $f(x) = \frac{3x+4}{2x-7}$.	(4)
(v)	Find derivative of $g(x) = \frac{\sin x^2}{\cos x}$.	(4)

SECTION – III

	LONG QUESTIONS	
Q.3	Solve $z^6 + z^3 + 1 = 0$, where z is a complex number. (Hint: Let $z^3 = u$)	(6)
Q.4	Find the curvature at any point of the curve $x^2 + y^2 = a^2$.	(6)
Q.5	Derive the reduction formula for the function $\sec^n x$ and evaluate $\int \sec^3 x\,dx$.	(6)
Q.5	Determine the extreme values of the function $g(x) = \frac{\ln x}{x}$, $0 < x < \infty$.	(6)
Q.6	$\int \frac{2x+3}{(x+1)^2(x^2+1)}dx = ?$	(6)

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Roll No. ....

First Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Mathematics A-I [Calculus(I)] **TIME ALLOWED: 30 mins.**

Course Code: MATH-101 / MTH-11309/11010 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

SECTION – I

Q.1	**MCQs (1 mark each)**			
(i)	$\int \sin^2 x \, dx = ?$			
	(a) $-\cos^2 x + c$	**(b)** $x - \sin 2x + c$	**(c)** $\frac{1}{2}x - \frac{1}{4}\sin 2x + c$	**(d)** $\frac{1}{2}(x - \sin 2x) + c$
(ii)	If $f(x) = \cos x$, then $f'(\pi) = ?$			
	(a) $-\sin x$	**(b)** -1	**(c)** 1	**(d)** 0
(iii)	If $f(x)$ has a local minimum at "c", then			
	(a) $f''(x) > 0$	**(b)** $f''(x) < 0$	**(c)** $f''(x) = 0$	**(d)** $f(c) = 0$
(iv)	Every polynomial function is ________________.			
	(a) linear	**(b)** trigonometric	**(c)** differentiable	**(d)** exponential
(v)	A point of inflexion of a function f is always occurring where f'' is			
	(a) positive	**(b)** negative	**(c)** zero	**(d)** undefined
(vi)	$(\sqrt{3} - i)^3$ is equal to			
	(a) $3\sqrt{3}$	**(b)** $8i$	**(c)** $-8i$	**(d)** none of these
(vii)	$\lim_{x\to\infty} \frac{2 - 3x}{\sqrt[3]{3 + 8x^3}} = ?$			
	(a) $\frac{2}{3}$	**(b)** $\frac{-3}{2}$	**(c)** $\frac{-2}{81}$	**(d)** none of these
(viii)	$\cos 45° - i^2 \sin 45°$ is equal to			
	(a) $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$	**(b)** $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$	**(c)** $\sqrt{2}$	**(d)** none of these
(ix)	$\lim_{x\to 2} \frac{x^2 - 4}{x - 2} = ?$			
	(a) 0	**(b)** 1	**(c)** 2	**(d)** 4
(x)	If z is a complex number, then $z + \bar{z}$ is			
	(a) real	**(b)** complex	**(c)** 0	**(d)** undefined

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Mathematics B-I [Vectors & Mechanics (1)] **TIME ALLOWED: 30 mins.**

Course Code: MATH-102 / MTH-11310 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

MULTIPLE CHOICE QUESTIONS. ($1\times10=10$)

Q# 1: Encircle the correct answer.\

i. For what value of a, the vector $V = (x+3y)i + (y-2z)j + (x+az)k$ is solenoidal

a) -2 b) 0 c) 2 d) 4

ii. $\nabla.\vec{r} = ---$, where $\vec{r}$ is a position vector

a) 0 b) 1 c) 2 d) 3

iii. If $\vec{A} = 4i + j - 2k$, then $|\vec{A}| = --$

a) 3 b) 13 c) $\sqrt{21}$ d) 21

iv. If the vectors $\vec{A} = ai - 2j + k$ and $\vec{B} = 2ai + aj - 4k$ are perpendicular, then $a = -$

a) -1, 1 b) -2, 2 c) -1, 2 d) -2, 1

v. If two forces P and Q act at such an angle that their resultant R=P, then the new resultant is -----------angles to Q, when the P is doubled

a) Zero degree b) 90 degree c) 180 degree d) 270 degree

vi. If a particle is in equilibrium under the action of three forces, each force has a magnitude proportional to the sine of the between the other two correspond to

a) λ, μ theorem b) Polygon of forces c) Varignon' Theorem d) Lamy's Theorem

vii. If one body slides along the other. The friction force in such a case which opposes motion is known as

a) Non-limiting friction b) limiting c) No friction d) Kinetic Friction

viii. For a particle to be in limiting equilibrium on an inclined plane under its own weight, if the inclination of the plane ------- the magnitude of the angle of friction

a) Equals b) greater c) less d) none of these

ix. Let $(F, -F)$ be the couple, then the sum of the moments of the components about the origin is $pF\vec{h}$, where p is the perpendicular distance from

a) F b) $-F$ c) $(F, -F)$ d) origin

x. The equation $F_a.\delta r = 0$, represents

a) Virtual displacement b) Principle of Virtual Work c) Workless Constraint d) system in equilibrium

UNIVERSITY OF THE PUNJAB

First Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics B-I [Vectors & Mechanics (1)] **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-102 / MTH-11310 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SHORT QUESTIONS. (2×10=20)

Q.#2: i) Find $\nabla(\phi)$, if $\phi = \ln|\vec{r}|$

ii) Prove that if $\vec{A}$ *and* $\vec{B}$ are non-collinear, then $x\vec{A} + y\vec{B} = 0$ *implies* $x = y = 0$

iii) Show that $\nabla(\varphi + \mu) = \nabla\varphi + \nabla\mu$.

iv) If $\vec{A} = t^2 i - tj + (2t+1)k$ and $\vec{B} = (2t-3)i + j - tk$, then find $\frac{d(\vec{A}\times\vec{B})}{dt}$.

v) Show that the resultant R of three concurrent non-coplanar forces acting at origin of a parallelepiped with the given forces for its edges is represented by the diagonal.

vi) State and prove Varigonon's Theorem.

vii) Find the angle of friction when two bodies have a rough contact at a point O.

viii) Find the least force to drag a particle on a rough horizontal plane.

ix) Show that the magnitude of the moment is the product of the perpendicular distance between the components of the couple and magnitude of either component.

x) State the principle of virtual work for a rigid body or a set of rigid bodies.

SUBJECTUVE QUESTIONS (5X6=30)

Q. #3: Forces of magnitude $P, 2P, 3P, 4P$ act respectively along the sides AB, BC, CD, DA of a square ABCD, of a side a and forces each of magnitude $(8\sqrt{2})$P act along the diagonals BE, AC. Find the magnitude of the resultant force and the distance of its line of action from A.

Q. #4: Forces $P_1, P_2, P_3, P_4, P_5, P_6$ act along the sides of a regular hexagon taken in order. Show that they will be in equilibrium if $\Sigma P = 0$ *and* $P_1 - P_4 = P_3 - P_6 = P_5 - P_2$.

Q. #5: Find the most general function differentiable function $f(r)$ so that $f(r)\vec{r}$ is solenoidal.

Q. #6: Evaluate $\nabla^2(lnr)$.

Q. #7: A weightless tripod consisting of three legs of equal length l, smoothly jointed at the vertex, stands on a smooth horizontal plane. A weight W hangs from the apex. The tripod is prevented from collapsing by three inextensible strings, each of length $\frac{l}{2}$, joining the mid-points of the legs. Show that the tension in each string is $\frac{\sqrt{2}W}{3\sqrt{3}}$.

Q. #8: A uniform semi-circular wire hangs on a rough peg, the line joining its extremities making an angle of 45 degree with the horizontal. If it is just on the point of slipping, find the coefficient of friction between the wire and the peg.

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First Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Business Mathematics
Course Code: MATH-112 /BUS-11132

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Subjective Type

Q-2 Answer the following short Questions. (20)

i. Which term of the sequence 5, 8, 11.... Is 320?
ii. Define arithmetic series.
iii. Write down formula for sum of n terms of geometric series.
iv. In how many ways the letters of the word PAKPATTAN can be arranged?
v. What is multiplicative inverse of $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$?
vi. Find two consecutive integers whose sum is 45.
vii. What is meant by common difference?
viii. What is singular matrix?
ix. At what rate Rs.10,000 double itself in 5 years?
x. If 6 is added to a certain number the result is 13. What is the number?

Long Questions: (30)

Q-3 Solve $\frac{2x-10}{x+4} + 3 = \frac{x-2}{x-3} + 4$ (6)

Q-4 Find three numbers in A.P. if their sum is 18 and product is 192. (6)

Q-5 Find the amount of Rs.5, 000 payable at the end of each of the 4 years at 3% compounded annually. (6)

Q-6 If A= $\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$ and B= $\begin{bmatrix} p & 2 \\ q & 4 \end{bmatrix}$ Find p and q if AB=BA (6)

Q-7 Prove that ${}^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$ (6)

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Business Mathematics | **TIME ALLOWED: 30 mins.**

Course Code: MATH-112 /BUS-11132 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Objective Type

Q-1 Encircle Correct Answer. **(10x1=10)**

1. Any matrix in which numbers of rows and columns are equal is called .... matrix.
   a) Identity b) triangular c) Diagonal d) Square

2. If |A| = 0, then A is said to be
   a) Singular matrix b) Non-singular matrix c) Zero matrix d) Identity matrix

3. What will be the simple interest earned on an amount of Rs. 16,800 in 9 months at the rate of % p.a?
   a) Rs. 787.50 b) Rs. 812.50 c) Rs 860 d) Rs. 887.50

4. Which of the following is linear equation?
   a) 2x-3y=-6 b) x+x+x c) $520x^2y^2$ d) y^2 -3=0

5. If every payment is made at the beginning then annuity is called
   a) Perpetuity b) Annuity due c) Ordinary Annuity d) None of these

6. Formula to calculate compound interest is
   a) $P = S(1+i)^s$ b) $P = S(1+i)^p$ c) $S = P(1+i)^n$ d) $S = P(1+i)^p$

7. If a = 3, r = 2, then nth term of G.P is
   a) 2.3n-1. b) 3.2n. c) 3.2n+1. d) 3.2n-1.

8. Evaluate ${}^{100}C_{100}$
   a) 1000 b) 1000 c) 100 d) 1

9. ______ is an ordered arrangement of numbers generated by a specific rule
   a) Sequence b) Common Ratio c) Function d) Common Difference

10. Find the value of $\log_4 (\log_3 5)$.
    a) 1.460 b) 0.275 c) 1.273 d) 0.165

UNIVERSITY OF THE PUNJAB

First Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus-I
Course Code: MATH-121

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Question no. 2

Briefly give the short answers of the asked questions. **(2 X 10 = 20)**

1. Find the coordinates of the point(s) on the given curve at which its gradient has the given value: **$y=(x+2)(x-2)$;1**
2. Write five properties of limit.
3. Find the derivative of $y = x\ln(\sin 2x+\cosh x)$ w.r.t. x.
4. Derive the equation of conic in polar coordinates i.e. $l = r(1-e\cos\theta)$ where e is the eccentricity.
5. *Evaluate* $\lim_{x\to 1}(1-x).\tan(\pi x/2)$.
6. Define continuity of a function and convergence sequence?
7. *Find the unit vector perpendicular to the plane P(1,-1,0), Q(2,1,-1) and R(-1,1,2).*
8. *Find the following limit using the L-Hospital rule.*

$$\lim_{x\to a}(x-a).\text{cosec}(\pi x/a)$$

9. *Suppose that* $f'(x) = 0$ for all x in some open interval (a,b). Then f is constant on (a,b). Is this statement true? Prove it.
10. *Suppose that* $F'(x) = x$ for all x and F(3)=2. What is F(x).

Question no. 3 **(3 x 10 = 30)**

Q1(a): Find the first 4 terms in the Maclaurins series for xe^{-x} (5+5)

Q1(b): Find the tangent and normal to the given curve at the points (-1, 2)

$$x^2 - xy + y^2 = 7$$

Q2(a): State and prove the mean value theorem (6+4)

Q2(b): Differentiate the following expression w.r.t. x.

$$y(x) = \frac{(x+2)^2}{(x-1)(x^4-3)^{1/2}}$$

Q3(a): Find the first 4 terms of the Taylor series for lnx centered at a = 1. (4+6)

Q3(b): Sketch the graph of the function **$r = 2a\cos 2\theta$**

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Calculus-I **TIME ALLOWED: 30 mins.**

Course Code: MATH-121 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Question no.1

Encircle the correct option form the given multiple choice questions. (1 X 10 = 10)

1) The correct representation of a function of x is:

A. $f = (x)y$
B. $x = f(y)$
C. $f = (y)x$
D. $y = f(x)$

2) Significant models to explain mathematical relationships are represented by

A. constant function
B. exponent function
C. model function
D. functions

3) In solving mathematical problems, a mathematical function work as

A. output-input device
B. solving function
C. terminating function
D. input-output device

4) $|z_1 + z_2| =$

A. $> |Z_1| + |Z_2|$
B. $\leq Z_1 + Z_2$
C. $> Z_1 + Z_2$
D. $\leq |Z_1| + |Z_2|$

5) The Polar form of a complex number is:

A. $r(\tan\theta + i\cot\theta)$
B. $r(\sec\theta + i\operatorname{cosec}\theta)$
C. $r(\sin\theta + i\cos\theta)$
D. $r(\cos\theta + i\sin\theta)$

P.T.O.

6) $\text{Tanh}^{-1}x =$

A. $\ln(x+\sqrt{(x^2+1})$
B. $\ln(x+\sqrt{(x^2+1})$
C. $1/2\ln(x+1/x-1)$
D. $1/2\ln(1+x/1-x)$

7) $\text{Cosh}^2x + \sinh^2x =$

A. $-\cosh(2x)$
B. $\sinh(2x)$
C. $\tanh(2x)$
D. $\cosh(2x)$

8) The system of linear equations$2x+2y-3z=1, 4x+4y+z=2, 6x+6y-z=3$ has
A. a unique solution
B. no solution
C. two solutions
D. infinite solutions

9) Let **f:X→Y** be a one-to-one mapping, which of the following is not correct?
A. X may be a subset of Y
B. Y may be a subset of X
C. cardinality of X should be equal to cardinality of Y
D. X should be equal to Y

10) The derivative of $\sin^2(30^0)$ w.r.t. x is
A. $\text{Cos } 30^0$
B. $2 \sin^{(30^0)} \text{Cos } 30^0$
C. $- \text{Cos } 30^0$
D. Zero

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Applied Mathematics **TIME ALLOWED: 30 mins.**

Course Code: MATH-122 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

(OBJECTIVE TYPE)

Q.1. Choose the correct answer. Cutting or over writing is not allowed.

1. The number 3 0.01850 $\times 10^3$ has – – – significant digits
   (a) 3 (b) 4 (c) 5 (d) 6
2. Order of convergence of Secant method is
   (a) 1 (b) 1.618 (c) 2 (d) none of these
3. Gauss-Seidel method is a – – – method
   (a) Non-iteration (b) iteration (c) infinite (d) algebraic
4. If $f(x)$ is a real continuous function in $[a,b]$, and $f(a)f(b) < 0$, then for $f(x) = 0$, there is (are) – – – in the domain $[a,b]$.
   (a) one root (b) an undeterminable number of roots (c) no root (d) at least one root
5. If $x = 3$ is a root of $f(x) = 0$, then the factor of $f(x)$ is – – –.
   (a) $x+3$ (b) 3 (c) $x-3$ (d) x
6. The value of k for the density function $f(x)=kx$, $0 \le x \le 2$ is
   (a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{1}{2}$ (d) None
7. Bag contains 10 black and 20 white balls, one ball is drawn at random. What is the probability that ball is white
   (a) 1 (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{3}{4}$
8. Variance of a binomial distribution is
   (a) np (b) $\frac{np}{q}$ (c) $np(1-q)$ (d) $np(1-p)$
9. There are 50 persons and we have to make a committee of 10 persons, then we have
   (a) $\frac{50!}{50!(50-10!)}$ (b) $\frac{50!}{10!(50-10!)}$ (c) $\frac{10!}{50!(50-10!)}$ (d) None
10. The convergence of which of the following method is sensitive to starting value?
    (a) False position (b) Gauss seidal method (c) Newton-Raphson method (d) All of these

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First Semester 2017
Examination: B.S. 4 Years Programme Roll No. ....

PAPER: Applied Mathematics **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-122 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

(SUBJECTIVE TYPE)

Part II Marks= 20

1. If A^c is the complement of an event A relative to sample space S, then prove that $P(A^c)=1-P(A)$. (2)
2. Prove that the total area under the normal distribution function curve is 1. (3)
3. 200 passengers have made a reservation for an airplane flight. If the probability that a passenger will not show up is 0.01. Find the probability that exactly 3 will not show up. (2)
4. Find the value of A in the following p.d.f of a continuous r.v"x":
$$f(x)=\begin{cases}A(3-x)(3+x), & 0\le x\le 3,\\ 0, & elsewhere.\end{cases}$$ (3)
5. Find the root of the function up to three decimal places by applying any numerical method $f(x)= x^3-x-1$, taking an initial value $x=1.5$. (3)
6. Evaluate $\int_0^4 \frac{dx}{1+x^2}$ using Trapezoidal Rule, for $n=6$. (3)
7. If $S_x^2=10.0$, $S_y^2=485,578.8$, $\sum(X-\overline{X})=159.45$, $\sum(Y-\overline{Y})=7,767,660$ and $\sum(X-\overline{X})(Y-\overline{Y})=28,768.4$, then find $Cov(x,y)$ and r_{xy}. (2)
8. State the axioms of probability. (2)

Part III Marks= 30

1. (a) Solve the following system of equations by using jacobi iterative method up to five iterations (6)
$$\begin{aligned}2x_1+x_2+x_3&=7\\ x_1+4x_2-x_3&=6\\ x_1+x_2+x_3&=6\end{aligned}$$
(b) For any two events A and B, prove that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$. (4)
2. (a) Find the root of the function correct up to three decimal places by applying the Newton Raphson method $f(x)=x^3-1$. take $x_o=1$. (6)
(b) Write the algorithm for the Newton Raphson Method for solving a non linear equation. (4)
3. Prove that correlation coefficient is independent of origin and scale. (10)

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2017
Examination: B.S. 4 Years Programme

PAPER: Calculus (IT)-I **TIME ALLOWED: 30 mins.**
Course Code: MATH-131 / MTH-11392 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

SECTION – I

Q.1	**MCQs (1 mark each)**
(i)	If $f(x)$ has a local extremum at "c", then **(a)** $f'(x) > 0$ **(b)** $f'(x) < 0$ **(c)** $f'(x) = 0$ **(d)** none of these
(ii)	The instantaneous rate of change at $t = 1$ for the function $f(t) = te^{-t} + 9$ is **(a)** -1 **(b)** 9 **(c)** 0 **(d)** 2
(iii)	$\lim_{x\to 0} \frac{x-5}{\sqrt{x}-\sqrt{5}} = ?$ **(a)** 0 **(b)** 5 **(c)** $\sqrt{5}$ **(d)** $2\sqrt{5}$
(iv)	Every differentiable function is ________________. **(a)** differentiable **(b)** integrable **(c)** continuous **(d)** exponential
(v)	A point of inflexion of a function f is always occurring where f'' is **(a)** positive **(b)** negative **(c)** zero **(d)** undefined
(vi)	For what value of x, the inequality $6x - 13 \leq 2x + 14$ is satisfied **(a)** 7 **(b)** 6 **(c)** 0 **(d)** 5
(vii)	$\lim_{x\to\infty} \frac{2-3x}{\sqrt[3]{3+8x^3}} = ?$ **(a)** $\frac{2}{3}$ **(b)** $\frac{-3}{2}$ **(c)** $\frac{-2}{81}$ **(d)** none of these
(viii)	First order Differential equation has almost ____independent solutions **(a)** 3 **(b)** 1 **(c)** 2 **(d)** 0
(ix)	$\lim_{x\to 3} \frac{x^2-x-6}{x-3} = ?$ **(a)** 0 **(b)** 1 **(c)** 5 **(d)** ∞
(x)	Domain of $\sqrt{x+3}$ is **(a)** $x \leq 3$ **(b)**) $x \leq 0$ **(c)**) $x \leq -3$ **(d)** none of these

UNIVERSITY OF THE PUNJAB

First Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus (IT)-I
Course Code: MATH-131 / MTH-11392

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

SECTION – II

Q. 2	SHORT QUESTIONS	
(i)	Evaluate the integral: $\int (5x^2e^{4x^3} + \frac{lnx}{x})\, dx$	(4)
(ii)	Evaluate $\int_1^e x^3 lnx\, dx$.	(4)
(iii)	If $x^2 - y^2 = 1$, Find $\frac{dy}{dx}$ and show that $d^2y/dx^2 = -1/y^3$.	(2+2)
(iv)	Find the area bounded by $y^2 - 4x = 4,\ 4x - y = 16$.	(4)
(v)	(a) Find $\frac{dy}{dx}$ of the equation $x\sqrt{1+y} + y\sqrt{1+x} = a$	(4)

SECTION – III

	LONG QUESTIONS	
Q.3	Find the equation of normal line at (2,0) of the function: $f(x) = 2x^2 - 3x + \frac{2}{x+1}$	(6)
Q.4	Find the extreme values and inflection point of the function: $f(x) = 12x^{\frac{1}{2}} - 2x^{\frac{3}{2}}$	(6)
Q.5	Derive the reduction formula for the function $\sec^n x$ and evaluate $\int \sec^3 x\, dx$.	(6)
Q.5	Solve the integral: $\int \frac{x}{(x-1)^2(x^2+1)} dx$	(6)
Q.6	Solve the differential equation: $(e^{-y} + 1)sinxdx = (1 + cosx)dy, \quad y(0) = 0$	(6)

UNIVERSITY OF THE PUNJAB

Roll No. ....

Second Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Mathematics A -II, [Plane Curves & Analytic Geometry] **TIME ALLOWED: 30 mins.**

Course Code: MATH-103 / MTH-12309 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Multiple Choice Questions ***(1×10=10)***

1. If the curve $x = z^2$, $y = 0$ is revolved about $x-$axis, then the surface of revolution is
(a) $x^2 + y^2 = z^2$ (b) $x = y^2 + z^2$ (c) $x^2 + y^2 = z^4$ (d) $x^2 + z^2 = y^4$.

2. If the curve $y = f(x)$ is real and $y \to a$ as $x \to \infty$, then the curve has a horizontal asymptote
(a) $y = 0$ (b) $x = 0$ (c) $y = a$ (d) $x = a$

3. Equation of straight line passing through origin and perpendicular to plane $x+y+z-4 = 0$ is
(a) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ (b) $\frac{x}{2} = \frac{y}{2} = \frac{z}{2}$ (c) $\frac{x-4}{1} = \frac{y-1}{1} = \frac{z-2}{1}$ (d) $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$

4. Radius of curvature, ρ, of a straight line is
(a) 0 (b) 1 (c) ∞ (d) -1

5. If the two tangents at a double point of a curve coincide and real the double point is called
(a) node (b) critical point (c) cusp (d) isolated point

6. The curve $x^3 + y^3 = 3axy$ is symmetric about the
(a) x-axis (b) the line $x = y$ (c) y-axis (d) both x and y axes

7. Parametric equations $x = at^2$ and $y = 2at$ represent the parabola
(a) $y^2 = 4ax$ (b) $x^2 = 4ay$ (c) $y^2 = 2ax$ (d) $x^2 = 8ay$

8. The locus of centers of curvatures for a given curve is called its
(a) involute (b) envelope (c) diameter (d) evolute

9. A surface defined by an equation of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$ has no
(a) trace in xy-plane (b) trace in yz-plane (c) z-intercept (d) x- and y-intercepts

10. A surface defined by an equation of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$ is called
(a) elliptic paraboloid (b) elliptic cone (c) ellipsoid (d) hyperbolic paraboloid

UNIVERSITY OF THE PUNJAB

Second Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics A -II,
[Plane Curves & Analytic Geometry]
Course Code: MATH-103 / MTH-12309

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Short Questions $10 \times 2 = 20$

1. Find the position of possible multiple points on the curve $x^2(x-y)+y^2=0$.
2. For what value of λ, the equation, $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents pair of straight lines.
3. Find the point of intersection (if it exists) of the pair of lines L: $\frac{x-2}{4} = \frac{y+1}{3} = \frac{z}{-2}$ and M: $\frac{x+1}{1} = \frac{y-3}{7} = \frac{z-5}{3}$.
4. Find equation of the straight line through the point $(2, 4, -3)$ and perpendicular to the plane $3x + 3y - 7z - 9 = 0$.
5. Show that equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at point (x_1, y_1) is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$
6. Find the surface of revolution obtained by revolving the curve $z^2 + y^2 = 5z, \quad x = 0$ about z-axis.
7. Convert the equation, $\rho = 7\sin\theta\sin\phi$, from spherical co-ordinates (ρ, θ, ϕ) into cartesian co-ordinates.
8. Find the traces of the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ in $xy-$ plane and $yz-$ plane.
9. Find equation of the sphere with center at $(-2, 4, -6)$ and tangent to $zx-$plane.
10. Find centre and radius of the sphere $x^2 + y^2 + z^2 + 2x - 4y - 6z + 5 = 0$.

Subjective Questions $6 \times 5 = 30$

1. Define pedal equation for a curve and show that the pedal equation for the curve $r = a\sin(m\theta)$ is $r^4 = p^2\left[r^2 + m^2(a^2 - r^2)\right]$.
2. Find the point on the line $2x - 7y + 5 = 0$ that is closest to the origin.
3. Find the area of the region bounded by the curve $xy^2 = 4(2-x)$ and the $y-$axis.
4. Prove that normals to a curve are tangents to its evolute.
5. Find the co-ordinates of the point on the line $\frac{x+3}{1} = \frac{y-7}{2} = \frac{z+13}{-1}$ which is nearest to the intersection of the planes $2x - y - 3z + 32 = 0 = 3x + 2y - 15z - 8$.
6. If ρ_1 and ρ_2 are radii of curvature at the extremities of any chord of a cardioid $r = a(1+\cos\theta)$ which passes through the pole, then prove that $\rho_1^2 + \rho_2^2 = \frac{16}{9}a^2$.

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Roll No. ....

Second Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Mathematics B-II [Mechanics(II)] **TIME ALLOWED: 30 mins.**

Course Code: MATH-104 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

NOTE: Attempt all questions from each section.

(1×10=10)

SECTION I

1. The number of oscillations which the particle completes in a unit of time, is known as .... (1 mark)

   (a) amplitude of oscillation

   (b) wavelength of oscillation

   (c) frequency of oscillation

   (d) time period

2. The slope of velocity-time curve of a particle moving in a straight line gives its .... (1 mark)

   (a) distance

   (b) velocity

   (c) acceleration

   (d) displacement

3. Every set of particles has .... center of mass. (1 mark)

   (a) one and only one

   (b) more than one

   (c) infinite

   (d) all of the above

4. The necessary and sufficient condition for a particle to be in equilibrium at a point is that .... at that point. (1 mark)

   (a) $\nabla V = 1$

   (b) $\nabla V = 2$

   (c) $\nabla V = 0$

   (d) $\nabla V \neq 0$

(P.T.O.)

5. If $\vec{F} = 0$ in Newton's second law then .... (1 mark)

(a) $\frac{d}{dt} m\vec{v} = 0$

(b) mv = constant

(c) m = constant

(d) both (a) and (b)

6. If T is the kinetic energy and V is the potential energy and $E = T + V$ is the total energy of the simple harmonic oscillator then .... (1 mark)

(a) $E = \frac{1}{2}mv^3 + \frac{1}{2}kx^2$

(b) $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

(c) $E = \frac{1}{2}mv^2 - \frac{1}{2}kx^3$

(d) $E = \frac{1}{2}mv^3 - \frac{1}{2}kx^3$

7. The center of mass of a triangular lamina is the point of intersection of its .... (1 mark)

(a) medians

(b) chords

(c) corners

(d) none of the above

8. The motion of a particle projected in air with velocity V_0 in the direction making an angle α with the horizontal is .... (1 mark)

(a) parabola

(b) circle

(c) ellipse

(d) straight line

9. The center of mass of uniform solid sphere lies .... (1 mark)

(a) on the radial segment

(b) on the geometric center

(c) on the perpendicular from the vertex

(d) none of the above

10. $\frac{d^2x}{dt^2}$ and $\frac{d^2y}{dt^2}$ are components of acceleration along..................... (1 mark)

(a) x-axis and y-axis respectively

(b) radial and tangential directions respectively

(c) normal directions

(d) all of the above

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Second Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics B-II [Mechanics(II)]
Course Code: MATH-104

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SECTION II-Questions with Short Answers

1. Define center of mass and center of gravity of a rigid body. (2 marks)

2. State principle of conservation of energy. (2 marks)

3. Define parabola of safety. (2 marks)

4. What is the maximum range possible for a projectile fired from a canon having muzzle velocity 1 mile/sec. (3 marks)

5. Differentiate between the harmonic oscillator and damped harmonic oscillator. (3 marks)

6. The position of a particle moving along an ellipse is $\vec{r} = a\cos t\hat{i} + b\sin t\hat{j}$. If $a > b$, find the position of the particle where its velocity has maximum value. (4 marks)

7. A particle describing simple harmonic motion has velocities $5ft/sec$ and $4ft/sec$ when its distances from the center are $12ft$ and $13ft$, respectively. Find the time period of motion. (4 marks)

SECTION III-Questions with Brief Answers

8. Find the tangential and normal components of the acceleration of a point describing the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with uniform speed v when the particle is at (0,b). (5 marks)

9. Determine whether $F = (x^2y - z^3)i + (3xyz + xz^2)j(2x^2yz + yz^4)k$ is conservative. (5 marks)

10. A rod of length 5a is bent so as to form 5 sides of a regular hexagon. Show that the distance of its center of mass from either end of the rod is $\frac{\sqrt{133}}{10}a$. (6 marks)

11. Two particles start simultaneously from a point O and move in a straight line; one with a velocity of 45 miles per hour and an acceleration of 2 ft/sec^2 and the other with a velocity of 90 miles per hour and a retardation of 8 ft/sec^2. Find the time after which the velocity of the particles are the same and the distance of O from the point where they meet again. (7 marks)

12. Discuss the motion of a particle projected in air with velocity V_0 in the direction making an angle α with the horizontal. (7 marks)

UNIVERSITY OF THE PUNJAB

Roll No. ....

Second Semester - 2017

<u>Examination: B.S. 4 Years Programme</u>

PAPER: Discrete Mathematics **TIME ALLOWED: 30 mins.**

Course Code: MATH-105 / MTH-12311 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

<u>OBJECTIVE</u>

Q1.Encircle the correct answer (1x10=10)

1. The contra-positive of the conditional statement $p \to q$ is
   a. $\neg p \to q$ b. $\neg p \to \neg q$ c. $q \to p$ d. $\neg q \to \neg p$

2. If A = {1, 2, 3, 4}, then the number of elements in P (A) =.....
   a. 2^4 b. 2^5 c. 2^6 d. 2^7

3. Consider the relation R={(1,1),(1,2),(1,4),(2,1),(2,2),(3,3),(4,4)} on set A={1,2,3,4} is
   a. Symmetric b. Reflexive c. Transitive d. All of these

4. *If x=1, x>2* "*x>2*" is a statement with truth-value
   a.. Ture b. False c. None of these

5. 1, 10, 10^2, 10^3, 10^4, 10^5,...is
   a. Arithmetic series b. Geometric series c. Arithmetic sequence
   d. Geometric sequence

6. The total number of one-to-one functions, from a set with three elements to a set with two elements is...........
   a. Zero b. 6 c. 9 d. None of these

7. The order pairs which are not present in a relation, must be present in
   a. Inverse of that relation b. Composition of relation c. Complementary relation of that relation d. None of these

8. Which of the given statement is incorrect?
   a. The process of defining an object in terms of smaller versions of itself is called recursion.
   b. A recursive definition has two parts: base and recursion.
   c. Functions cannot be defined recursively.
   d. Sets can be defined recursively.

9. A collection of rules indicating how to form new set objects from those already known to be in the set is called
   a. Base b. Restriction c. Recursion d. Fallacy

10. The statement of the form p∨~ p is.............
    a. Tautology b. Contradiction c. Fallacy d. All of these

UNIVERSITY OF THE PUNJAB

Second Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Discrete Mathematics | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-105 / MTH-12311 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE

Q2. Solve the following short questions (2x10=20)

1. Using Laws of Logic, verify the logical equivalence
   $\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p$.
2. Rewrite the statement form $\sim p \vee q \rightarrow r \vee \sim q$ to a logically equivalent form that uses only $\sim$ and $\wedge$.
3. State Binomial theorem.
4. Define partially ordered set.
5. Let R be a binary relation on a set A. Prove that
   If R is reflexive, then R^{-1} is reflexive.
6. Define a binary relation P from R to R as follows:
   for all real numbers x and $y (x, y) \in P \Leftrightarrow x = y^2$. Is P a function? Explain.
7. Find four binary relations from X = {a,b} to Y = {u,v} that are not functions.
8. Define inverse of function.
9. Find the sum of first n natural numbers.
10. Suppose that f is defined recursively by $f(0) = 3, f(n+1) = 2f(n) + 3$.
    Find $f(1)$.

Q3. Solve the following **Long Questions.** (5x6=30)

1. Let f: R → R be defined by the rule $f(x) = x^3$. Show that f is a bijective.
2. Find the 36^{th} term of the arithmetic sequence whose 3^{rd} term is 7 and 8^{th} term is 17.
3. Find the sum of all two digit positive integers which are neither divisible by 5 nor by 2.
4. Use Mathematical Induction to prove that
   $1 + 2 + 3 + 4 + \ldots + n = \dfrac{n(n+1)}{2}$ for all integers $n \geq 1$
5. Among 200 people, 150 either swim or jog or both. If 85 swim and 60 swim and jog, how many jog?
6. Suppose f: X → Y and g: Y → Z and both of these are one-to-one and onto. Prove that $(gof)^{-1}$ exists and that $(gof)^{-1} = f^{-1}og^{-1}$.

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Roll No. ....

Second Semester - 2017
Examination: B.S. 4 Years Programme

PAPER: Elementary Mathematics-I (Algebra) **TIME ALLOWED: 30 mins.**
Course Code: MATH-111 / MTH-12107 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Q.1 Tick on the correct option. **[10]**

i) If $A = \{1,2,3\}$ and $B = \{2,3,4\}$, then $A \backslash B =$

a) $\{0\}$ b) $\{1\}$ c) $\{4\}$ d) $\{2,3\}$

ii) The order of the matrix $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ is

a) 3×1 b) 1×3 c) 3×3 d) 1×1

iii) If $4x - 3 = 2x + 7$, then find x

a) $x = 1$ b) $x = 2$ c) $x = -1$ d) $x = -2$

iv) The $z = i - 2$, then find $|\bar{z}|$.

a) $\sqrt{2}$ b) $\sqrt{3}$ c) $\sqrt{5}$ d) $\sqrt{7}$

v) The domain of the function $f(x) = \sqrt{x}$ is

a) $(0, \infty)$ b) $[0, \infty)$ c) $(0, \infty]$ d) $[0, \infty]$

vi) If $f(x) = \sqrt{x + 7}$, then find $f(2)$

a) 1 b) 2 c) 3 d) 4

vii) The power set $\{1,2,\{3,4,5\}\}$ has elements

a) 4 b) 8 c) 12 d) 16

viii) If $f(x) = a - bx$, where a and b are constants, then $f^{-1}(x) =$

a) $\frac{a-x}{b}$ b) $\frac{x-a}{b}$ c) $\frac{x-b}{a}$ d) $\frac{b-x}{a}$

ix) The product of the roots of equation $5x^2 - x + 4 = 0$ is

a) $\frac{5}{4}$ b) $-\frac{5}{4}$ c) $\frac{4}{5}$ d) $-\frac{4}{5}$

x) If $9^{\frac{1}{x}} = \frac{1}{3}$, then x equals

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 2 d) -2

UNIVERSITY OF THE PUNJAB

Second Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Elementary Mathematics-I (Algebra) **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-111 / MTH-12107 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Q.2 Answer the following short questions. [20]

i. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then show that $A^2 - 5A - 2I = 0$.

ii. Solve the equation $2x^2 + 7x + 5 = 0$.

iii. Find the domain and range of the function $f(x) = \sqrt{3x - x^2}$.

iv. Find the multiplicative inverse of $\frac{7}{i-5}$.

v. Solve $\frac{\sin 45^0}{\cos 45^0 + \tan 45^0}$

vi. Which term of the sequence $\frac{5}{2}, \frac{3}{2}, \frac{1}{2}, \ldots$ is $-\frac{105}{2}$?

vii. If $\begin{vmatrix} k+4 & 5 \\ -1 & k-2 \end{vmatrix} = 0$, then find the value\s of k.

viii. Find four A.Ms between -9 and 1.

ix. Solve $\sin(x + 30^0)\ \sin(x - 30^0)$.

x. Expand $(x - 2y)^6$.

Answer the long questions.

Q. 3 Prove that the sum of the first n odd numbers is n^2. [6]

Q.4 The sum of three numbers in A.P is 27 and their product is 585. Find the numbers. [6]

Q. 5 Solve $\frac{9}{x+4} + \frac{3}{x-4} = \frac{5}{x-8}$ [6]

Q.6 In a survey of 60 people, it was found that: [6]

25 read *Newsweek* magazine, 26 read *Time*, 26 read *Fortune*, 9 read both *Newsweek* and *Fortune*, 11 read both *Newsweek* and *Time*, 8 read both *Time* and *Fourtune* and 3 real all three magazines.

a) Find the number of people who read at least one of the three magazines.

b) Draw the Venn diagram, where N, T and F denote the set of people who read *Newsweek*, *Time* and *Fortune*, respectively.

c) Find the number of people who read exactly one magazine.

Q.7 Find the coefficient of x^3in the expansion of $\left(x - \frac{2}{x^2}\right)^{12}$. [6]

UNIVERSITY OF THE PUNJAB

Roll No. ....

Second Semester - 2017

<u>Examination: B.S. 4 Years Programme</u>

PAPER: Calculus-II | **TIME ALLOWED: 30 mins.**

Course Code: MATH-123 / MTH-12333 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Objective Type

Q. 1 Choose the best answer. **[10]**

(i) $\int_2^\infty \frac{2}{x^2} dx =$

(a) 1 (b) 0 (c) -1 (d) 1/2 (e) Divergent

(ii) $\int_{-1}^{2} \frac{1}{x^2} dx =$

(a) 1 (b) 0 (c) -5/2 (d) Divergent

(iii) $\Gamma(n+1) =$

(a) (n+1)! (b) n! (c) (n-1)! (d) 0

(iv) If $f(x,y,z) = \cos(x - y + z)$, then $f(0,1,1) =$

(a) 0 (b) 1 (c) undefined (d) Could be any real number

(v) What information can you deduce for the function f at (0,0) satisfying $f_{xx}(0,0) = 3, f_x(0,0) = 0$

(a) local maxima at (0,0) (b) local minima at (0,0) (c) saddle point at (0,0) (d) insufficient information

(vi) If $f(x,y) = xy$, then $df =$

(a) $x + y$ (b) $xdx + ydy$ (c) $xdy + ydx$ (d) 0

(vii) The series $\sum_{n=1}^{\infty} n$

(a) Converges absolutely (b) converges conditionally (c) Diverges

(viii) The geometric series $\sum_{n=0}^{\infty} r^n$ converges if

(a) r < 1 (b) r > 1 (c) -1 < r < 1 (d) r = 1

(ix) The volume of a surface generated by revolving $y = x$ and $x = 1$ about x axis is

(a) π (b) 2π (c) $\sqrt{2}\pi$ (e) $\frac{\pi}{2}$

(x) $\int_{-\infty}^{\infty} x dx =$

(a) 0 (b) undefined (c) -1 (d) 1

UNIVERSITY OF THE PUNJAB

Second Semester - 2017
Examination: B.S. 4 Years Programme | **Roll No. ....**

PAPER: Calculus-II | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-123 / MTH-12333 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Q2. Short Question [2x10]

(i) Find the area under the curves $y = x$ and $y = x^2$.
(ii) Find the volume by revolving the surface $y = \sin(x)$ from 0 to π about x-axis.
(iii) Determine if the function $f(x, y, z) = xyz$ satisfies Laplace equation.
(iv) What are the critical points of $f(x, y) = x^2 + y^2 - xy$?
(v) Evaluate $\int \sin(x)\cos(x)\,dx$.
(vi) Use fundamental theorem of calculus to evaluate $\frac{d}{dx}\int_0^{x^2} \sin(t)\,dt$.
(vii) Determine the convergence or divergence of $\sum_{n=0}^{\infty} n$.
(viii) Find the change in area of a square if the side of length 6 is increasing at the rate of 1 m/s.
(ix) Evaluate $\int_{-\infty}^{\infty} x^3\,dx$.
(x) State Ratio test of convergence for a series.

Q3. Long Questions [3x10]

1. (a) Find maximum and minimum values of the function $f(x, y) = x + y$ on the circle $x^2 + y^2 = 1$.

(b) Evaluate $\int_0^1 \frac{\ln(x)}{x}\,dx$.

2. (a) Determine whether the series $\sum_{n=1}^{\infty} \frac{n+1}{n^3+2n+1}$ converges or diverges. Explain your reasoning.

(b) Use integral test to check the convergence of $\sum_{n=1}^{\infty} \frac{1}{n}$.

3. Find the volume of solid of revolution obtained by revolving the curves $y = x$ and $y = x^2$ about
   (a) x-axis
   (b) y-axis

UNIVERSITY OF THE PUNJAB

Roll No. ....

Second Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Analytical Geometry | **TIME ALLOWED: 30 mins.**
Course Code: MATH-124 / MTH-12118 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Objective

Q.1. Choose the best answer. **[10]**

i. The distance between $P_1(x_1, y_1, z_1)$ *and* $P_2(x_2, y_2, z_2)$ is
 a) $(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2, -z_1)^2$
 b) $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2 + (z_2, +z_1)^2}$
 c) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2, -z_1)^2}$

ii. The co-ordinates of the midpoint of a line segment are found by
 a) Dividing b) Averaging c) Integrating

iii. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$ represents a surface of the type
 a) An ellipsoid b) Elliptic cone c) Circular parabolic

iv. The angle between two planes is defined as the angle between their
 a) Projection b) Normal vectors c) Cross product

v. The intercept form of equation of plane is
 a) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ b) $\frac{x}{A} + \frac{y}{B} + \frac{z}{C} = 0$ c). $\frac{x}{A} + \frac{y}{B} + \frac{z}{C} \neq 1$

vi. The figure in space obtained by joining four non-coplanar points in pairs is called
 a) Tetrahedron c) Circle
 b) Plane d) Parabola

vii. The graph of $v\cos\theta = -4$is line passing through
 a) y = 4
 b) x= - 4
 c) x = 4
 d) y = -4

viii. The set of all points (x,y,z) satisfying the second degree equation

$ax^2 + by^2 + cz^2 + fyz + gxz + hxy + ux + vy + wz + l = 0$ represents a.

 a) Plane b) Quadric surface c) Cone

ix. If $f(x, y, z) = 0$ implies $f(-x, y, z) = 0$, the surface is symmetric with respect to
 a) x-axis b) yz-plane c) Origin

x. The set of all points in space that are equidistant from a fixed point is called
 a) Parabola
 b) Sphere
 c) Cone
 d) Circle

UNIVERSITY OF THE PUNJAB

Second Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Analytical Geometry **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-124 / MTH-12118 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Short Questions

Q.2. [20]

i. Find the intercepts and traces for the surface
$$x^2 + 4y^2 + 5xz - 2x + y - 3 = 0$$
ii. Fine the center and radius of the sphere
$$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$$
iii. Find the distance from the point S(1,1,5) to the line
$$L: x = 1 + t, y = 3 - t, z = 2t$$
iv. Find the distance from the point (2,2,3) to the plane 3x+2y+6z=6

v. Find spherical coordinates equation for the sphere
$$x^2 + y^2 + (z - 1)^2 = 1$$

Long Questions

Q.3. [10]

a) Find the point of intersection of the lines $x = 2t + 1, y = 3t + 2, z = 4t + 3, and\ x = s + t, y = 2s + 4, z = -4s - 1$, and then find the plane determined by these lines.

b) Find the distance between the parallel planes.
$$2x + 2y - 4z + 3 = 0$$
And
$$3x + 3y - 6z + 1 = 0$$

Q.4. [10]

a) Find an equation of the straight line through the point (1,2,3) and parallel to the line $x - y + 2z - 5 = 0 = 3x + y + z + 6$

b) Find the point in which the line meets the plane.
$$x = -1 + 3t, y = -2, z = 5t; 2x - 3z = 7$$

Q.5. [10]

a) Sketch the surface $4x^2 + y^2 = 36$

b) Find the vector projection of $\underline{u} = 6\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ on to $\underline{v} = \hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ and the scalar component of $\underline{u}$ in the direction of $\underline{v}$.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Second Semester - 2017
Examination: B.S. 4 Years Programme

PAPER: Calculus (IT)-II | **TIME ALLOWED: 30 mins.**
Course Code: MATH-132 / IT-12392 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Objective Type

Q. 1 Choose the best answer. **[10]**

(i) Which of the following equations describe a plane parallel to $2x - y + 4z + 4 = 0$.

(a) $x - y + z + 2 = 0$ (b) $y = 2(x+z)$ (c) $-x + \frac{y}{2} - 2z = 0$ (d) $2x + y + 4z = 4$

(ii) What is the scalar projection of $< -6, 1, 7 >$ on $< 1, 4, -2 >$.

(a) $\frac{16}{\sqrt{21}}$ (b) $-\frac{16}{\sqrt{21}}$ (c) $\frac{16}{21}$ (d) $-\frac{16}{21}$

(iii) Find the volume of the parallelepiped determined by $< 1, 2, 7 >, < 0, -3, 4 >$ and $< 0, 0, 6 >$.

(a) 18 (b) 12 (c) 0 (d) 6 (e) 20

(iv) Find the distance between (-1,-1,-1) and the plane $x + 2y + 2z - 1 = 0$.

(a) 2 (b) 0 (c) 6 (d) -2 (e) -6

(v) Which vector is always orthogonal to $\vec{b} - proj_{\vec{a}}\vec{b}$.

(a) $\vec{a}$ (b) $\vec{b}$ (c) $\vec{a} - \vec{b}$ (d) $\vec{a} \cdot \vec{b}$

(vi) Arc length of the curve $r(t) =< \cos(t), \sin(t) >$ from 0 to 2π is

(a) π (b) 2π (c) 1 (d) 0

(vii) If $f(x, y) = x^2 + y$, then $f_{xx} + f_y$ is

(a) $x + 1$ (b) 0 (c) 3 (d) 2

(viii) Convert to spherical coordinates the equation $x^2 + y^2 + z^2 = 9$

(a) $\rho = 3$ (b) $\rho = 9$ (c) $\rho\cos(\phi) = 3$ (d) $\rho\cos(\phi) = 9$

(ix) If a vector field f is conservative then $\oint f dr$ for any closed curve is

(a) Undefined (b) 0 (c) 1 (d) Not enough information

(x) Divergence of $f = x + y + z$ is

(a) 0 (b) 1 (c) 2 (d) 3

UNIVERSITY OF THE PUNJAB

Second Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus (IT)-II | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-132 / IT-12392 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Short Questions **[20]**

Q 2. (a) Find parametric equation of line passing through $P(1,0,0)$ and $Q(3,2,1)$.

(b) Show that curvature of a line is always 0.

(c) Find the arc length of the curve $r(t) =< t, \sqrt{t+2} >$ from $1 \leq t \leq 7$.

(d) Determine whether the vector field $f =< 1, y, x >$ is conservative?

(e) Show that divergence of a curl of a vector field is always zero.

Long Questions **[30]**

Q 3. (a) Find unit tangent and unit normal vector for the curve $r(t) =< \cos(t), \sin(t) >$ at $t = \frac{\pi}{2}$.

(b) Find potential function for the vector field $f =< x, y, z >$.

Q 4. (a) Calculate the flow through the closed surface using Divergence theorem.

$$f(x, y, z) =< x + y, y, z >: z = 16 - x^2 - y^2, \qquad z = 0$$

(b) Find the projection of $< 1, -1, 3 >$ on $< 3, 3, 0 >$.

Q 5. Find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$ for $w = x + y^3 - 2z^2 + 2xy - 5xz - 4yz$, where $x = 2s - 3t$, $y = 4s + t$, $z = s + 5t$.

Write your answers in terms of t.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2017
Examination: B.S. 4 Years Programme

PAPER: Mathematics A-III **TIME ALLOWED: 30 mins.**
Course Code: MATH-201/MTH-21309 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE

Multiple Choice Questions (1x10=10)

Q1. Encircle the correct answer.

i) A square matrix A is said to be symmetric, if:
a) $A^T = A^{-1}$ b) $A^T = A$ c) $A^T = 0$ d) $A^T = -A$

ii) The empty set of a vector space V(F) is always taken as
a) Linearly independent b) linearly dependent c) basis d) subspace

iii) Which of the following set is a vector space?
a) Set of real numbers b)Set of natural numbers c) Set of whole numbers d) Set of integers

iv) For a nonsingular matrix A, $(A^n)^{-1}$ = ...
a) $(A^{-1})^n$ b) A^{-1} c) A^n d) $(A^{-1})^T$
Where, n is a positive integer.

v) A system of linear equations $Ax=b$, with m=n has a unique solution, if A is
a) Singular b) Non singular c) Periodic d) Idempotent

vi) For the direct sum of two sub spaces U and W of a vector space V,
then $U \cap W$ = ...
a) $\{0\}$ b) U c) W d) empty set

vii) A linear transformation that is both one-one and onto is called:
a) Bijective b) Homomorphism c) Isomorphism d) Basis

viii) A set of linearly independent vectors spanning a vector space V is called ....
a) Basis for V b) dimension of V c) column space d) row space

ix) The eigen values of a symmetric matrix are:
a) Orthogonal b) Real and equal c) Complex d) Real

x) A linear transformation $T: U \rightarrow V$ is one-one if and only if
a) R(T)=$\{0\}$ b) N(T) = $\{0\}$ c) R(T)=N(T) d) R(T)=V

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Third Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics A-III **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-201/MTH-21309 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE

Q2. Short Questions (2x10=20)

i) Let A and B be 3x3 matrices with $det(A)=2$ and $det(B)=4$. Find the values of $det(AB)$ and $det(A^{-1}B)$.

ii) Consider a linear system whose augmented matrix is of the form

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ -1 & 4 & 3 & 2 \\ 2 & -2 & a & 3 \end{bmatrix}$$ For what value(s) of a will the system have a unique solution?

iii) Define Undetermined system

iv) Define determinant of a matrix.

v) Check whether the transformation $T: R^2 \to R^2$, defined as $T(x,y)=(-y,x)$ is linear or not.

vi) Check whether $W=\{(x,0,z): x,y,z \in R\}$ is a subspace of R^3?

vii) Find the eigen values of linear transformation $T: R^2 \to R^2$, defined as $T(x,y)=(3x+3y, x+5y)$.

viii) Without expansion, show that

$$\begin{vmatrix} a+b & c & 1 \\ b+c & a & 1 \\ c+a & b & 1 \end{vmatrix} = 0$$

ix) Show that if A is a symmetric nonsingular matrix, then A^{-1} is also symmetric.

x) Let $A=\begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$

Compute A^2 and A^3. What will A^n turn out to be?

Subjective Questions (5x6=30)

Q3. Determine the solution of the following system of linear equations

$$x-y+z=1$$
$$x-2y-3z=6$$
$$2x-y-z=6$$

Q4. Find equations defining the subspace W of R^3 spanned by the vector $(2,3,4)$.

Q5. Check whether the following transformation $T: R^3 \to R^2$ is one-one or not $T(x,y,z)=(x-y,z)$.

Q6. Let $T: R^3 \to R^3$ be a linear transformation defined by $T(a,b,c)=(a+2b-c,\ b+c,\ a+b-2c)$. Find basis and dimension of R (T) and N (T).

Q7. Prove that a one-one linear transformation preserves basis and dimension.

Q8. If A is a matrix over R and $AA^T=0$, then show that A=0.

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Roll No. ....

Third Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Mathematics B-III [Calculus (II)] **TIME ALLOWED: 30 mins.**

Course Code: MATH-202/MTH-21310 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE

	QUESTION 1	
	Tick the correct option	(10)
	(i) The sequence $\{n^{1/n}\}$ a) Converges to 0 b) Converges to 1 c) Diverges d) None of these	
	(ii) A Series that converges but does not converge absolutely a) Diverges b) Diverges conditionally c) Diverges absolutely d) Converges conditionally	
	(iii) An interior point of the domain of a function f where f' is zero or undefined is a ___________ point. a) Saddle b) Critical c) Inflection d) None of these	
	(iv) The normal line of the surface at P_0 ___________ to $\nabla f\|_{P_0}$. a) Parallel b) Perpendicular c) Equal d) None of these	
	(v) At any point (a,b), $f_{xx}f_{yy} = f_{xy}^2 \Rightarrow$ a) Maxima b) Minima c) Saddle point d) Critical	
	(vi) Let C be a curve in a plane and L be a line not in the plane. The union of all lines that intersect C and are parallel to L is called a __________. a) Sphere b) Cylinder c) Cone d) None of these	
	(vii) The Curve of intersection of a surface and a plane is called the ________ of the surface. a) Trace b) Revolution c) Continuous image d) All of these	
	(viii) The set of all points (x,y,z) which satisfy an equation of the form $f(x,y,z)=0$ is called a ____________. (a) Curve (b) Cylinder (c) Surface (d) All of these	
	(ix) The area of a closed and bounded region R in the polar coordinate plane is a) $\iint_R dr d\theta$ b) $\oiint \theta dr d\theta$ c) Zero d) $\iint_R r dr d\theta$	
	(x) Moment of inertia about a line L is given as a) $\iiint r^2 dV$ b) $\iiint r\delta^2 dV$ c) $\iiint r^2 \delta dV$ d) $\iint_R x^2 y^2 dV$	

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Third Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics B-III [Calculus (II)] | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-202/MTH-21310 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE

Short Questions

QUESTION 2	Marks
Answer the following short questions (i) Find the local extreme values of $f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$. (ii) Find the $\underset{(x,y)\to(0,0)}{Limit} \frac{x^3 - y^3}{x^2 + y^2}$. (iii) Show by example that $\sum_{n=1}^{\infty} a_n b_n$ may diverge even if $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge. (iv) Find an equation of the trace of the surface $xy + yz + zx = 1$ in the xz-plane. (v) Find the limits of integration for integrating $f(r,\theta)$ over the region R that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$.	5*4 = 20

Long Questions

QUESTION 3	
Determine whether the series $\sum_{1}^{\infty} (-1)^n \frac{n+2}{n(n+1)}$ converges absolutely, converges conditionally or diverges.	(10)
QUESTION 4	
If $f(x,y) = x^2 \arctan\left(\frac{y}{x}\right) - y^2 \arctan\left(\frac{x}{y}\right)$, show that $\frac{\partial^2 f}{\partial x \partial y} = \left(\frac{x^2 - y^2}{x^2 + y^2}\right)$.	(10)
QUESTION 5	
A solid trough of constant density is bounded below by the surface $z = 4y^2$ above the plane z = 4 and on the ends by the planes x = 1 and x = -1. Find the center of mass and moments of inertia with respect to the three axes.	(10)

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Roll No. ....

Third Semester 2017
Examination: B.S. 4 Years Programme

PAPER: Graph Theory | **TIME ALLOWED: 30 mins.**
Course Code: MATH-205/MTH-21312 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

(OBJECTIVE)

Q#1: Tick or circle the correct answer of the following. Multiple choice questions.

i) The number of edges in a path graph P_n is
(a) n (b) n-1 (c) n+1 (d) none

ii) The sum of degrees of all vertices of non-trivial graph is equal to:
(a) e (b) $2e$ (c) e -1 (d) none

iii) An example of regular graph is
(a) P_n (b) W_n (c) C_n (d) none

iv) A vertex of degree one is called __________ vertex
(a) pendant (b) isolated (c) loop (d) none

v) In a bipartite graphs each vertex has__________ degree
(a)even (b) odd (c) zero (d) one

vi) Any simple graph with 8 vertices and more than 21 edges is always
(a) tree (b) connected (c) disconnected (d) none

vii) A connected graph which does not contain a cycle is called
(a) tree (b) circuit (c) loop (d) forest

viii) The number of edges of K_5 is
(a) 10 (b) 12 (c) 14 (d) 16

ix) The complement of simple graph is
(a) simple (b) multi graph (c) Pseudograph (d) none

x) The number of edges of K_n is:

(a) n (b) $\frac{n(n-1)}{2}$ (c) n^2 (d) *None*

UNIVERSITY OF THE PUNJAB

Third Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Graph Theory **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-205/MTH-21312 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

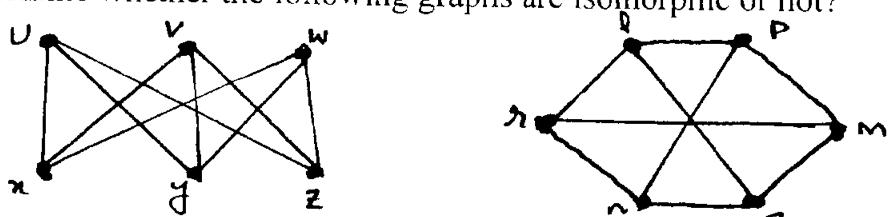
SUBJECTIVE

Q#2: Solve the following Short Questions. **(2×10=20)**

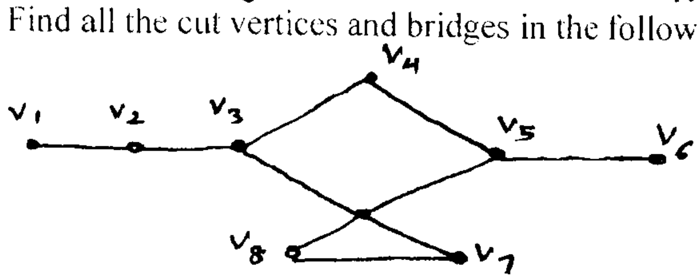
i) Define Bipartite graphs and give example.
ii) Define two matrix representation of a graph with example.
iii) Is it possible to construct a graph with degree sequence{1,2,2,2,3,3}?
iv) Define complete graph and show that it has $\frac{n(n-1)}{2}$ number of edges.
v) Construct a spanning tree of wheel graph W_5.
vi) Define handshaking lemma for digraphs.
vii) Define cut vertex and give example.
viii) Define Hamiltonian graphs.
ix) Determine whether a graph with degree sequence {1,2,2,3} is Eulerian or not?
x) What is difference between forest and a tree?

Q# 3: Solve the following Long Questions. **(5×6=30)**

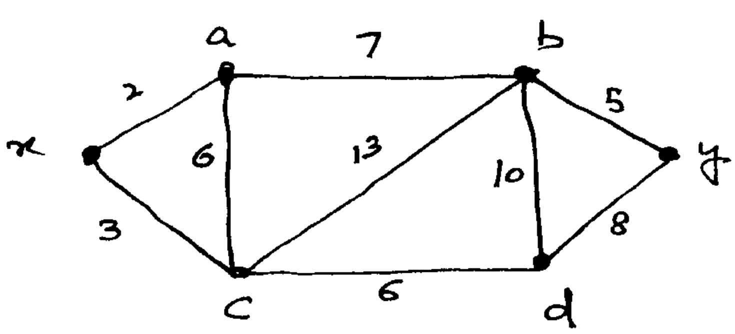
i) Determine whether the following graphs are isomorphic or not?



ii) Find all the cut vertices and bridges in the following graph.



iii) Draw all the simple cubic graphs with atmost 6 vertices.
iv) Prove that a tree must has atleast two vertices of degree one.
v) If a graph has only vertices of degree 2 or 3 and the number of vertices of degree 2 are 6 and total number of edges are 16 then find the number of vertices with degree 3.
vi) Find the shortest path between the vertices x and y in the following graph.



UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2017
Examination: B.S. 4 Years Programme

PAPER: Elementary Mathematics-II (Calculus) **TIME ALLOWED: 30 mins.**
Course Code: MATH-211/MTH-21107 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Objective type

Q1 Tick on the correct option (10).

(i) if f: x→y is a function then domain of f is

a)X b) -X c) Y d) -Y

(ii) The solution of the inequality $7 \leq 2-5x < 9$ is

a)[-7/5,-1) b) [-7/5,-1] c) (-7/5,-1) d) (-7/5,-1]

(iii) $\lim_{x\to\infty}(1+1/x)^x =$

a)e b) -e c) 0 d) ∞

(iv) $d(e^x)/dx =$

a)$e^x \ln e$ b) $e \ln e^x$ c) $e^x/\ln a$ d) $a/\ln e^x$

(v)$-1/\sqrt{1+x^2}$ is the derivative of

a)$\sin^{-1}x$ b) $\cos^{-1}x$ c) $\tan^{-1}x$ d) $\cot^{-1}x$

(vi)$\int \sqrt{x}dx$=

a) $2/3\sqrt{x^3}+c$ b) $3/2\sqrt{x^3}+c$ c) $1/3\sqrt{x^3}+c$ d) $2/3\sqrt{x^2}+c$

(vii) $\int \cot x\, dx$

a) lnsecx+c b) lncosx+c c) lnsinx+c d) lncscx+c

(viii) $\int 1/1+x^2\, dx$=

a)$\sin^{-1}x$ b) $\cos^{-1}x$ c) $\tan^{-1}x$ d) $\cot^{-1}x$

ix) $\int_0^1 1/1+x^2\, dx$=

a)45° b) 30° c) 60° d) 90°

(x) $\int \csc x \cot x\, dx$ =

a)-cscx +c b) secx +c c) cotx+c d) cosx+c

UNIVERSITY OF THE PUNJAB

Third Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Elementary Mathematics-II (Calculus) **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-211/MTH-21107 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Short Questions

Q2 **(5x4)**

I. Check the continuity of $f(x)=|x-3|$ at $x=3$.

II. Find dy/dx if $\sqrt{x}+\sqrt{y}=\sqrt{a}$

III. Evaluate $\int \tan x / \cos x+\sec x \, dx$.

IV. Evaluate $\int_1^3 \ln x \, dx$

V. Evaluate $\int_0^1 \tan^{-1} x \, dx$.

Long Questions

Q3 **(10)**

Let $f(x)= x \sin(1/x)$ if $x\neq 0$ and $f(0)=0$. Discuss the continuity and differentiability of **f** at $x=0$.

Q4 **(10)**

Differentiation w.r.t X

$$Y= X^x e^x \sin x (\ln x)$$

Q5 **(10)**

(i)Find $d(e^{\sqrt{x}})/dx$ and hence evaluate $\int_0^1 e^{\sqrt{x}}/\sqrt{x} \, dx$.

(ii)Evaluate $\int \tan^2 x \sec^4 x \, dx$.

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Roll No. ....

Third Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Differential Equations-I | **TIME ALLOWED: 30 mins.**

Course Code: MATH-221/MTH-21334 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION – I (Objective)

Fill in the blank or answer true/false.

1. The piecewise-defined function $y = \begin{cases} \sqrt{25-x^2}; & -5 < x < 0 \\ -\sqrt{25-x^2}; & 0 \le x < 5 \end{cases}$ is a solution of the differential equation $\frac{dy}{dx} = -\frac{x}{y}$ on the interval $(-5, 5)$. (True/False)
2. $x\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^4 + y = 0$ is a fourth order non-linear ordinary differential equation. (True/False)
3. The ordinary differential equation $\frac{dy}{dx} - ky = A$, where k and A are constants, is autonomous. $y =$ .... is a critical point of the equation.
4. The differential equation $\frac{dy}{dx} = p(x) + q(x)y + r(x)y^2$ is known as ....
5. If $F(x, y, y') = 0$ is a first-order linear ordinary differential equation if and only if $F(x, y, y')$ is a linear function ....
6. If wronskian $W(y_1, y_2) = \det(y_1y_2' - y_1'y_2) = 0$, then the set of functions $\{y_1, y_2\}$ is linearly independent. (True/False)
7. The general solution of $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$ is $y(x) = c_1y_1(x) + c_2y_2(x)$ if $W(y_1, y_2) = \det(y_1y_2' - y_1'y_2) \neq 0$ (True/False)
8. The only solution of the initial-value problem $\frac{d^2y}{dx^2} + x^2y = 0, y(0) = 0, y'(0) = 0$ is ....
9. A constant multiple of a solution of a linear differential equation is also a solution. (True/False)
10. $\frac{d^4y}{dx^4} + y^3 = 0$ is a linear ordinary differential equation. (True/False)

UNIVERSITY OF THE PUNJAB

Third Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Differential Equations-I | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-221/MTH-21334 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Section-II (Short Questions) Marks=20

1. Verify that the piecewise-defined function

$$y = \begin{cases} 0, & x < 0 \\ x, & x \geq 0 \end{cases}$$

satisfies the condition $y(0) = 0$. Determine whether this function is also a solution of the initial-value problem $x\frac{dy}{dx} - y = 0$, $y(0) = 0$.

2. Find a continuous solution satisfying

$$\frac{dy}{dx} + y = f(x), \qquad \text{where} \qquad f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$$

and the initial condition $y(0) = 0$.

3. Using an appropriate substitution, solve the Bernoulli equation

$$x\frac{dy}{dx} - (1+x)y = xy^2.$$

4. Verify that the set of functions $\{\cos(\ln x), \sin(\ln x)\}$ forms a fundamental set of solutions of the differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0,$$

on the interval $(0, \infty)$.

5. The function $y_1 = x^2$ is a solution of

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0.$$

Find the general solution of the differential equation on the interval $(0, \infty)$.

Section-III Marks=30

1. Solve the differential equation by using undetermined coefficients

$$\frac{d^2y}{dx^2} + y = 4x + 10\sin x, \qquad y(\pi) = 0,\ y'(\pi) = 2.$$

2. Solve the given initial-value problem

$$\frac{dy}{dx} = (-2x + y)^2 - 7, \qquad y(0) = 0.$$

3. Solve

$$\left(\frac{1}{1+y^2} + \cos x - 2xy\right)\frac{dy}{dx} = y(y + \sin x), \qquad y(0) = 1.$$

4. Solve the system of linear differential equations

$$\begin{aligned} \frac{d^2x_1(t)}{dt^2} - x_1(t) - x_2(t) &= 0, \\ \frac{dx_1(t)}{dt} - x_1(t) + \frac{dx_2}{dt} &= 0. \end{aligned}$$

5. Solve

$$2x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + y = x^2 - x,$$

by using variation of parameters.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Pure Mathematics
Course Code: MATH-222/MTH-21119

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Q.1 Tick on the correct option. **[10]**

i) If $A = \{1,2,3\}$ and $B = \{2,3,4\}$, then $A \backslash B =$

a) $\{0\}$ b) $\{1\}$ c) $\{4\}$ d) $\{2,3\}$

ii) If $S = \{1,2,3\}$, then $Power(S)$ has elements

a) 2 b) 3 c) 6 d) 8

iii) If $X = \{1,2,3,4\}$, then partition of X is

a) $\{1\},\{2\},\{3,4\}$ b) $\{1\},\{1,2\},\{3,4\}$ c) $\{1,2\},\{2\},\{3,4\}$ d) $\{1\},\{2,3\},\{3,4\}$

iv) The converse of the conditional $\sim p \to q$ is

a) $p \to \sim q$ b) $q \to p$ c) $q \to \sim p$ d) $\sim p \to \sim q$

v) The domain of the function $f(x) = \sqrt{x}$ is

a) $(0,\infty)$ b) $[0,\infty)$ c) $(0,\infty]$ d) $[0,\infty]$

vi) For the discrete metric space $(\boldsymbol{Z}, d), x \in \boldsymbol{Z}$ and $r \in \boldsymbol{R}, r > 1$, then open ball $B(x,r) =$

a) $\{x\}$ b) $Z - \{x\}$ c) $\{\pm 1\}$ d) Z

vii) Let $X = \{1,2,3,4,5\}$ with $\tau = \{X, \phi, \{2\}, \{4\}, \{2,4\}\}$. Let $A = \{1,2,5\}$. Then $A^o =$

a) $\{x\}$ b) $Z - \{x\}$ c) $\{\pm 1\}$ d) Z

viii) If $f(x) = a - bx$, where a and b are constants, then $f^{-1}(x) =$

a) $\frac{a-x}{b}$ b) $\frac{x-a}{b}$ c) $\frac{x-b}{a}$ d) $\frac{b-x}{a}$

ix) The contrapositive of $p \to q$ is

a) $q \to p$ b) $\sim q \to \sim p$ c) $q \to \sim p$ d) $\sim q \to p$

x) A set X with one element has ...... topology (topologies).

a) 1 b) 2 c) 3 d) 4

UNIVERSITY OF THE PUNJAB

Third Semester 2017

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Pure Mathematics **TIME ALLOWED: 2 hrs. & 30 mins.**

Course Code: MATH-222/MTH-21119 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Q.2 Answer the following short questions. [20]

i. Find two topologies on $X = \{1,2,\dots,8\}$, each contain four members.

ii. What is the difference between Symmetric and Anti-symmetric Relations? Also give a suitable example.

iii. Find the domain and range of the function $f(x) = \sqrt{3x - x^2}$.

iv. Evaluate $\lim_{n\to\infty} \frac{\sin n}{n}$.

v. What is the difference between conjunction and disjunction? Also give a suitable example.

vi. Define interior of a set in a metric space. Also give a suitable example.

vii. Define absurdity with suitable example.

viii. Sketch the piecewise function $f(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ 2 - x, & x \geq 1 \end{cases}$.

ix. Define discrete metric space. Also give an example.

x. Define homeomorphism. Also give an example

Answer the long questions.

Q. 3 Prove that the sum of the first n odd numbers is n^2. [6]

Q.4 Determine the validity of the argument: $p \to q, \sim q \vdash \sim p$. [6]

Q. 5 Let (X, τ) be a topological space. Then show that [6]

(i) a subset A is closed $\Leftrightarrow$ $\bar{A} = A$.

(ii) a subset A is open $\Leftrightarrow$ $A^o = A$.

Q.6 In a survey of 60 people, it was found that: [6]

25 read *Newsweek* magazine, 26 read *Time*, 26 read *Fortune*, 9 read both *Newsweek* and *Fortune*, 11 read both *Newsweek* and *Time*, 8 read both *Time* and *Fourtune* and 3 real all three magazines.

a) Find the number of people who read at least one of the three magazines.

b) Draw the Venn diagram, where N, T and F denote the set of people who read *Newsweek*, *Time* and *Fortune*, respectively.

c) Find the number of people who read exactly one magazine.

Q.7 Prove that any open ball in the usual metric space $\mathbb{R}$ is open interval. [6]

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Discrete Mathematics (IT) **TIME ALLOWED: 30 mins.**
Course Code: MATH-231/IT-21404 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Multiple Choice Questions (1x10=10)

Q1. Encircle the correct answer.

i) If 2+2=4, then $x:=x+1$. For $x=0$, the value of x is ....
a) 0 b) 1 c) 2 d) 3

ii) If the set $X = \{x \in N : x = 1/x\}$, then
a) $X = \phi$ b) $X = \{1,-1\}$ c) $X = \{1\}$ d) none of these

iii) $\forall x \ (x \in A \leftrightarrow B)$ means
a) $A \subseteq B$ b) $B \subseteq A$ c) $A = B$ d) none of these

iv) $p \wedge q$ is equivalent to
a) $q \to \neg p$ b) $\neg p \to q$ c) $\neg(q \to p)$ d) $\neg(q \to \neg p)$

v) How many edges are there in a graph with 10 vertices each of degree 6
a) 30 b) 40 c) 50 d) 60

vi) The example of directed graph is
a) Hollywood Graph b) Acquaintance graph c) Influence graph
d) None of these

vii) How many relations are there on a set with 3 elements
a) 8 b) 512 c) 64 d) None of these

viii) The range of the function $f(x) = \frac{1}{x-2}, x \neq 2$ is
a) $\Re \setminus \{1\}$ b) $\Re \setminus \{0\}$ c) $\Re$ d) None of these

xi) The domain of the function $f(x) = \sqrt{|x|}$ is
a) $(-\infty, 0]$ b) $[0, \infty)$ c) $(-\infty, \infty)$ d) all of these

x) If both f and g are one-to-one functions, then fog is
a) One-to-one b) onto c) a&b d) none of these

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Third Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Discrete Mathematics (IT) **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-231/IT-21404 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Q2. Short Questions (2x10=20)

i) Prove that an undirected graph has even number of vertices of odd degree.

ii) Show that $p \rightarrow q = \neg p \vee q$.

iii) Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology.

iv) Determine whether the function $f(x) = x^2$ from $N \rightarrow N$ is one-to-one. Is this function onto?

v) Let $f : R \rightarrow R$ be defined by $f(x) = \frac{2x-7}{4}$. Find $f^{-1}(x)$.

vi) How many non-isomorphic unrooted trees are there with three vertices?

vii) Give an example of a relation which is symmetric and anti-symmetric.

viii) Show that $\neg(p \vee q)$ *and* $\neg p \wedge \neg q$ are logically equivalent.

ix) Let $P(x)$ denote the statement "$x > 3$". What are the truth values of $P(4)$ and $P(2)$?

x) Construct the truth table for $(p \wedge q) \rightarrow (p \vee q)$.

Subjective Questions (30)

Q3. **a)** State and prove the pigeon hole principal.

b) Show that $\neg(p \leftrightarrow q)$ *and* $p \leftrightarrow \neg q$ are logically equivalent.

Q4. **a)** If T is a tree with *n* vertices then prove that T contains no cycles and has *n-1* edges.

b) An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

Q5. **a)** Draw a graph whose adjacency matrix is given by $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

b) Show that among any group of five integers, there are two with the same remainder when divided by 4.

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Roll No. ....

Fourth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Mathematics A-IV | **TIME ALLOWED: 30 mins.**

Course Code: MATH-203 / MTH-22309 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION – I

Q.1	**MCQs (1 mark each)**			
(i)	$2xy'' + y^2 = 2x^2$ is a______ order differential equation.			
	(a) 2	**(b)** 1	**(c)** 0	**(d)** none of these
(ii)	$\frac{dy}{dx} = \frac{y^2+2y}{2x^2+x}$ is ___________differential equation			
	(a) Homogenous	**(b)**Nonhomogeneous	**(c)** Exact	**(d)** none of these
(iii)	The initial value problem $y' = y,\ y(0) = 1$ has the solutions $y = e^x$ and y=_____			
	(a) 0	**(b)** 1	**(c)** Ce^x	**(d)** none of these
(iv)	Classify the following differential equation $\frac{du}{dt} = 1 + u + t + ut.$			
	(a) Separable	**(b)** Linear	**(c)** Exact	**(d)** Reducible to Linear
(v)	The function *P(x)* in the given linear 1st order ODE. $\frac{dx}{dt} = \frac{x+t^2-x\sqrt{t}}{t}$			
	(a) x	**(b)** 0	**(c)** 1	**(d)** none of these
(vi)	If y_1 and y_2 be the solutions of a differential equation then $y_1 + 10\,y_2 = 0$ is			
	(a) Solution	**(b)** Not a Solution	**(c)** Singular	**(d)** none of these
(vii)	$y' + P(x)y = f(x)$ is ______________			
	(a) Homogeneous	**(b)** Inhomogeneous	**(c)** Linear	**(d)** none of these
(viii)	The Singular points of $(x^3 - 8)y'' - 2xy + y = 0$ are given by_________			
	(a) 0	**(b)** 2	**(c)** $\sqrt{5}$	**(d)** none of these
(ix)	The particular solution of a Non-homogenous differential equation has__________ arbitrary constants.			
	(a) 1	**(b)** 0	**(c)** 2	**(d)** none of these
(x)	The General solution of a Non-homogenous 2nd order differential equation has___________arbitrary constants.			
	(a) 3	**(b)** 2	**(c)** 1	**(d)** none of these

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Fourth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics A-IV | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-203 / MTH-22309 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION – II

Q. 2	SHORT QUESTIONS	
(i)	Solve $y'' + 2y(y')^3 = 0.$	(4)
(ii)	Find the solution of the following differential equations. $$x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0.$$	(4)
(iii)	Determine the appropriate form for a particular solution of the following fifth order differential equation. $$(D-2)^3(D^2+9)\,y = x^2e^{2x} + x\sin 3x.$$	(4)
(iv)	Solve $\frac{y}{x^2}\frac{dy}{dx} + e^{2x^3+y^2}=0.$	(4)
(v)	Find the general solution of the following differential equation. $[D^2+6D+13]^2y = 0.$	(4)

SECTION – III

	LONG QUESTIONS	
Q.3	Solve the following differential equations $$xy'' - y' + \frac{y}{x} = x^2.$$	(6)
Q.4	Use reduction of order to find a second solution of differential equation. $$(3x+1)y'' - (9x+6)y' + 9y = 0, \quad y_1 = e^{3x}.$$	(6)
Q.5	Find the orthogonal trajectories of the given family of curves. $$x^2 + 4y + 1 + ce^{2y} = 0.$$	(6)
Q.6	Find the general solution of the following differential equation. $\frac{d^3y}{dx^3} + \frac{dy}{dx} = \csc x$	(6)
Q.7	Solve by power series method $$(x^2+1)y'' + xy' - y = 0.$$	(6)

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Roll No. ....

Fourth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Mathematics B-IV
(Metric Spaces & Group Theory)
Course Code: MATH-204 / MTH-22310

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Section - I

Q1) Encircle the correct answer. All questions carry equal marks. (10 marks)

(i) The self conjugate index is
a) 3 b) 2 c) 4 d) none

(ii) The distance between the sets in the discrete metric space is
a) 1 or 0 b) greater than 1 c) less than 1 d) none

(iii) The derived set of $N=\{1,2,3,\ldots\}$ in real line R with usual metric space is
a) Empty set b) R c) N d) none

(iv) In a metric space every convergent sequence is
I. Cauchy
II. bounded
a) I only b) II only c) both I & II d) none

(v) If $f: X \to Y$ is continuous and a sequence $\{x_n\}$ converges to a, then the sequence $\{f(x_n)\}$ converges to
a) $f(a)$ b) $b \neq f(a)$ c) may or may not $f(a)$ d) none

(vi) The orders of a and its inverse a^{-1} in a group is
a) Always different b) always same c) may be same or not d) none

(vii) A group of even order always contain an element of order
a) Odd b) even greater than 2 c) 2 d) none

(viii) The set of all nonzero integer is a ______ under multiplication
a) Group b) monoid c) semi-group d) none

(ix) A group of order 17 is always
a) Non-abelian b) abelian & cyclic c) abelian & non-cyclic c) none

(x) The order of permutation (1 2 4 5) is
a) 4 b) 5 c) 2 d) 3

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Fourth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics B-IV
(Metric Spaces & Group Theory)
Course Code: MATH-204 / MTH-22310

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Section - II

Q2. Solve the following short question. (2 marks for each question)

(i) Define open ball and determine open balls with center a and radius r in real line with usual metric space and discrete metric space.
(ii) Define closure and find the closure of interval $(2,6]$ in real line with usual metric space.
(iii) Prove that union of a finite number of closed sets is closed in a metric space (X,d).
(iv) Does every cauchy sequence in a metric space is convergent. If yes then prove this and if no then give an example.
(v) Define continuous function and give an example.
(vi) State and prove the cancellation laws of group.
(vii) Define group and show that $\{1,2,3\}$ under multiplication modulo 4 is not a group.
(viii) Find all subgroups of cyclic group of order 40.
(ix) If H is a subgroup of a group G. Then prove that $H.H = \{h_1h_2 \colon h_1, h_2 \in H\} = H$
(x) Define permutation and find the order of the permutation $\begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8 \\ 8\,2\,6\,3\,7\,4\,5\,1 \end{pmatrix}$

Section-III

Q3. Let $x = (x_1, x_2, \ldots, x_n)$, $y = (y_1, y_2, \ldots y_n)$ be elements of R^n. If p and q are conjugate indices then prove that

$$\sum_{k=1}^{n} |x_k y_k| \le \left(\sum_{k=1}^{n} |x_k|^p\right)^{1/p} \left(\sum_{k=1}^{n} |y_k|^q\right)^{1/q} \quad (6)$$

Q4. Let (X,d) be a metric space . Prove the following
(i) $A^o \cap B^o = (A \cap B)^o$
(ii) $A^o \cup B^o \subseteq (A \cup B)^o$. Does equality hold? Justify your answer. (6)

Q5. Define order of a group G and its element. For $a \in G$, show that order of a and its conjugate $b^{-1}ab$ is same. (6)

Q6. State and prove Lagrange Theorem. (6)

Q7. Prove that every permutation of degree n can be written as product of cyclic permutation acting on mutually disjoint sets. (6)

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Roll No. ....

Fourth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Elementary Number Theory | **TIME ALLOWED: 30 mins.**
Course Code: MATH-206 / MTH-22313 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

(Objective)

Q#1: Tick or circle the correct answer of the following. MCQs(Marks=10).

i)If a number N is divisible by 16 then which of the following is not true
a) N is divisible by 4 b) N is divisible by 8 c) N is divisible both by 4 & 8 d) N is odd

ii) If $\gcd(a,b)=8$, then
a) Both a & b are even b) a is even but b is odd c) both a & b are odd d) none

iii) If gcd $(a,b)=1$ and $c|a$ then
a) $c\nmid b$ b) $c \mid b$ c) $b = c$ d) none

iv) $\text{lcm}(a,b) = ab$ if and only if
a) $a| b$ b) $a\nmid b$ c) $\gcd(a,b)=1$ d) $\gcd(a,b)\neq 1$

v) The solution of Diophantine equation $6x+51y=22$
a) exist and unique b) exist but not unique c) does not exist d) $x=1, y=1$

vi) If $7\,|ab$, then
a) $7|a$ or $7|b$ b) 7 must divides both a and b c) 7 neither divides a nor b d) none

vii) If 5 is the solution of a linear congruence $P(x) \equiv 0 \pmod n$ and $b \equiv 5 \pmod n$, then
a) $P(b)\not\equiv 0 \pmod n$ b) $P(b)\equiv 0 \pmod n$ c) $P(b)\equiv 1 \pmod n$ d) none

viii) The number of incongruent solutions of $25x\equiv 15 \pmod{29}$ is
a) 29 b) unique c) 2 d) 10

ix) The number 63893548 is divisible by
a) 2 but not by 4 b) both 2 and 4 c) neither by 2 nor by 4 d) divisible by 6

x) The linear congruence $ax\equiv b \pmod n$ has a unique solution if
a) gcd(a,n)=1 b) n is prime c) a| n d) a|b

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Fourth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Elementary Number Theory
Course Code: MATH-206 / MTH-22313

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q#2: Solve the following Short Questions. **(2×10=20 Marks)**

i) Prove that the square of any integer is either of the form $3k$ or $3k+1$.

ii) Prove that if $a \mid b$ and $c \mid d$ then $ac \mid bd$.

iii) Define relatively prime integers and show that if $\gcd(a,b) = d$ then $\gcd(a/d, b/d)=1$.

iv) If $\gcd(a,b)=1$ then prove that $\gcd(a+b, a-b) = 1$ or 2.

v) State Chinese Remainder Theorem.

vi) If $a \equiv b \pmod n$, then show that $a^k \equiv b^k (mod\ n)$ for any positive integer k.

vii) Verify that for any arbitrary integer a, $3|a(a+1)(a+2)$.

viii) Prove that two integers a and b are relatively prime if and only if there exist integers x and y such that $ax+by=1$.

ix) Find gcd(143,227) and lcm(3054,12378).

x) Determine whether the equation 33x+14y=115 can be solved or not? Justify your answer.

SECTION-III

Long Questions (5×6=30 Marks)

Q#3: Prove $21|4^{n+1} + 5^{2n-1}$ by using induction on n.

Q#4: Show that linear Diophantine equation $ax+by = c$ has a solution if and only if $d \mid c$, where $d = \gcd(a,b)$. Also show that if x_0 and y_0 is any particular solution then all other solutions are of the form

$$x = x_0 + \left(\frac{b}{d}\right)t, \qquad y = y_0 - \left(\frac{a}{d}\right)t,$$

where t is any arbitrary integer.

Q#5. Solve the linear congruence $6a \equiv 15 \pmod{21}$.

Q#6. Use Euclidean Algorithm to obtain integers x and y satisfying $\gcd(24,138)=24x+138y$.

Q#7. Use Chinese remainder Theorem to solve the following set of congruences

$$x \equiv 1(mod\ 3),\ \ x \equiv 2(mod\ 5), x \equiv 3(mod\ 7)$$

UNIVERSITY OF THE PUNJAB

Roll No. ....

Fourth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Differential Equations-II
Course Code: MATH-223 / MTH-22334

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Instructions. Attempt all questions

Section-I (Objective) Marks=10

Fill in the blank or answer true/false.

1. If $f(t)$ is not piecewise continuous on $[0,\infty)$, then $\mathcal{L}\{f(t)\}$ will exist. (True/False)
2. The general solution of $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-1)y=0$ is $y=c_1J_1(x)+c_2J_{-1}(x)$. (True/False)
3. $\mathcal{L}^{-1}\left\{\frac{1}{3s-1}\right\}=$ ....
4. $\mathcal{L}\{f(t-a)U(t-a)\}=$ ....
5. If $\mathcal{L}\{f(t)g(t)\}=\mathcal{L}\{F(s)\}\,\mathcal{L}\{G(s)\}$. (True/False)
6. $P_n(-x)=(-1)^nP_n(x)$ (True/False)
7. $\mathcal{L}^{-1}\left\{\frac{k}{s^2-k^2}\right\}=$ ....
8. $x=0$ is an ordinary point of $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$. (True/False)
9. $y=2x$ is a solution of the differential equation $\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+4y=0$. (True/False)
10. If $\mathcal{L}\{f(t)\}=F(s)$ and $\mathcal{L}\{g(t)\}=G(s)$, then $\mathcal{L}^{-1}\{F(s)G(s)\}=\mathcal{L}^{-1}\{F(s)\}\,\mathcal{L}^{-1}\{G(s)\}$. (True/False)

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Fourth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Differential Equations-II
Course Code: MATH-223 / MTH-22334

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Section-II (Short Questions) Marks=20

1. Use the change of variable $y = x^{1/2}v(x)$, find the general solution of the differential equation

$$x^2\frac{d^2y}{dx^2} + (\alpha^2x^2 - \beta^2 + \frac{1}{4})y = 0.$$

2. If $\mathcal{L}\{f(t)\} = F(s)$ and $a > 0$ is a constant, show that

$$\mathcal{L}\{f(at)\} = \frac{1}{a}F(\frac{s}{a}).$$

3. $x = 0$ is a regular singular point of the differential equation

$$3x\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0,$$

find indicial equation and recurrence relation only.

4. Find singular points (if exists) of the following differential equations

$$\begin{aligned}(x^2-a^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 6y &= 0,\\ 2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y &= 0,\\ x\frac{d^2y}{dx^2} + \frac{dy}{dx} - y &= 0,\\ x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-\nu^2)y &= 0,\\ (1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y &= 0.\end{aligned}$$

5. Use integration by parts to show that

$$\mathcal{L}\{\ln t\} = s\mathcal{L}\{t\ln t\} - \frac{1}{s}.$$

Section-III Marks=30

1. Find two power series solutions of the differential equation

$$\frac{d^2y(x)}{dx^2} - 2x\frac{dy(x)}{dx} + 2ay(x) = 0.$$

2. Use the substitution $y(x) = -\frac{1}{u(x)}\frac{du(x)}{dx}$, show that the first-order differential equation

$$\frac{dy(x)}{dx} = x^2 + y^2,$$

transforms into second-order differential equation

$$\frac{d^2u(x)}{dx^2} + x^2u(x) = 0.$$

3. Using Laplace transformation find solution of the following initial-value problem

$$t\frac{d^2y}{dt^2} + \frac{dy}{dx} + ty = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

4. If $\mathcal{L}\{f(t)\} = F(s)$ and $a > 0$, then show that

$$\mathcal{L}\{f(t-a)U(t-a)\} = e^{-as}F(s),$$

where $U(t-a)$ is unit step function.

5. Use the Laplace transformation to solve

$$f(t) + 2\int_0^t f(\tau)\cos(t-\tau)d\tau = 4e^{-t} + \sin t.$$

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Fourth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Linear Algebra **TIME ALLOWED: 30 mins.**

Course Code: MATH-224 / MTH-22120 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Section (1)

Objective

Q. 1	MCQs
(i)	If a set $S = \{v_1, v_2, \ldots v_n\}$ in R^n contains repeated vectors, then the set S is .... a) Linearly dependent b) Linearly independent c) Basis d) None of these
(ii)	The set of all solutions to the homogeneous equation $Ax = 0$ always form a a) Row Rank b) Column Rank c) Subspace d) None of a), b) and c).
(iii)	Which one of the following is not a vector space, a) $C(R)$ b) $R(R)$ c) $Z(Q)$ d) $R(Q)$
(iv)	If a vector space V has a basis of $n+1$ vectors, then every basis of V must contain exactly .... vectors. a) $n-1$ b) n c) $n+1$ d) $n+2$
(v)	The set of vectors $\{(1, 2, 3), (2, 3, 4), (3, 6, 9)\}$ is .... a) Linearly independent b) Linearly dependent c) Basis d) Subspace
(vi)	Let $T : R^5 \rightarrow R^5$ be a linear transformation. Then T is one-one if and only if T is a) Independent b)Onto c) Singular d) Trivial
(vii)	In the group (Z, o) of all integers where $aob = a + b - 3$ for $a, b \in Z$, the inverse of 2 is a) 1 b) 2 c) 3 d) 4 e) Not given
(viii)	Diagonalization of a matrix is possible only if all eigen values are a) Imaginary b) Real c) Repeated d) Not given
(ix)	The product of even and odd permutation is a) odd b) even c) prime d) both a) and b) e) Not given
(x)	The demission of vector space $R(Q)$ is a) 1 b) 2 c) 3 d) infinite e) Not given

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Fourth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Linear Algebra
Course Code: MATH-224 / MTH-22120

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q. 2	Short Questions (4x5 = 20 Marks)
(i)	Define linearly dependent and linearly independent vectors. Determine whether the vectors (3, 0, -3), (-1, 1, 2), (4, 2,-2) and (2,1,1) are linearly dependent or linearly independent?
(ii)	Let $G = < a, b \mid a^2 = b^2 = (ab)^2 = e >$ and $C_2 = < g \mid g^2 = e >$ be two groups, then show that G is isomorphic to $C_2 \times C_2$.
(iii)	Prove that, if S, T are subspaces of a vector space V, then S+T is a subspace of V containing both S and T. Further S+T is the smallest subspace containing both S and T.
(iv)	Find the eigenvalues and eigenvectors of the matrix (if possible) $\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$
(v)	Prove that a linear transformation T: U → V is injective if and only if $N(T) = \{0\}$.

Section-III

	Long Questions (6x5 = 30 Marks)
Q.3	Let $T : R^2 \to R^2$ be a linear transformation defined by $T(x, y) = (2x + 3y, x + y)$. Find the matrix of linear transformation T with respect to the basis $\{(1, 2), (1, 1)\}$.
Q.4	Prove that the vectors v_1, v_2, v_3 are linearly independent if and only if the vectors $v_1 + v_2, v_2 + v_3$ and $v_1 + v_3$ are linearly independent.
Q.5	Show that $\{(1, 1,1), (0,1,1), (0,0,1)\}$ is a basis of R^3. Using Gram-Schmidt orthonormalization process, transform this basis into an orthonormal basis.
Q.6	Prove that the eigenvalues of a symmetric matrix are all real.
Q.7	Let H be a finite subset of a finite group G. Prov that H is a subgroup of G if and only if H is closed.

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Fifth Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Real Analysis-I **TIME ALLOWED: 30 mins.**

Course Code: MATH-301 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Q.1: Encircle the correct answer.

i. If $A \subset B$ then

a) A is proper subset of B b) A is improper subset of B

c) A is superset of B d) None

ii. Between any two rational numbers their lie rational numbers:

a) One b) Two c) Three d) Infinite

iii. Let $x,y,z \in R$, if $x < y$ and $y < z$ then $x < z$ is called.

a) Reflexive property b) Symmetric property c) Transitive property d) None

iv. A real sequence $Sn \leq Sn+1$ is, for all $n \geq 1$ is called:

a) Strictly increasing b) Discontinuous c) Undefined d) None

v. Every Continuous function is

a) Define b) Undefine c) Uniform Cont. d) None

vi. A bounded monotonic sequence ____________ must be convergent.

a) The integers b) Real numbers c) Rational numbers d) Irrational numbers

vii. If $\{t_n\}$ is bounded and $\{s_n\}$ is null sequence then $\{t_n s_n\}$ is:

a) Also a null sequence b) Bounded sequence c) Not a sequence d) None

viii. Every subsequence of a convergent sequence is convergent and coverage to the ____________ limit.

a) Same b) Different c) Infinite d) None

ix. Ordering property does not exist in

a) Real no. b) Complex no. c) Rational no. d) Irrational no.

x. There are types of discontinuity:

a) 1 b) 2 c) 3 d) 4

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Fifth Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Real Analysis-I **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-301 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Q.2: Do you following "Short Questions" **(5x4=20)**

(i) If r is rational and $r \neq 0$ and x is an irrational number then prove that $r + x$ is an irrational number.

(ii) Let F be an ordered field then if $0 < x < y \Rightarrow 0 < \frac{1}{y} < \frac{1}{x}$

(iii) Show that the series $\sum_{n=0}^{\infty} x^n$ is convergent is is $|x| < 1$ and divergent if $|x| \geq 1$

(iv) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ convergent if $p > 1$ and divergent if $p \leq 1$.

(v) Show that $f(x) = x \sin \frac{1}{x}$

$= 0$ at $x = 0$ is continuous

Q. 3: Do the following long questions. **(5x6=30)**

(i) State and prove CAUCHY SCHWARZ inequality

(ii) Let $\underline{x}, \underline{y}, \underline{z} \in R^k$, then prove that $\|\underline{x} \cdot \underline{y}\| \leq \|\underline{x}\| \cdot \|\underline{y}\|$

(iii) If $P > o$, then $\lim_{n \to \infty} \frac{1}{n^p} = 0$

(iv) Define $f(x) = \begin{bmatrix} x+2 & -3 < x < -2 \\ -x-2 & -2 \leq x < 0 \\ x+2 & 0 \leq x < 1 \end{bmatrix}$ discuss the continuity at $x = 0$

(v) Let f be a differentiable real valued function on [a, b], Such that $f'(a) < \lambda < f'(b)$ show that there is a point $x \in (a, b)$ with $f'(x) = \lambda$

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Fifth Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Group Theory-I **TIME ALLOWED: 30 mins.**

Course Code: MATH-302 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

i. Let G be a cyclic group and a be its element then $|a|$ _______ $|G|$

a. $>$ b. $<$ c. $=$ d. none

ii. The order of identity element is

a. 3 b. 2 c. 1 d. 0

iii. In case of abelian group the centralizer of a subset is

a. Subset itself b. $\{e\}$ c. Group itself d. Both a & c

iv. A group of order 4 is always

a. abelian b. cyclic c.non-abelian d. none

v. A one-one homomorphism is called

a. Epimorphism b. Isomorphism c. Monomorphism d. none

vi. The orders of Conjugate elements are

a. Equal b. Different c. one d. none

vii. If for any group G, $Z(G) = G$, where $Z(G)$ is the center of G, then G is

a. Abelian b. non-abelian c. cyclic d. trivial

viii. Let G be a group with $|G|=15$ then G cannot have a subgroup of order

a. 3 b. 7 c. 15 d. 5

ix. Every group is

a. Monoid b.Groupoid c. Semi group d. all of these

x. The center of a group of order 9 is

a. $\{e\}$ b. proper c. group itself d. none

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Fifth Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Group Theory-I
Course Code: MATH-302

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Q.1: Solve the following "Short Questions" **(2x10=20)**

i. Does every abelian group is cyclic? Justify your answer.
ii. Show that the intersection of two subgroups is again a subgroup.
iii. Differentiate between Centralizer and Normalizer.
iv. Define cyclic groups and find all the proper subgroup of order 21.
v. Define double cosets.
vi. Give an example of a group whose all subgroups are cyclic.
vii. Define the Kernel of a group homomorphism with example.
viii. Write down the class equation of $(Z_6, +)$
ix. Give an example of a non-abelian group whose all proper subgroups are abelian.
x. Give an example of a subgroup which is not normal.

Q.2: Solve the following "Long Questions

i. State and Prove Lagrange's Theorem. (6)
ii. Find all the conjugacy classes of a group $V_4 = \{e,a,b,ab\}$. (4)
iii. Prove that a group of even order has at least one element of order 2. (6)
iv. Prove that group of permutation is non-abelian. (4)
v. Let H be a normal subgroup and K a subgroup of group G. Then show that (10)
   a. HK is a subgroup of G.
   b. $H \cap K$ is normal in G.
   c. $HK/H = \cong K/(H \cap K)$

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Fifth Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Complex Analysis-I
Course Code: MATH-303

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Question I. Circle the correct answer to each question. 1 x 10=10

1. An analytic function with constant modulus is
   (a) variable (b) constant
   (c) may be variable or constant (d) none of these

2. The value of $\left(\frac{i^{12}+1}{i}\right)\left(\frac{i^{17}}{i^{18}-1}\right)-7$ is
   (a) $i-7$ (b) $-i-7$ (c) -7 (d) -8

3. A transformation of the type $w=\alpha z+\beta$ where α and β are complex constants is known as
   (a) translation (b) magnification
   (c) linear transformation (d) bilinear transformation

4. $Log(i)=$
   (a) $i\frac{\pi}{2}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) None of these

5. $|e^z|=$
   (a) e^y (b) e^x (c) $e^x e^y$ (d) e^{x+y}

6. The mapping $w=e^z$ is _______ through out the entire $z-$plane.
   (a) Isogonal (b) Conformal (c) Linear (d) None of these

7. A continuous curve which does not have a point of self intersection is called _______.
   (a) Simple curve (b) Multiple curve (c) Integral curve (d) None of these

8. For $C:|z|=1$, the value of $\int_C \frac{dz}{z^2-4}=$ _______ is
   (a) 2π (b) $2\pi i$ (c) 0 (d) None of these

9. Let $f(z)$ be analytic function on and within the boundary of C of a simply connected region D and a be any point within C then $\int_C \frac{f(z)dz}{(z-a)^{n+1}}=$
   (a) $\frac{2\pi i}{n!}$ (b) $\frac{2\pi i}{n!}f(a)$ (c) $\frac{2\pi i}{n!}f^{(n)}(a)$ (d) $\frac{2\pi i}{n!}f^{(n+1)}(a)$

10. Every entire bounded function is constant.
    (a) Cauchy-Goursat theorem (b) Morera's theorem
    (c) Liouville's theorem (d) Cauchy-Fundamental theorem

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Fifth Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Complex Analysis-I
Course Code: MATH-303

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Question II. Write the answer of the following short questions. **5 x 4=20**

1. Prove that $\left|\frac{az+b}{b\bar{z}+a}\right| = 1$ for $|z| = 1$.

2. Find the radius of convergence of the series $\sum_{k=0}^{\infty}(k+1)^k z^k$.

3. Evaluate $\int_C \frac{\cos 2z + \cosh 4z}{z} dz$ where, $C : |z| = 2$.

4. If $z_1 = -1$ and $z_2 = -1$ then prove that $Log(z_1 z_2) = 2\pi i$.

LONG QUESTIONS **10x3=30**

Question III. Prove that if $w = f(z)$ is an analytic function then $\frac{\partial f}{\partial \bar{z}} = 0$.

Question IV. Prove that a line $y = x - 1$ is mapped onto a circle $u^2 + v^2 - u - v = 0$ under the transformation $w = \frac{1}{z}$. Locate the center and radius of the circle.

Question V. Prove that radius of convergence of the series $\sum_{n=0}^{\infty} a_n z^n$ after differentiation and integration remains the same as the original series.

UNIVERSITY OF THE PUNJAB

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Fifth Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Vector and Tensor Analysis **TIME ALLOWED: 30 mins.**

Course Code: MATH-304 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Q.1 Tick the correct option.

(i) A unit vector vector normal to the surface $2x^2 + 4xy - 5z^2 = -10$ at the point (3,-1,2) is

(i) $12\hat{\imath} + 8\hat{\jmath} - 24\hat{k}$ (ii) $\frac{1}{7}(12\hat{\imath} + 8\hat{\jmath} - 24\hat{k})$ (iii) $\frac{1}{7}(3\hat{\imath} + 2\hat{\jmath} - 6\hat{k})$ (iv) $12\hat{\imath} + 8\hat{\jmath}$

(ii) A field $\vec{F}$ is conservative if

(i) $\nabla \times \vec{F} = 0$ (ii) $\nabla \times \vec{F} \neq 0$ (iii) $\nabla \cdot \vec{F} = 0$ (iv) None of these

(iii) How many components does a tensor of rank 2 in a 3-dimensional space?

(i) Zero (ii) Six (iii) Eight (iv) Nine

(iv) A contraction in a tensor of rank 2 yields

(i) A vector (ii) A scalar (iii) A tensor of rank 3 (iv) Zero vector

(v) A vector is solenoidal if its __________ is zero

(i) Gradient (ii) Curl (iii) Divergence (iv) Directional angle

(vi) The scalar product of $3\hat{\imath} - \hat{\jmath}$, $\hat{\jmath} + 2\hat{k}$, $\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$ is ____________.

(i) -10 (ii) 20 (iii) 10 (iv) None of these

(vii) $\vec{A} = 18z\hat{\imath} - 12\hat{\jmath} + 3y\hat{k}$, $z = \frac{12-2x-3y}{6}$ *and* $\hat{n} = \frac{1}{7}(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$, a unit normal to the surface which has the projection in the xy-plane for which $0 \le x \le 6, 0 \le y \le \frac{12-2x}{3}$.

Then the surface integral $\iint \vec{A} \cdot \hat{n}\, dS =$ ____________.

(i) 24 (ii) 12 (iii) Zero (iv) None of these

(viii) The line integral $\int_{P_1}^{P_2} \vec{A} \cdot d\vec{r}$ appears to be independent of the curved path C in a region R joining the two points P_1 & P_2. Then what is true about the vector field $\vec{A}$?

(i) $\nabla \times \vec{A}=0$ (ii) $\nabla \cdot \vec{A}=0$ (iii) $\nabla \times \vec{A} \neq 0$ (iv) None of these

(ix) ____________ theorem converts line integral to surface integral.

(i) Green (ii) Gauss (iii) Divergence (iv) Stokes

(x) The ____________ law is used for determining a quantity whether a tensor or not.

(i) Contraction (ii) Quotient (iii) Kronecker (iv) Product

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Fifth Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Vector and Tensor Analysis
Course Code: MATH-304

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Section – I (Short Questions)

Q.2 Solve the following Short Questions. **(5 × 4 = 20)**

(i) Determine whether $\frac{\partial A_p}{\partial x^q}$ is a tensor or not, where A_p is a covariant tensor of rank one.

(ii) Evaluate $\int_{(0,0)}^{(\pi,2)} (6xy - y^2)dx + (3x^2 - 2xy)dy$ along the cycloid

$$x = \theta - sin\theta, y = 1 - cos\theta.$$

(iii) Find scale factors for cylindrical coordinates.

(iv) Evaluate $\int \vec{A} \times \frac{d^2\vec{A}}{dt^2}$.

(v) Define Christoffel symbol of first and second kind.

Section – II (Long Questions) **(3 × 10 = 30)**

Q.3 Evaluate $\int_{(1,0)}^{(-1,0)} \frac{-ydx+xdy}{x^2+y^2}$ along the straight line segments from (1,0) to (1,-1), then to (-1,-1), then to (-1,0). Show that although $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, the line integral is dependent on the path joining (1,0) to (-1,0) and explain.

Q.4 Represent the vector $\vec{A} = 2y\hat{\imath} - z\hat{\jmath} + 3x\hat{k}$ in spherical coordinates and determine $A_r,\ A_\theta \ and \ A_\varphi$. Also show that spherical coordinate system is orthogonal.

Q.5 Find $g \ and \ g^{jk}$ corresponding to the metric

$$ds^2 = 3(dx^1)^2 + 2(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2.$$

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Roll No. ....

Fifth Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Topology **TIME ALLOWED: 30 mins.**

Course Code: MATH-305 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

SECTION-I

Q. 1 **MCQs (1 Mark each)**

(i) Let $(\mathbb{R},\tau)$ be topological space with usual topology on $\mathbb{R}$ then the set $\mathbb{Q}$ of rational numbers is -----

(a) open (b) closed (c) both open and closed (d) neither open nor closed

(ii) The closure of the subset $(0,1)\cup\{2,3\}$ on the real line $\mathbb{R}$ under the usual topology is ------

(a) $(0,1)\cup\{2,3\}$ (b) $(0,1)$ (c) $[0,1]\cup\{2,3\}$ (d) $[0,1]$

(iii) The interior of the subset $(0,1)\cup\{2,3\}$ on the real line $\mathbb{R}$ under the usual topology is ------

(a) $(0,1)\cup\{2,3\}$ (b) $(0,1)$ (c) $[0,1]\cup\{2,3\}$ (d) $[0,1]$

(iv) In $(\mathbb{R},\tau)$ with usual topology τ on $\mathbb{R}$, then the derived set of $\mathbb{N}=\{1,2,3,...\}$ is -----

(a) $\{0\}$ (b) $\mathbb{N}$ (c) $\mathbb{R}$ (d) ϕ

(v) Let $(\mathbb{R},\tau)$ be topological space with usual topology τ on $\mathbb{R}$, then the boundary of the set $(-1,1)$ is -----

(a) $(-1,1)$ (b) $[-1,1]$ (c) $\{-1,1\}$ (d) ϕ

(vi) Let $(\mathbb{R},\tau)$ be a topological space with usual topology τ on $\mathbb{R}$ then the set $\{0,\pi,e\}$ is -------

(a) open (b) closed (c) both open and closed (d) neither open nor closed

(vii) A space is separable if it contains a ---------

(a) open dense subset (b) countable dense subset

(c) both open and closed subset (d) neither open nor closed subset

(viii) The set $\{\mathbb{Q}\cap(-\infty,r),\mathbb{Q}\cap(r,\infty)\}$ is a disconnection for $\mathbb{Q}$ where r is ---------

(a) integer (b) rational number (c) irrational number (d) real number

(ix) Let (X,τ) be a topological space and $A\subset X$. Show that A is closed if and only if -------

(a) $b(A)\subset A$ (b) $b(A)\supset A$ (c) $A^{\circ}=A$ (d) None of these

(x) Let $A_n=\left[a+\frac{1}{n},b-\frac{1}{n}\right]\subseteq\mathbb{R}$ then $\bigcup_{n=1}^{\infty}A_n=$ -------

(a) (a,b) (b) $[a,b]$ (c) $(a+1,b-1)$ (d) None of these

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Fifth Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Topology | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-305 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

SECTION-II

Q. 2 **SHORT QUESTIONS**

(i) Find the closure of set $A = [2,5)$ and $B = \{2,5\}$ and under the usual topology on R. (4)

(ii) Find the interior of set $A = [0,1)$ and $B = \{0,1\}$ and under the usual topology on R. (4)

(iii) Prove that every metric space (X,d) is normal space. (4)

(iv) Prove that every closed subspace of a normal space is a normal. (4)

(v) Let X be countably compact space. Show that every infinite subset of X has a limit point in X. (4)

SECTION-III

LONG QUESTIONS

Q.3 Let $X = \{x,y,z\}, \tau_X = \{\phi,\{y\},\{x,y\},\{y,z\},X\}$, $Y = \{1,2,3\}, \tau_Y = \{\phi,\{1\},Y\}$, and $f : X \to Y$ be defined by $f(x) = 2, f(y) = 1, f(z) = 3$. Then prove that f is continuous but not open. (6)

Q.4 Prove that closed subspace of a Lindelöf space is Lindelöf. (6)

Q.5 Let X be a topological space and Y be a T_2 space. Let $f : X \to Y$ be a continuous function. Then Show that the graph $G = \{(x,y) : y = f(x), x \in X\} \subseteq X \times Y$ is closed in $X \times Y$. (6)

Q.6 Let X be Hausdorff space, C a compact subset of X and x an element of X which is not in C. Then there are disjoint open sets U_x and V_x in X such that $x \in U_x$ and $C \subseteq V_x$. (6)

Q.7 Prove that continuous image of a connected space is connected. (6)

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Fifth Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Differential Geometry | **TIME ALLOWED: 30 mins.**
Course Code: MATH-306 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Objective Part

Each MCQ carries 1 mark. Encircle and clearly mark the correct option only. Cutting, overwriting and use of ink remover are not allowed. [1x10=10]

1. (i) For the curve $\mathbf{x} = \mathbf{x}(s)$ parameterized by the natural parameter s, the magnitude of the vector $d\mathbf{t}/ds$ is called
(A) curvature (B) torsion (C) geodesic curvature (D) radius of curvature.

(ii) A vector perpendiculat to the rectifying plane is parallel to the
(A) principal normal (B) tangent (C) binormal (D) more information is needed.

(iii) Locus of the centre of curvature is an evolute only when the curve is a
(A) skew curve (B) plane curve (C) space curve (D) twisted curve.

(iv) Position vector $\mathbf{c}$ of the centre of curvature is given by
(A) $\mathbf{r} + \rho\mathbf{n}$ (B) $\mathbf{r} + \kappa\mathbf{n}$ (C) $\mathbf{r} + \tau\mathbf{n}$ (D) none of these.

(v) The tangent lines along the principal sections at a point are called
(A) tangential lines (B) principle directions (C) horizontal lines (D) normal lines.

(vi) The surface is called minimal surface if at all points on the surface the mean curvature is
(A) positive (B) negative (C) zero (D) infinite.

(vii) If radius of curvature is constant for a given curve then tangent to the locus of the centre of curvature is parallel to
(A) the binormal (B) the tangent (C) the normal (D) the rectifying plane.

(viii) If the curvature κ is zero at all points of a space curve then the curve is a
(A) planar curve (B) straight line (C) circle (D) sphere.

(ix) A point P of a smooth surface is umbilical iff the Gaussian curvature K and the mean curvature H satisfy the relation
(A) $H^2 - K = 0$ (B) $H - K^2 = 0$ (C) $H - K = 0$ (D) $H + K = 0$.

(x) At a point of inflection on a curve $\mathbf{x} = \mathbf{x}(s)$, the radius of curvature ρ is
(A) zero (B) constant (C) infinite (D) more information is needed.

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Fifth Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Differential Geometry | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-306 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Subjective Part

Note: Attempt this part of the Paper on Separate Sheet(s). Question 2 is worth a total of 20 marks and question 3 is worth a total of 30 marks.

SECTION-I (SHORT QUESTIONS)

Attempt the following questions. [4x5=20]

2. (i) Derive the expression for the three planes namely the osculating plane, the normal plane and the rectifying plane at a given point $s = s_0$ on a curve $\mathrm{x} = \mathrm{x}(s)$. Workout these planes for the helix $x = a\ cos\varphi,\ y = b\ sin\varphi,\ z = b\ \varphi$ at $\varphi = \pi/2$.
   (ii) Show that the torsion τ of the twisted curve $\mathrm{r} = \mathrm{r}(s)$ satisfies the relation $\tau\kappa^2 = [r', r'', r''']$.
   (iii) Find the singular and non singular points of the epicycloid given by $x = 4\cos\vartheta - \cos 4\vartheta,\ \ y = 4\sin\vartheta - \sin 4\vartheta$ and determine its intrinsic equations.
   (iv) Prove that the tangent at any point P_1 of the involute C_1 is parallel to the normal at a corresponding point to the curve C.
   (v) State Fundamental Existence and Uniqueness Theorem for space curves. Derive the equation of a curve whose intrinsic equations are $\kappa = \kappa(s)$ and $\tau(s) = 0$ and hence find the curve for which $\kappa(s) = \frac{1}{\sqrt{2as}}$ and $\tau(s) = 0$, where a is constant and s, the arc-length parameter.

SECTION-II (LONG QUESTIONS)

Attempt the following questions. [3x10=30]

3. Show that along a regular curve $\mathrm{x} = \mathrm{x}(s)$ of class ≥ 4, $\left[\mathrm{x}^{(2)}, \mathrm{x}^{(3)}, \mathrm{x}^{(4)}\right] = \kappa^5 \frac{d}{ds}\left(\frac{\tau}{\kappa}\right)$ and hence is a general helix iff $\left[\mathrm{x}^{(2)}, \mathrm{x}^{(3)}, \mathrm{x}^{(4)}\right] = 0$, where $\mathrm{x}^{(j)}$denote the derivatives $\frac{d^j\mathrm{x}}{ds^j}$ for $j = 2, 3, 4$.
4. Prove that the coefficients of the first and second fundamental forms satisfy the Codazzi-Mainardi equations and the Gauss equations as the normal and tangential components of the compatibility conditions.
5. Show that Monge patch $\mathrm{x}(u, v) = (u, v, h(u, v))$ is a regular surface parametric representation of class C^m if $h(u, v)$ is of class C^m. Find the expression for the normal curvature k_n and the geodesic curvature κ_g. What is vanishing condition for κ_g at a point on the surface? What does it tell us about the nature of the surface?

UNIVERSITY OF THE PUNJAB

Roll No. ....

Sixth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Real Analysis-II — **TIME ALLOWED: 30 mins.**

Course Code: MATH-307 — **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Q.1: Choose the best option. **Marks: 10**

(i) A convergent integral whose absolute integral is divergent is called?

(a) absolutely convergent (b) divergent

(c) conditionally convergent (d) none

(ii) Let p be a statement such that: p: A function which is function of bounded variation is always a continuous function.

(a) p is always false (b) p is always true

(c) p is not statement (d) not sure

(iii) The Gamma function $\int_0^\infty x^{m-1}e^{-x}\,dx$ converges if

(a) $m = 0$ (b) $m < 0$

(c) $m > 0$ (d) $m = -1$

(iv) The integral $\int_1^\infty x^{-p}dx$ divergent if,

(a) $p > 1$ (b) $p \le 1$

(c) $p = -1$ (d) $p = 0$

(v) Riemann Steiltjes integral becomes Riemann integral if the monotonically increasing function α becomes.

(a) bounded (b) continuous

(c) identity function (d) discontinuous

(vi) A partition $\dot{P}$ is said to common refinement of partitionsP_1 & P_2, if

(a) $\dot{P} = P_1 \cap P_2$ (b) $\dot{P} = P_1 \cup P_2$

(c) $\dot{P} = P_1 - P_2$ (d) $\dot{P} = P_2 - P_1$

(vii) The integral $\int_1^\infty \frac{1}{x^p}dx$ is not divergent if

(a) $p > 1$ (b) $p = 1$

(c) $p < 1$ (d) $p = 0$

(viii) A partition Q is said to be a refinement of a partition P if

(a) $Q \subset P$ (b) $P \subset Q$

(c) $P \not\in Q$ (d) none

(ix) An infinite integral which oscillates finitely becomes ________ after the insertion of a bounded monotonic factor which tends to zero as a limit.

(a) divergent (b) convergent

(c) oscillate (d) unbounded

(x) If $f \in R(\alpha_1)$ and $f \in R(\alpha_2)$ then

(a) $f \in R(\alpha_1) + R(\alpha_2)$ (b) $f \in R\left(\frac{\alpha_1}{\alpha_2}\right)$

(c) $f \in R(\alpha_1+\alpha_2)$ (d) none of all

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Real Analysis-II
Course Code: MATH-307

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Questions with Short Answers. **Marks: 20**

Q.2: Answer the following short questions. All questions carry equal marks. (5×4=20)

(i) Test Convergence of the integral $\lim_{\lambda\to 0}\int_1^x t^{\lambda-1}dt = \log x$

(ii) State and prove weirstress M – test for uniform convergence of functions.

(iii) Let $f \in$ R (α) on [a,b], then prove that $|f| \in R(\alpha)\, on [a,b]$

(iv) Prove that a monotone function f is a function of bounded variation.

(v) Discuss the convergence of integral $\int_1^\infty x^{-p}dx$

Q.3: Answer these Long questions. All questions carry equal marks. (6×5=30)

(i) Prove that integral $\int_1^0 x^{m-1}(1-x)^{n-1}dx$ is convergent if and only m , n > 0.

(ii) Let f be continuous function on [a,b], then prove that $f \in$ R (α) on [a,b].

(iii) State and prove first fundamental theorem for integral calculus.

(iv) Prove that a function f is a function of bounded variation on [a,b], if and only if f can be expressed as a difference of two increasing functions.

(v) Test the convergence of the integral $\int_0^\infty \frac{\cos x}{\sqrt{x^2+x}}$

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Sixth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Rings and Vector Spaces | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-308 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJETIVE TYPE

Q. 2 **SECTION-II**

(i) Define Maximal Ideal of a ring R and give one example. (4)

(ii) Let R be a commutative ring and $a \in R$, then show that $aR = \{ar : r \in R\}$ is an ideal in R. (4)

(iii) Define similar matrices and prove that eigen values of the similar matrices are same. (4)

(iv) Let R be a commutative ring with 1 as its identity element. Then R / I is integral domain if I is prime ideal. (4)

(v) Prove that one-to-one linear transformation preserves the basis and dimension. (4)

SECTION-III

Q.3 Let R be a commutative ring with identity. The ideal P is prime ideal *iff* the quotient ring R / P is an integral domain. (6)

Q.4 Prove that a finite integral domain is a field. (6)

Q.5 Distinguish between integral domain and division ring. (6)

Q.6 Prove that quotient ring is a ring. (6)

Q.7 Find the eigen values and eigen vectors of $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$. (6)

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Roll No. ....

Sixth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Rings and Vector Spaces | **TIME ALLOWED: 30 mins.**

Course Code: MATH-308 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION-I

Q. 1 **MCQs (1 Mark each)**

(i) A ring R is a Boolean Ring if

a) $x^2 = x \ \forall x \in R$ b) $x^2 = -x \ \forall x \in R$ c) $x = x \ \forall x \in R$ d) None of these

(ii) nZ is a maximal ideal of a ring Z if and only if n is ....

a) Prime number b) Composite number c) natural number d) None of these

(iii) Let R^3 be the vector space of all ordered triples of real numbers. Then the transformation $T : R^3 \to R^3$ defined by $T(x, y, z) = (x, y, 0)$ is

a) Linear b) Not Linear c) Rational d) None of these

(iv) What are Zero divisors in the Ring of integers modulo 5

a) $\bar{2}$ b) $\bar{2}$ and $\bar{3}$ c) no zero divisor d) None of these

(v) The number of proper ideals of R is............................

(a) 0 (b) 1 (c) 2 (d) 3

(vi) Which of the following is vector space

(a) $Q(Q)$ (b) $Q(R)$ (c) $R(C)$ (d) $C(Z)$

(vii) If λ is an eigenvalue of a matrix A and x is a corresponding eigenvector, and if k is any positive integer, then .... is an eigenvalue of A^k and x is a corresponding eigenvector.

a) λ^k b) λ^{k-1} c) λ^{2k} d) None of these

(viii) The dimension of $\operatorname{Im} T$ is called ....

(a) Rank (b)Nullity (c) basis (d)} none of these

(ix) The dimension of $KerT$ is called ....

(a) Rank (b)Nullity (c) basis (d)} none of these

(x) The set $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ is a .... for the vector space V of all 2 x 2 matrices.

(a) Linearly dependent (b) Null space

(c) Basis (d) None of these

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Sixth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Complex Analysis-II | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-309 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SHORT QUESTIONS

Q.2. Write the answer of the following short questions. **5 x 4=20**

I. Show that when $0 < |z| < 4$, $\frac{1}{4z - z^2} = \sum_{n=0}^{\infty} \frac{z^{n-1}}{4^{n+1}}$.

II. Find the residue of the functions at given singularities.

(a) $\exp\left(\frac{2}{3z}\right)$ at $z = 0$ (b) $\frac{z^2 + 2}{(z+2)^3}$ at z= -2.

III. Prove that the series $\sum_{n=0}^{\infty} \frac{z^n}{2n+1}$ and $\sum_{n=0}^{\infty} \frac{(z-i)^n}{(2-i)^{n+1}}$ are analytic continuations of each other.

IV. Evaluate $\int_C \frac{z^2 - z + 1}{(z-1)(z-4)(z+3)} dz$ where C is the circle $|z| = 5$.

V. Investigate the zeros, poles and singularities of the following functions at $z = \infty$.
(a) z^2 (b) $\exp(2z)$ (c) $z^2(z+1)$

LONG QUESTIONS **10x3=30**

Q.3. (a) Find the Laurent series of the function

$$f(z) = \frac{1}{(z+1)(z+3)} \quad \text{for the region } |z| < 1.$$

(b) Expand the function $\frac{1}{z^2}$ about $z = 2$ using Taylor series.

Q.4. (a) Evaluate $\int_C \frac{e^{ez} dz}{\cosh(\pi z)}$ where $C : |z| = 1$.

(b) Expand $f(z) = \cot(\pi z)$ by Mittag-Leffler's theorem and prove that

$$\frac{\pi^2}{(\sin \pi z)^2} = \sum_{n=-\infty}^{\infty} \frac{1}{(z-n)^2}.$$

Q.5. Prove that $\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$ where $a > b > 0$.

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Roll No. ....

Sixth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Complex Analysis-II
Course Code: MATH-309

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Attempt all questions. Use of Scientific Calculators and Statistical tables is allowed but exchange of anything i.e., calculators etc. is not allowed.

OBJECTIVE TYPE

Q.1. Tick the correct answer to each question. 1 x 10=10

1. The function $f(z) = \log z$ has _______ singularity at $z = 0$.

   (a) essential isolated (b) removable (c) non-isolated (d) none of these

2. The value of $\left(\frac{i^{12}+1}{i}\right)\left(\frac{i^{17}}{i^{18}-1}\right) - 7$ is

   (a) $i - 7$ (b) $-i - 7$ (c) -7 (d) -8

3. The function $f(z) = 1 - \cos z$ has a zero of order _______ at $z = 0$.

   (a) 1 (b) 2 (c) 3 (d) 4

4. The zeros of $f(z)$ at $z = a$ are _______ of $\frac{1}{f(z)}$ at $z = a$.

   (a) zeros (b) critical points (c) residue (d) poles

5. $|e^z| =$

   (a) e^y (b) e^x (c) $e^x e^y$ (d) e^{x+y}

6. A function $f(z)$ which has no singularity in the finite part of the plane or at infinity is called _______ function.

   (a) an analytic (b) entire (c) a constant (d) meromorphic

7. An analytic function with constant modulus is

   (a) variable (b) constant
   (c) may be variable or constant (d) none of these

8. The function $f(z) = \frac{e^z}{\sinh \pi z}$ has poles of order

   (a) 0 (b) $n, n \in \mathbb{Z}$ (c) 1 (d) none of these

9. If $f(z)$ is analytic function except at a finite number of poles $a_1, a_2, \ldots, a_n$ within a closed contour C and continuous on the boundary of C then, $\int_C f(z)dz =$ _______.

   (a) $2\pi i \sum_{i=1}^{n^2} R_i$ (b) $2\pi i \sum_{i=1}^{n} R_i$ (c) $\pi i \sum_{i=1}^{n} R_i$ (d) $2\pi \sum_{i=1}^{n} R_i$

10. If $f(z)$ is entire function having zeros at $a_1, a_2, \ldots, a_n$ which can be arranged as _______ then,
    $f(z) = f(0)\exp\left(\frac{f'(0)}{f(0)}\right)\prod_{n=-\infty}^{\infty}\left(1 - \frac{z}{a_n}\right)\exp(\frac{z}{a_n})$.

    (a) $|a_1| = |a_2| = \ldots = |a_n|$ (b) $|a_1| < |a_2| < \ldots < |a_n|$
    (c) $|a_1| \le |a_2| \le \ldots \le |a_n|$ (d) $|a_1| \ge |a_2| \ge \ldots \ge |a_n|$

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mechanics | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-310 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

2. Write Short Answers: *20 marks*

(a) (2 marks) What is principal axis and principal M.I.

(b) (2 marks) Calculate angular speed of the earth about its axis

(c) (2 marks) Write down the equilibrium conditions for a rigid body.

(d) (2 marks) Find M.I of rod of mass M and length $2L$ about a line through end points and perpendicular to rod.

(e) (2 marks) Define inertia and write its units.

(f) (2 marks) State Euler's theorem.

(g) (2 marks) Write down the necessary and sufficient conditions for two systems S_1, S_2 to be equimomental.

(h) (2 marks) Let $\vec{r}'_v$ and $\vec{v}'_v$ be position vector and velocity of a particle v relative to centre of mass, show that

$$\sum_v m_v \vec{r}'_v = 0$$

(i) (4 marks) For a system of N particles, show that the components L_x, L_y and L_z of angular momentum L in terms of moments and products of inertia are:
$L_x = \omega_x I_{xx} + \omega_y I_{xy} + \omega_z Ixz$, $L_y = \omega_x I_{xy} + \omega_y I_{yy} + \omega_z Iyz$ and $L_z = \omega_x I_{xz} + \omega_y I_{yz} + \omega_z Izz$

3. Write Brief Answers: *30 marks*

(a) (5 marks) Find a set of three rotation matrices for Euler angles and express the components of angular velocity in terms of these angles.

(b) (5 marks) Using euler equation of motion for a rigid body having zero external torque show that $I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2$ and $I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$ are conserved quantities. What do they represent?

(c) (5 marks) A system consist of three particles, each of unit mass, with positions and velocities as follows:

$$\begin{aligned} r_1 &= i + j\,, \quad v_1 = 2i \\ r_2 &= j + k\,, \quad v_2 = j \\ r_3 &= k\,, \quad v_3 = i + j + k \end{aligned}$$

Find the position and velocity of the center of mass. Find also the linear momentum of the system. Find the kinetic energy of the above system.

(d) (5 marks) Find the angular momentum about the origin in part (c).

(e) (5 marks) Discuss Torque free motion of a rigid body symmetric about an axis with one point fixed.

(f) (5 marks) State and prove CHASLE's theorem.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Sixth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Mechanics **TIME ALLOWED: 30 mins.**

Course Code: MATH-310 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

1. Encircle the Correct Option Only. *10 marks*

(a) (1 mark) The moment of inertia of a circular section of diameter 'd' about its centroidal axis is given by
A. $\frac{\pi d^4}{16}$ B. $\frac{\pi d^4}{32}$ C. $\frac{\pi d^4}{64}$ D. $\frac{\pi d^3}{32}$

(b) (1 mark) Calculate gyroscopic couple acting on a disc which has radius of 135 mm. Angular and precessional velocities are 15 rad/sec and 7 rad/sec respectively. Assume density = 7810 kg/m^3 and thickness of disc = 30 mm
A. 12.83 N-m B. 10.99 N-m C. 11 N-m D. Incomplete data

(c) (1 mark) A surface having no thickness is called
A. Ellipsoid B. Cuboid C. Lamina D. Sphere

(d) (1 mark) Radius of Gyration is
A. $\frac{1}{M}$ B. $\sqrt{M}$ C. $\sqrt{\frac{I}{M}}$ D. $\frac{I}{G}$

(e) (1 mark) Relationship between the time rate of change of angular momentum of a rigid body relative to axes fixed in space and in the body respectively given by
A. $\frac{d\Omega}{dt}|_s = \frac{d\Omega}{dt}|_b + 2\omega \times \Omega$ B. $\frac{d\Omega}{dt}|_s = \frac{d\Omega}{dt}|_b + \omega \times \Omega$ C. $\frac{d\Omega}{dt}|_s = \frac{d\Omega}{dt}|_b$ D. $\frac{d\Omega}{dt}|_s = 2\frac{d\Omega}{dt}|_b + \omega \times \Omega$

(f) (1 mark) The degrees of freedom for a system consisting of N particles with m constraints are:
A. 3N B. 3N+m C. 3N-m D. 6

(g) (1 mark) The kinetic energy of rotation of a rigid body with respect to its principle axes of inertia is given by
A. $T = \frac{1}{2}(I_{xx}\omega_x^2 + I_{yy}\omega_y^2 + Izz\omega_z^2)$ B. $T = \frac{1}{2}(I_{xx}\omega_x + I_{yy}\omega_y + Izz\omega_z)$ C. $T = (I_{xx}\omega_x^2 + I_{yy}\omega_y^2 + Izz\omega_z^2)$ D. $T = (I_{xx}\omega_x + I_{yy}\omega_y + Izz\omega_z)$

(h) (1 mark) For a circular wire of uniform density ρ, radius a and mass m, the moment of inertia $I = \int_0^{2\pi} (a\sin\theta)^2 (\rho a d\theta)$ about one of its diameters simplifies to

A. $\frac{1}{2}ma^2$ B. $\frac{3}{2}ma$ C. $\frac{1}{3}ma^2$ D. $\frac{1}{12}ma^2$

(i) (1 mark) When a body is in rest position or moving with constant velocity, then force required to change the state of motion is called
A. Centripetal force B. Inertia C. Equimomental force D. Angular Momentum

(j) (1 mark) $\omega \times \vec{r}$ is called
A. Coriolis acceleration B. Apparent acceleration C. Transverse acceleration D. Angular acceleration

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Functional Analysis-I — **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-311 — **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q.2 (i) Show that in a metric space (X,d) every convergent sequence is Cauchy. (4)

(ii) Suppose that $\|.\| \sim \|.\|_1$ be equivalent norms defined on a linear space N. Then prove that every Cauchy sequence in $(N,\|.\|)$ is also Cauchy sequence in $(N,\|.\|_1)$. (4)

(iii) Find the norm of the linear functional f on $C[-1,1]$ defined by (4)

$$f(x)=\int_{-1}^{0} x(t)dt-\int_{0}^{1} x(t)dt.$$

(iv) Show that the norm $\|.\|: N \to \mathbb{R}$ is uniformly continuous. (4)

(v) For any complete subspace A of an inner product space V, prove that $A = A^{\perp\perp}$. (4)

SECTION-III

Q.3 Prove that a subspace Y of complete metric space (X,d) is complete if and only if Y is closed in X. (6)

Q.4 Suppose $\|.\|$ and $\|.\|_0$ are equivalent norms on N_1, let N be a finite dimensional subspace of $(N_1,\|.\|_0)$, then prove that N is complete subspace of $(N_1,\|.\|_0)$. (6)

In particular $(N,\|.\|_0)$ is complete.

Q.5 Show that any two norms defined on a finite dimensional normed space are equivalent. (6)

Q.6 Consider the space $B(N,M)$ of all bounded linear operators with the norm $\|T\| = \sup_{\|x\|=1} |Tx|, x \in N$. Show that if M is Banach space then so is $B(N,M)$. (6)

Q.7 Let A be non-empty complete convex subset of an inner product space V, and $x \in V \setminus A$. Then there is a unique $y \in A$ such that $\|x-y\| = \inf_{y' \in A}\|x-y'\|$. (6)

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Sixth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Functional Analysis-I | **TIME ALLOWED: 30 mins.**

Course Code: MATH-311 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION-I

Q. 1 **MCQs (1 Mark each)**

(i) In the real line $\mathbb{R}$, an example of nowhere dense subset is ---------

(a) $\mathbb{Z}$ (b) $\mathbb{Q}$ (c) $\mathbb{Q}'$ (d) $\mathbb{R}$

(ii) Let $f: R \to R$ given by $f(x) = cx$, then for any $\varepsilon > 0$, function f is uniformly continuous for $\delta =$ -----

(a) ε (b) $c\varepsilon$ (c) $\varepsilon / 3$ (d) c / ε

(iii) A complete subspace of $\mathbb{R}$ being a closed subspace is --------

(a) $(0,1)$ (b) $(0,1]$ (c) $[0,1)$ (d) $[0,1]$

(iv) In $(\mathbb{R}, d)$ with usual metric d on $\mathbb{R}$ the boundary of the set $\mathbb{Q}$ is ---------

(a) $\mathbb{Q}$ (b) $\mathbb{R}$ (c) $\mathbb{Q}'$ (d) None of these

(v) In $(\mathbb{R}, d)$ with usual metric d on $\mathbb{R}$ the closure of $(0,1] \cup \{2,3\}$ is ---------

(a) $(0,1] \cup \{2,3\}$ (b) $(0,1) \cup \{2,3\}$ (c) $[0,1] \cup \{2,3\}$ (d) None of these

(vi) Let $A = \{1,2,3,\ldots\}, B = \left\{n - \frac{1}{n}; n \geq 2, n \in N\right\}$. Then $d(A,B) =$ ---------

(a) n (b) $2n$ (c) 0 (d) N

(vii) An example of convex combination of the vectors x, y, z in a linear space V is ---------

(a) $2x + 3y + z$ (b) $\frac{1}{2}x + \frac{1}{3}y + \frac{1}{4}z$

(c) $\frac{1}{3}x + \frac{1}{3}y + \frac{1}{3}z$ (d) $\frac{1}{2}x + \frac{1}{2}y + \frac{1}{2}z$

(viii) A subset A of a linear space N is convex if for any $x, y \in A, \alpha \in [0,1]$, ___________

(a) $\alpha x + (1+\alpha)x \in A$ (b) $\alpha x + (1-\alpha)x \in A$

(c) $\alpha x + \alpha x \in A$ (d) $\alpha + (1-\alpha)x \in A$

(ix) Let **c** be the space of all convergent sequences. Then the sequences $e = (1,1,1,\ldots)$ and $e_i = \{\delta_{ij}\}$ form a base for **c**. Then each $x \in$ **c** be a sequence, which converges to a, can be uniquely written as ---------

(a) $x = \sum_{k=1}^{\infty} x_k e_k$ (b) $x = \sum_{k=1}^{\infty} (x_k - a)e_k$

(c) $x = ae + \sum_{k=1}^{\infty} x_k e_k$ (d) $x = ae + \sum_{k=1}^{\infty} (x_k - a)e_k$

(x) Every linear operator in a finite dimensional normed space is --------

(a) open (b) closed (c) bounded (d) unbounded

UNIVERSITY OF THE PUNJAB

Roll No. ....

Sixth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Ordinary Differential Equations **TIME ALLOWED: 30 mins.**

Course Code: MATH-312 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Multiple Choice Questions

Q1. Encircle the correct choice of the following questions

1. For Legendre polynomial $P_3(x)$ is given by

   a) $\frac{1}{2}(5x^2-3x)$ b) $\frac{1}{2}(x^3-5x)$ c) $\frac{1}{2}(5x^3-x)$ d) $\frac{1}{2}(x^3-x)$

2. $y=0$ is called ------------------ solution of $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$

   a) trivial b) complementary c) particular d) singular

3. Eigen values of S-L system are always

   a) Real b) complex c) rational d) integral

4. First order differential equation $\frac{dy}{dx}=x\sqrt{1-y^2}$ is of the type

   a) Linear differential equation b) exact c) homogenous d) separable

5. A differential equation $\frac{dy}{dx}=Py^2+Qy+R$ where P,Q and R functions of x (constants) is called

   a) Clairauts equation b) Ricatti equation c) Bernoulli's equation d) None of these

6. The solutions of S-L equation are called the

   a) S-L functions b) particular solutions c) eigen functions d)None of these

7. $\frac{dy}{dx}[x^n J_n(x)]=$ ....

   a) $nx^{n-1}J_n(x)$ b) $-x^{-n}J_{n+1}(x)$ c) $x^n J_{n-1}(x)$ d) $J_{n-1}(x)$

8. $J_{1/2}(x)=$ ....

   a) $\sqrt{\frac{2}{\pi x}}\sin x$ b) $\sqrt{\frac{2}{\pi x}}\cos x$ c) $\sqrt{\frac{2}{\pi x}}(\frac{\sin x}{x}-\cos x)$ d) $-\sqrt{\frac{2}{\pi x}}(\frac{\cos x}{x}+\sin x)$

9. $P_2(x)=$ ....

   a) $\frac{1}{2}(x^2-3)$ b) $\frac{1}{2}(3x^2-2)$ c) $\frac{1}{2}(3x^2-1)$ d) $\frac{1}{2}(3x^2-x)$

10. $P_n(-x)=$ ....

    a) $(-1)^n P_n(x)$ b) $(-1)^{n/2}P_n(x)$ c) $-P_n(x)$ d)None of these

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Sixth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Ordinary Differential Equations — **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-312 — **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION I (Short Questions)

Q2. Solve the following short questions **(4x5=20)**

1. Show that if $u(x)$ and $v(x)$ are periodic solutions of the Mathieu equation with period π having distinct eigenvalues, then $\int_0^\pi u(x)\, v(x)\, dx = 0$

2. Drive the following recurrence relation for Legendre polynomials

$$nP_n(x) = x\frac{d}{dx}P_n(x) - \frac{d}{dx}P_{n-1}(x)$$

3. Determine the eigen values of the system

$$u''(x)+\lambda u(x)=0 \text{ with } u(0) = u(\pi),\ u'(0) = 2u'(\pi).$$

4. Solve the differential equation $\frac{d^2y}{dx^2} - y = \cosh x$

SECTION II (Long Questions) **(3x10=30)**

Q3. Prove that the eigen values of regular S-L system are real.

Q4. Use appropriate recurrence relations to express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.

Q5. Find the series solution of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 2y = 0$ with centre of expansion at $x_0 = 1$

UNIVERSITY OF THE PUNJAB

Roll No. ....

Seventh Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Set Theory | **TIME ALLOWED: 30 mins.**

Course Code: MATH-401 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION-I

Q 1. Encircle the correct option.

i. An ordered set S is said to be well ordered if every subset of S contains ------------- element.
a) maximal b) minimal c) last d) first

ii. In a partially ordered set S, for $a \in S$ is a/an ----------------- of b if $a \ll b$.
a) predecessor b) immediate predecessor c) immediate successor d) successor

iii. Let α be the cardinality of a non-empty set A. Then by Cantor Theorem
a) $\alpha = 2^{\alpha}$ b) $\alpha < 2^{\alpha}$ c) $\alpha \leq 2^{\alpha}$ d) $\alpha \nless 2^{\alpha}$

iv. Cardinal numbers of infinite sets are called ------------- cardinal numbers.
a) finite b) countable c) transfinite d) continuum

v. A set S is ------------ if it has the same cardinality as a proper subset of itself.

a) finite b) infinite c) countable d) uncountable

vi. Let a and b are elements of partially ordered set S. We say a and b are ---------- if neither preceeds nor dominates other.

a) non comparable b) comparable c) divisible d) maxmial

vii. An element $a \in S$ is called ------------ element of S, if no element of S strictly precedes a, i.e. if $x \leq a$ implies $x = a$.

a) maximal b) last c) minimal d) first.

viii. Every ----------- set contains a subset which is countable.

a) finite b) countable c) infinite d) uncountable

ix. Let A = {3, 4}, B = {a, b, c}. Then A^B consists exactly -------- functions.

a) 5 b) 6 c) 8 d) 16

x. If a relation R satisfy the reflexive, symmetric and transitive properties then R is a/an-------- relation.

a) equivalence b) partial ordered c) linearly ordered d) well ordered

UNIVERSITY OF THE PUNJAB

Seventh Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Set Theory
Course Code: MATH-401

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q.2 **SHORT QUESTIONS (5*4)**

(i) Prove that every infinite set contains a countable subset.
(ii) Let α, β be finite cardinal numbers. If $\alpha + \beta$ represents the usual addition in N then show that $n + \aleph_0 = \aleph_0$; $\aleph_0 + \aleph_0 = \aleph_0$ and $c + c = c$.
(iii) Define totally ordered set and partially ordered sets with examples.
(iv) Let A = $\{A_i \mid i \in I\}$ be a set of pairwise disjoint intervals in the line R. Show that A is countable.
(v) Prove that in an ordered set first and last elements are unique.

SECTION-III

LONG QUESTONS (6*5)

Q.3. State and prove Schroeder Bernstein theorem. (6)

Q.4. (a) Prove that $[0,1] \sim (0,1)$

(b) State (i) well ordering theorem (ii) Russell's Paradox (iii) cantor's Paradox (3+3)

Q.5. Draw a Hasse Diagram on $P(S)$, under set inclusion, where $S = \{1,2,3\}$ and $P(S)$ denote the power set of S, then find the followings.

(i) Maximal element
(ii) Minimal element
(iii) Upper and lower bounds of $X = \{2\}$.
(iv) All chains and anti-chains of the Hasse Diagram (6)

Q.6. Prove that every element in a Well-ordered set has a unique immediate predecessor except the first element.

Q.7 (a) State Zorn's Lemma. Apply it to deduce that every vector space admits a basis. (3+3)
(b) Let $s(\lambda)$ be the set of ordinal numbers which precede λ. Then show that $\lambda = ord(s(\lambda))$.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Seventh Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Partial Differential Equations | **TIME ALLOWED: 30 mins.**

Course Code: MATH-402 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

NOTE: Attempt all questions from each section.

SECTION I

1. $u_{xx} + u_{yy} + u_{zz} = 0$ is called (1 mark)

(a) heat equation

(b) Laplace equation

(c) wave equation

(d) none of the above

2. Auxiliary equation for $Pp + Qq = R$ is (1 mark)

(a) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

(b) $\frac{dx}{p} = \frac{dy}{q} = \frac{dz}{r}$

(c) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{r}$

(d) none of the above

3. To convert $u_{xx} - 5u_{xy} + 6u_{yy} = 0$ into canonical form, we use (1 mark)

(a) $\xi = 2x + y, \eta = 3x + y$

(b) $\xi = x + y, \eta = x$

(c) $\xi = x - y, \eta = y$

(d) none of the above

4. $\frac{\partial^2 T}{\partial x^2} - K\frac{\partial T}{\partial x} = 0$, where T is temperature is called (1 mark)

(a) heat equation in one dimension

(b) Laplace equation

(c) wave equation

(d) none of the above

5. $\frac{\partial^2 z}{\partial x \partial y}$ =——— (in usual notation) (1 mark)

(a) p

(b) q

(c) r

(d) s

P.T.O.

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Seventh Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Partial Differential Equations
Course Code: MATH-402

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

NOTE: Attempt all questions from each section.

SECTION II-Questions with Short Answers

1. Find the PDE for $u = ax + (1-a)y + b$. (4 marks)

2. Find the integral surface for $\frac{dx}{cy-bz} = \frac{dy}{az-cx} = \frac{dz}{bx-cy}$. (4 marks)

3. Find the canonical form for $z_{xx} - 5z_{xy} + 6z_{yy} = 0$. (4 marks)

4. Evaluate the complete integral surface of the PDE, $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ containing $x + y = 0,\ z = 1$. (4 marks)

5. Define linear, non-linear and quasi linear PDE with atleast one example. (4 marks)

SECTION III-Questions with Brief Answers

6. Solve $(D_xD_y + D_x - D_y - 1)u = xy$. (6 marks)

7. Convert the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. into polar form. (6 marks)

8. Reduce into normal form and find an integral surface of $2u_{xx} - 2yu_{xy} - u_y = 0$ which contains $u(x,1) = x^2/2,\quad u_y(x,1) = 2$. (6 marks)

9. Solve (6 marks)

$$(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0.$$

10. Interpret and solve the following equation: (6 marks)

$$\frac{1}{c^2}u_t = u_{xx} + u_{yy},\quad u(0,y,t) = u(a,y,t) = 0,\quad u(x,y,0) = f(x,y),$$
$$u(x,0,t) = u(x,b,t) = 0,\ t > 0\quad x \in (0,a),\quad y \in (0,b).$$

UNIVERSITY OF THE PUNJAB

Roll No. ....

Seventh Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Numerical Analysis-I **TIME ALLOWED: 30 mins.**

Course Code: MATH-403 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Note: Attempt all questions.

OBJECTIVE

Q N0. 1:- Tick the correct answers $1 \times 10=10$

(i) Order of convergence of Newton-Rahpson method is
(a) 0 (b) 1 (c) 2 (d) 3

(ii) If $g(x)$ is differentiable on [a,b] such that $|g(x)'| \leq k < 1$ and $g(x)$maps [a,b] into itself then $g(x)$ has ----------------fixed point
(a) No (b) Unique (c) many (d) two

(iii)In Gauss Jordan Method, augmented matrix A_b is reduced to
(a) Echelon form (b) Reduced echelon form (c) Traiangular form (d) Diagonal form

(iv)In LU-factorization, we take diagonal element of L equal to I. The method is called

(a) Dolittle's (b) Crouts (c) Cholesky (d) None of these

(v) If $A = [a_{ij}]_{n\times n}$ & λ is a scalar $\neq 0$ & $X = [x_1, x_2, \dots, x_n]^t$ is a non zero vector, then $|A - \lambda I| = 0$ is called
(a) Eigen value (b) Eigen vector (c) Characteristic equation (d) None of these

(vi)In $D = Q^{-1}AQ$, diagonal elements of D are same as
(a) Eigen values of A (b) Eigen vector of A (c) Diagonal elements of A

(d) None of these

(vii) If $\nabla f(x) = f(x) - f(x-h)$, ∇ is called
(a) Forward difference operator (b) Backward difference operator

(c) Shift operator (d) Inverse shift operator

(viii) $\delta f(x) = f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right)$, δ is called
(a) Forward operator (b) backward operator

(c) Central difference operator (d) Average difference operator

(ix)Divided Difference interpolation formula is used for
(a) Equally spaced data (b) Unequally spaced data

(c) Both for equal & un equal intervals (d) None of these

(x) In interpolation, if the estimated value is required near the start of the table, we use ____________

(a) Newton backward difference formula
(b) Newton forward difference formula
(c) Central difference formula
(d) Lagrange's Interpolation Formula

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Seventh Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Numerical Analysis-I — **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-403 — **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE

SHORT QUESTIONS

Q N0. 2:- Solve the following short questions 4 × 5=20

i. Compute 1-norm and ∞-norm of the matrix

$$\begin{bmatrix} 4 & 4 & 5 \\ 0 & 6 & 5 \\ 1 & 3 & 1 \end{bmatrix}$$

ii. Derive the Newton-Gregory Formula for forward interpolation.

iii. Prove that $\Delta^r y_k = \nabla^r y_{k+r}$

iv. Diagonalize the matrix

$$\begin{bmatrix} -1 & 2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

v. Solve the following system of equations by Dolittles's decomposition method

$$4x_1 + x_2 - x_3 = 2$$
$$x_1 + 3x_2 + 5x_3 = 3$$
$$x_1 - x_2 + x_3 = 3$$

LONG QUESTIONS

Solve the following questions 6 × 5=30

Q N0. 3:- Prove that Newton-Raphson method is quadratically convergent.

Q N0. 4:- Solve the following system of equations by Jacobi's method

$$2x_1 + x_2 + 4x_3 = 12$$
$$8x_1 - 3x_2 + 2x_3 = 20$$
$$4x_1 + 11x_2 - x_3 = 33$$

Q N0. 5:- Find the solution of $f(x) = xe^x - 5$ upto 3 decimal places using Bisection method.

Q N0. 6:- Find the value of y at $x = 10$ using Lagrange's Interpolation formula from the table

x	5	6	9	11
y	12	13	14	16

Q N0. 7:- Prove that

$$\Delta^n x^n = n!$$

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Roll No. ....

Seventh Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Mathematical Statistics-I | **TIME ALLOWED: 30 mins.**

Course Code: MATH-404 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

SECTION – I

Q. 1	MCQs (1 Mark each)
(i)	The joint probability of two independent events A and B is (a) $P(A) + P(B)$ (b) $P(A) + P(B) - P(A \cap B)$ (c) $P(A)\, P(B)$ (d) $P(A)\, P(A \backslash B)$
(ii)	In a normal distribution, mean derivative is equal to (a) 1.0 σ (b) 0.8 σ (c) 0.6745 σ (d) 2.0 σ
(iii)	A continuous probability distribution is not represented by (a) a table (b) a mathematical function (c) a graph (d) a density function
(iv)	A random variable is known as (a) chance variable (b) stochastic variable (c) variate (d) all of these
(v)	The probability of continuous random variable X at $x = a$ is (a) between 0 and 1 (b) 1 (c) 0 (d) less than 1
(vi)	The normal distribution will be less spread out when (a) the mean is small (b) the median is small (c) the mode is small (d) the standard deviation is small
(vii)	The middle area under the normal curve with $\mu \pm 2\sigma$ is (a) 0.6827 (b) 1.0000 (c) 0.9545 (d) 0.9973
(viii)	In the standard normal distribution (a) Mean = 2 (b) Mean = - 1 (c) Mean = 0 (d) Mean = 10
(ix)	If X and Y are two independent random variables, then $var(X - Y)$ is equal to (a) $var(X) - var(Y)$ (b) $var(X) + var(Y)$ (c) $var(X) - var(Y) - 2cov(X, Y)$ (d) none of these
(x)	For a negative binomial distribution, mean and variance are related by (a) $\mu = \sigma^2$ (b) $\mu < \sigma^2$ (c) $\mu > \sigma^2$ (d) none of these

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Seventh Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematical Statistics-I
Course Code: MATH-404

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

SECTION – II

Q. 2	SHORT QUESTIONS	
(i)	Write down the Moment generating function of normal distribution and derive its mean and variance.	(4)
(ii)	The continuous random variable x has the probability density function $f(x)$ where $f(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ 0 & otherwise \end{cases}$ Find $P(x \geq 1)$ and the value of k.	(4)
(iii)	Find variance of the Binomial distribution.	(4)
(iv)	Let X_1 and X_2 be two independent random variables having variance k and 2 respectively. If $var(3X_2 - X_1) = 25$, find the value of k.	(4)
(v)	Prove that $E(cx) = cE(x)$, where c is a constant.	(4)

SECTION – III

	LONG QUESTIONS	
Q.3	Let x be a normal random variable with density given by $n(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, \quad -\infty < x < \infty$ Find mean deviation of the distribution	(10)
Q.4	Prove that the mean and variance of hypergeometric distribution are $\mu = \frac{nM}{N}, \quad \sigma^2 = \frac{nM(N-M)(N-n)}{N^2(N-1)}$	(10)
Q.5	State and prove Baye's theorem.	(10)

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Roll No. ....

Seventh Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Ring Theory | **TIME ALLOWED: 30 mins.**

Course Code: MATH-407 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

SECTION-I

Q. 1 **MCQs (1 Mark each)**

(i) The set $Z_3 = \{0,1,2\}$ under addition and multiplication modulo 3 forms

(a) Non commutative Division Ring (b) Field (c) Non commutative Ring (d) None of these

(ii) The maximal ideal ring in the ring Z of integers is

(a) Z (b) $5Z$ (c) $4Z$ (d) $\{0\}$

(iii) Which of the following is not a prime ideal of the ring Z of integers?

(a) $2Z$ (b) $3Z$ (c) $7Z$ (d) $\{0\}$

(iv) Units of $Z(i)$ are

(a) ± 1 (b) $\pm i$ (c) $\pm 1, \pm i$ (d) None of these

(v) Every .... is irreducible in an integral domain.

(a) Integer (b) Prime (c) Real number (d) none of these

(vi) A ring which is commutative with identity element and having no zero divisor is called

(a) Division Ring (b) Integral domain

(c) Prime Ring (d) nilpotent ring

(vii) If R & R' be arbitrary ring $\phi : R \rightarrow R'$ is ring homomorphism such that

$\varphi(a) = 0 \ \forall \ a \in R$ then $Ker\phi =$ --------------

(a) R' (b) $\{0\}$ (c) R (d) None of these

(viii) 12π is algebraic over

(a) Q (b) R (c) Z (d) None of these

(ix) If C is finite extension of R, then $[C : R] =$ ------------

(a) 2 (b) 3 (c) 4 (d) 5

(x) A ring with non zero characteristic is

(a) Z (b) Q (c) Z_3 (d) $Z \times Z$

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Seventh Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Ring Theory **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-407 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Q. 2

SECTION-II

(i) Let R be a ring with identity. Then show that the relation of being associates is an equivalence relation. (4)

(ii) Find all associates of $2+x-3x^2$ in $Z(x)$. (4)

(iii) If R is integral domain then prove that $R[x]$ the polynomial ring over R is integral domain. (4)

(iv) Define the following term: Euclidean Domain, divisor, units, associates, unique factorization domain. (4)

(v) Show that x^3-5 is irreducible polynomia. of $Q(\sqrt{3})$. (4)

SECTION-III

Q.3 Let *R* be an integral domain and *p* be non zero element of *R*. Then prove that *p* is prime in *R* if and only if $\frac{R}{pR}$ is integral domain. (6)

Q.4 Prove that in a unique factorization domain every irreducible element is prime? (6)

Q.5 Show that if $R[x]$ is commutative ring with identity and $f(x), g(x)$ are polynomia $R[x]$ with leading coefficients of $g(x)$, a unit in $R[x]$. Then there exists unique polynomials $q(x), r(x)$ such that $f(x)=q(x)g(x)+r(x)$ where either $r(x)=$ $\deg(r(x))<\deg(g(x))$. (6)

Q.6 Let R be commutative ring with identity prove that an ideal M of R is maximal if and R/M is a field. (6)

Q.7 Show that the Rings Z_{15} and $Z_5 \oplus Z_3$ are isomorphic. (6)

UNIVERSITY OF THE PUNJAB

Roll No. ....

Seventh Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Number Theory-I **TIME ALLOWED: 30 mins.**

Course Code: MATH-408 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION-I

Q. 1	**MCQs (1x10 = 10 Marks) Time: 30 min**
(i)	641 divides a) F_5 b) F_3 c) F_2 d) F_0
(ii)	Two integers a and b are incongruent to each other modulo an integer $m>0$ if m a) divides $a-b$ (b) equal $a-b$ (c) greater than $a-b$ (d) Not divide $a-b$
(iii)	What is the remainder when 5^{48} is divided by 12? a) 10 b) 1 c) 8 d) none
(iv)	The sum of positive divisors of 38 is a) 28 b) 16 c) 60 d) None of (a),(b),(c)
(v)	The number of primitive roots mod 80 are (a) 2 (b) - 1 (c) 1 (d) 0
(vi)	If 15x+7y = 210, then a) x=2, y=5 b) x=7, y=15 c) x=2, y=15 d) x=7, y=16
(vii)	If 2 has exponent 3 mod 7, then 2^6 has exponent (a) 1 (b) 3 (c) 5 (d) 7
(viii)	If $\sigma(n)$ is an odd integer, then n is a (a) Square free (b) Perfect square (c) a Prime (d) perfect number
(ix)	If p is a prime number and d is a factor of p-1 then the number of solutions of the congruence $x^{d-1} \equiv 0 \pmod{p}$ is a) p-1 b) p c) d-1 d) d
(x)	If p is an odd prime then $\tau(p^2) =$ (a) 2 (b) 3 (c) 5 (d) 25

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Seventh Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Number Theory-I | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-408 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q. 2	Short Questions (5x4 = 20 Marks)	
(i)	Let $m>0$ and a,b,c be integers such that $ac \equiv bc \pmod m$, $d=\gcd(c,m)$. Prove that $a \equiv b \pmod{\frac{m}{d}}$.	(4)
(ii)	Find all primitive roots of 49.	(4)
(iii)	State Wilson theorem and apply it to find the residue of 27! modulo 29.	(4)
(iv)	Prove or disprove that $\frac{m!}{a!b!c!}, a+b+c=m$ is an integer.	(4)
(v)	Prove or disprove that $\varphi(m)$ is always an even number.	(4)

Section-III

	Long Questions (6x5 = 30 Marks)	
Q.3	(i) Let $n>1$ be a composite integer then show that there exists a prime p such that $p \mid n$ and $p \le \sqrt{n}$ (ii) If integers $a_1, a_2, \ldots, a_k$ form a Reduced Residue System modulo m then show that $\varphi(m)=k$.	(2+3)
Q.4	State and prove the Chines Remainder theorem. Apply it to find an integer which leaves remainders 1, 2 and 4 when divided by 2, 3 and 5 respectively.	(3+2)
Q.5	State and prove Lagrange Theorem.	(2+3)
Q.6	Define multiplicative arithmetic function. Prove that the number theoretic Mobius function μ is multiplicative.	(2 +3)
Q.7	Let $m>0$ and a be a primitive root modulo m. Prove that $Ind\ a^k \equiv k\ Ind\ a \pmod{\varphi(m)}$.	(5)
Q.8	Prove that there exist no primitive root of mn, where, $m,n>2$ and $\gcd(m,n)=1$.	(5)

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Roll No. ....

Seventh Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Operations Research-I | **TIME ALLOWED: 30 mins.**
Course Code: MATH-412 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION – I

Q1. MCQs (Marks=10)	
(i)	If LP problem has an equality constraint then it can be solved by a) M-technique Method b) Two Phase Method c) both a & b d) none
(ii)	A dual variable corresponding to an equality constraint in primal problem is a) Restricted b) Unrestricted c) zero d) none of these
(iii)	The starting solution for simplex method must be a) optimal but may not be feasible b) feasible but may not be optimal c)neither optimal nor feasible (d) none
(iv)	Assignment model can be solved by a) Regular simplex method b) Northwest corner method c) Hungarian method d)none
(v)	The number of basic variables of transportation model with m rows and n columns is a) m b) $m+n$ c) $m+n$-1 d) $m-n+1$
(vi)	The dual of dual LP problem gives a) dual LP model b) Original LP Problem c) neither original nor dual d) both
(vii)	A transshipment model is a transportation model with supply a_i and demand b_i at each node such that
	a) $a_i = b_i$ b) $a_i = b_i = 1$ c) $a_i = b_i = 0$ d) $a_i \neq b_i$
(viii)	A loop for leaving variable in transportation table is constructed by drawing a) horizontal lines only b) vertical lines only c) only horizontal and vertical lines d) inclined lines
(ix)	The Dijkstra's Algorithm is used to solve a) any LP model b) shortest route problem c) both d) none
(x)	If objective function is parallel to one of the constraint then solution is a) degenerate b) infeasible c) alternate optima exist d) none

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Seventh Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Operations Research-I **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-412 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION – II

Q.2 Short Questions (5 × 4 = 20 Marks)

(i) Solve the following LP model by simplex method

Maximize z = 4x + 3y

Subject to

$$2x + 3y \le 6$$
$$-3x + 2y \le 3$$
$$2x \le 5$$
$$x, y \ge 0$$

(ii) Write a brief note on a) M-technique b) Degeneracy

(iii) The assignment cost of assigning any one worker to any one job is given in the following table. Determine the optimal solution

Job

Workers	1	2	3	4
A	1	4	6	3
B	9	7	10	9
C	4	5	11	7
D	8	7	8	5

(iv) Find the starting basic solution of the following transportation model by least cost method

P.T.O.

10	2	20	11	15
12	7	9	20	25
4	14	16	18	10
5	15	15	15	

Q.3 **Long Questions (10 × 3 = 20 Marks)**

(i) Write the dual of the following primal problem and find the values of dual variables by solving primal LP model

Maximize z = 5x + 12y + 4w

Subject to

$$x + 2y + w \leq 10$$
$$2x - y + 3w = 8$$
$$x, y, w \geq 0$$

(ii) A person requires 10,12, 12 units of chemicals A,B,C respectively. A liquid product contains 5, 2,& 1 units of chemicals A,B,C respectively per jar and a dry product contains 1,2, & 4 units of chemicals A,B, C respectively per carton. If the liquid product costs $3 per jar and the dry product costs $2 per carton.

a) Construct LP model.
b) Provide graphical solution.
c) How many of each should be purchased to minimize the cost and meet requirement.

(iii) Solve the following transportation model by using method of multipliers. Use Vogel approximation for starting basic feasible solution

7	6	4	5	9	40
8	5	6	7	8	30
6	8	9	6	5	20
5	7	7	8	6	10
30	30	15	20	5	

UNIVERSITY OF THE PUNJAB

Seventh Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Theory of Approximation & Splines -I **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-413 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION I

Short Questions

Q2. Answer the following short questions **(4x5=20)**

1. Define Barycentric Coordinates and Convex Coordinates with examples.
2. Define Euclidean Transformation. Show that rotation is distance preserving transformation.
3. Show that $\Delta = e^{hD} - 1$
4. Define data linearization technique of curve fitting. Find the exponential fit $y = Ce^{Ax}$ using data linearization technique.
5. Define polynomial interpolation. Write the Lagrange polynomial of degree three.

SECTION II (Long Questions)

Q3. Prove that Euclidean transformation is an equivalence relation. (8)

Q4. Determine the image of circle $x^2 + y^2 = 16$, under the transformation of the stretching along

a) Along x-axis by factor 2. (4+4)
b) Along y-axis by a factor 3.

Q5. Use the data linearization method and find the exponential fit $y = Ce^{Ax}$ for the five data points $(0, 1.5)$, $(1, 2.5)$, $(2, 3.5)$, $(3, 5.0)$, and $(4, 7.5)$. (7)

Q6. Fit a polynomial of third degree to the following data using Newton's divided difference method (7)

x	0	1	2	4	5	6
f(x)	1	14	15	5	6	9

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Seventh Semester 2017
Examination: B.S. 4 Years Programme

PAPER: Theory of Approximation & Splines -I **TIME ALLOWED: 30 mins.**
Course Code: MATH-413 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Multiple Choice Questions

Q1. Encircle the correct choice of the following questions

1. The composition of two reflections is a
   a) Reflection b) Rotation c) Translation d) Shear

2. .... is the method of finding the value outside the given data points.
   a) Interpolation b) Extrapolation c)Approximation d)Curve fitting

3. $\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta} =$ ....
   a) $\Delta - \nabla$ b) $\Delta + \nabla$ c) $\Delta = \nabla$ d) none of these

4. Every isometry is ....
   a) Into b) Onto c) One-one d) none of these

5. The Lagrange polynomial for the points (0,0), (1,1) is
   a) $2x$ b) x c) $x/2$ d) All of these

6. The operator used in the Gauss's backward interpolation formula is
   a) E b) δ c) ∇ d)All of these

7. The matrix of reflection transformation is
   a) $\begin{bmatrix} \cos 2t & \sin 2t \\ \sin 2t & \cos 2t \end{bmatrix}$ b) $\begin{bmatrix} \cos 2t & \sin 2t \\ \sin 2t & -\cos 2t \end{bmatrix}$ c) $\begin{bmatrix} -\cos 2t & \sin 2t \\ \sin 2t & \cos 2t \end{bmatrix}$ d) $\begin{bmatrix} \cos t & \sin t \\ \sin t & -\cos t \end{bmatrix}$

8. $\Delta y_3 =$ ....
   a) $y_2 - y_3$ b) $y_2 + y_3$ c) $y_4 - y_3$ d) None of these

9. $\mu =$ ....
   a) $E+1$ b) $\Delta + \nabla$ c) $\frac{E^{1/2} + E^{-1/2}}{2}$ d) $\frac{E^{1/2} - E^{-1/2}}{2}$

10. Lagrange interpolation formula for 2 points is
   a) $\left(\frac{x-x_1}{x_0-x_1}\right)y_1 + \left(\frac{x-x_0}{x_1-x_0}\right)y_0$ b) $\left(\frac{x-x_1}{x_0-x_1}\right)y_1 + \left(\frac{x-x_0}{x_1-x_0}\right)y_1$
   c) $\left(\frac{y-y_1}{y_0-y_1}\right)x_1 + \left(\frac{y-y_0}{y_1-y_0}\right)x_0$ d) $\left(\frac{y-y_1}{y_0-y_1}\right)x_0 + \left(\frac{y-y_0}{y_1-y_0}\right)x_1$

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Seventh Semester 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Fluid Mechanics-I **TIME ALLOWED: 2 hr. 30 min**
Course Code: MATH-415 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Section – I (Short Questions)

Q.2 Solve the following Short Questions. **(5 × 4= 20)**

(i) Define the following: Dynamic viscosity, Surface forces, Circulation, Uniform flows.

(ii) Discuss the equivalence of Equation of Continuity for Lagrangian and Eulerian specification.

(iii) State and prove the Kelvin's minimum energy theorem.

(iv) Show that the equipotential lines and the streamlines are orthogonal to each other .

(v) A flat plate having dimensions of $2m \times 2m$ slides down an inclined plane at an angle of one radian to the horizontal at a speed of 6 *m/s*. The inclined plane is lubricated by a thin film of oil having a viscosity of 30×10^{-3} *Pa.s*. The plate has a uniform thickness of 20 *mm* and a density of 40,000 kg/m^3. Determine the thickness of lubricating oil film.

Section – II (Long Questions) (3 × 10 = 30)

Q.3 The velocity components of a two dimensional fluid flow are given by

$$u = 3x + y, \qquad v = 2x - 3y.$$

Calculate the circulation around the circle $(x-1)^2 + (y-6)^2 = 4$.

Q.4 If every particle of fluid moves on the surface of a sphere, prove that the equation of continuity is $\frac{\partial \rho}{\partial t} \cos\theta' + \frac{\partial(\rho\omega' \cos\theta')}{\partial \theta'} + \frac{\partial(\rho\omega \cos\theta')}{\partial \varphi} = 0$, ρ being the density, θ', φ the latitude and longitude of any element, and ω', ω the angular velocities of the element in latitude and longitude respectively.

Q.5 Find the Cartesian equation of the streamlines when the fluid is streaming from three equal sources situated at the corners of an equilateral triangle.

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Seventh Semester 2017

Examination: B.S. 4 Years Programme

PAPER: Fluid Mechanics-I | **TIME ALLOWED: 30 mins.**

Course Code: MATH-415 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION – I (Objective)

Q.1 Tick the correct option. (1×10 = 10)

(i) Euler's equation of motion refers to conservation of ______________.

(a) Momentum (b) Mass (c) Newton's law (d) Force

(ii) The reciprocal of density is known as specific ______________.

(a) Gravity (b) Weight (c) Mass (d) Volume

(iii) Pascal-second is the unit of ______________ viscosity.

(a) Kinematic (b) Dynamic (c) Both (a) & (b) (d) None of these

(iv) $\frac{dx}{dt} = u, \frac{dy}{dt} = v, \frac{dz}{dt} = w$ represents the differential equation for the ______________ lines.

(a) Stream (b) Streak (c) Path (d) All of these

(v) The velocity potential function and the stream function are ______________ functions.

(a) Continuous (b) Orthogonal (c) Conjugate (d) All of these

(vi) For describing the motion in fluid mechanics, the ______________ method is commonly used.

(a) Eulerian (b) Lagrangian (c) Newtonian (d) Archimedes

(vii) In material derivative $\frac{DH}{Dt} = \frac{\partial H}{\partial t} + \vec{V} \cdot \nabla H$, the term $\frac{\partial H}{\partial t}$ is known as ______________ rate of change.

(a) Local (b) Convective (c) Stokes (d) Substantial

(viii) The Vorticity vector is ______________.

(a) Rotational (b) Irrotational (c) Orthogonal (d) Solenoidal

(ix) The ______________ of a two dimensional source is defined to be the volume of fluid which emits from it in unit time.

(a) Mass (b) Specific volume (c) Strength (d) Velocity

(x) A ______________ represents the type of flow in which the fluid particles move in circular paths about a central point.

(a) Doublet (b) Source (c) Sink (d) Vortex

UNIVERSITY OF THE PUNJAB

Roll No. ....

Eighth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Measure Theory and Lebesgue Integration **TIME ALLOWED: 30 mins.**

Course Code: MATH-416 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION-I

Q. 1 **MCQs (1 Mark each)**

(i) Let $f:\mathbb{R}\to\{1,2\}$ be defined by $f(x)=\begin{cases}1, x\in\mathbb{Q}'\\2, x\in\mathbb{Q}\end{cases}$ then f is ----------

(a) a constant function (b) a step function (c) $f=1$ *a.e.* (d) $f=2$ *a.e.*

(ii) The limit superior of the sequence $\{1+(-1)^n\}$ is ----------

(a) 0 (b) 2 (c) ∞ (d) None of these

(iii) A set G is said to be G_δ set if it is the countable ---------- sets.

(a) intersection of open (b) intersection of closed

(c) union of open (d) union of closed

(iv) The Lebesgue outer measure of the set $\mathbb{N}$ of natural numbers is ----------

(a) 0 (b) 1 (c) 2 (d) None of these

(v) Let $X=\{a,b,c\}, \Im=\{\varphi,\{a\},\{b\},\{a,b\},X\}$ then $\Im$ is ----------

(a) not a topology (b) both "topology and σ – algebra"

(c) not a topology and not σ – algebra (d) a topology but not σ – algebra

(vi) Let f be a function defined by $f(x)=-1, \forall x\in\mathbb{R}$, then $\{x:|f(x)|>1\}==$ ----------

(a) $\mathbb{R}$ (b) ϕ (c) $\{1\}$ (d) None of these

(vii) Let $T=\bigcup_{k=1}^{\infty}\left(\frac{1}{2^k},\frac{1}{2^{k-1}}\right)$ then $m^*(T)=$---------

(a) $\mathbb{R}$ (b) 1 (c) ∞ (d) None of these

(viii) Let $A=[3,5]\cup[-4,-2]$ then $m^*(A)=$ ---------

(a) 4 (b) 10 (c) 0 (d) None of these

(ix) The cantor set C is ---------

(a) uncountable with measure zero (b) non-measurable

(c) countable with measure zero (d) None of these

(x) Let $f_n(x)=\frac{1}{\left(1+\frac{x}{n}\right)^n}, x\in[0,1]$ then the value of Lebesgue integration of

$\int_0^1 \lim_n f_n =$ ---------

(a) $\frac{e+1}{e}$ (b) $\frac{e}{e-1}$ (c) $\frac{e-1}{e}$ (d) None of these

UNIVERSITY OF THE PUNJAB

Eighth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Measure Theory and Lebesgue Integration **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-416 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q. 2 **SHORT QUESTIONS**

(i) Define G_δ-set. Given any $A \subseteq \mathbb{R}$ and $\varepsilon > 0$, then there is a G_δ-set G with $A \subseteq G$. Show that $m^*(A) = m^*(G)$. (4)

(ii) Find the Lebesgue outer measure μ^* of the set $A = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$. (4)

(iii) Let Ω be any uncountable set and $\Delta = \{A \subset \Omega : A \text{ is countable or } A' \text{is countable}\}$. Determine whether Δ is $\sigma-$ algebra. (4)

(iv) Let f be a non-negative measurable function on E. Then $\int_E f = 0 \Leftrightarrow f = 0$ $a.e.$ on E. (4)

(v) If $f_n \to f$ in measure and $g_n \to g$ in measure, then prove that $f_n + g_n \to f + g$ in measure. (4)

SECTION-III

Q.3 Show that the Lebesgue outer measure of an interval is its length. (6)

Q.4 Define Cantor set and show that Cantor set C is uncountable. (6)

Q.5 Define characteristic function. Show that if $A, B \subseteq X$, then (i) $\chi_{A\cap B} = \chi_A \cdot \chi_B$ (6)

(ii) $\chi_{A\cup B} = \chi_A + \chi_B - \chi_A \cdot \chi_B$ (iii) $\chi_{A'} = 1 - \chi_A$.

Q.6 Show that the function $f(x) = \begin{cases} 1/x, 0 < x \le 1 \\ 0, x = 0 \end{cases}$ is measurable. (6)

Q.7 State and prove bounded convergence theorem. (6)

UNIVERSITY OF THE PUNJAB

Roll No. ....

Eighth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Methods of Mathematical Physics | **TIME ALLOWED: 30 mins.**

Course Code: MATH-417 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Multiple Choice Questions

1. Laplace transform of $\sin(2t) + \cos(2t)$ is
   (a) $\frac{s+2}{s^2+4}$ (b) $\frac{s+2}{s^2-4}$ (c) $\frac{s-2}{s^2+4}$ (d) $\frac{s+4}{s^2+4}$.

2. The Green's function $G(x,t)$ for the non-homogeneous differential equation $L(y(x)) = f(x)$ is given by the formula
   (a) $\sum_n \frac{y_n(x)y_n(t)}{\lambda_n}$ (b) $\sum_n \frac{y_n(t)}{\lambda_n y_n(x)}$ (c) $\sum_n \frac{y_n(t)}{\lambda_n y_n(x)}$ (d) $\sum_n \lambda_n y_n(x) y_n(t)$

3. Let $H(t)$ be a unit step function then $\mathcal{L}\{H(t-5)\sin(t-5)\}$ is
   (a) $\frac{s-5}{(s-5)^2+1}$ (b) $\frac{1}{(s-5)^2+1}$ (c) $\frac{e^{-5s}}{(s-5)^2+1}$ (d) $\frac{e^{-5s}}{s^2+1}$

4. If $F(s)$ and $G(s)$ are inverse Laplace transforms of the functions $f(t)$ and $g(t)$, respectively, then $\mathcal{L}^{-1}\{F(s)G(s)\}$
   (a) $\int_0^t f(\tau)g(t-\tau)\,d\tau$ (b) $f(t)g(t-\tau)$ (c) $f(g(t))$ (d) all of these

5. The inverse Laplace transform of s^2 is
   (a) $\frac{2}{t^3}$ (b) $\frac{1}{s}$ (c) $\frac{t}{t+1}$ (d) none of these

6. The inverse Laplace transform $\mathcal{L}^{-1}\{F'(s)\} =$
   (a) $t^2 f(t)$ (b) $-t^2 f(t)$ (c) $tf(t)$ (d) $-tf(t)$

7. The problem of finding a curve of minimum distance between two points on a given surface is called
   (a) Brachistochrone problem (b) Dido's problem (c) Geodesic problem (d) Plateau's problem

8. Let $F(k)$ be Fourier transform of an odd and real valued function $f(x)$. Then
   (a) $F(k)$ is real (b) $F(k)$ is pure imaginary (c) $F(k)$ is even (d) $F(k)$ is odd

9. The Fourier transform $\mathcal{F}\{f(x-a)\}$ is equal to
   (a) $e^{ika}\mathcal{F}\{f(x)\}$ (b) $e^{-ika}\mathcal{F}\{f(x)\}$ (c) $e^{-ka}\mathcal{F}\{f(x)\}$ (d) $e^{ka}\mathcal{F}\{f(x)\}$

10. The curve $y = f(x)$ along which a functional $J[y]$ takes the stationary value is called the
    (a) closed curve (b) concave curve (c) extremal (d) none of these

UNIVERSITY OF THE PUNJAB

Eighth Semester - 2017
Examination: B.S. 4 Years Programme

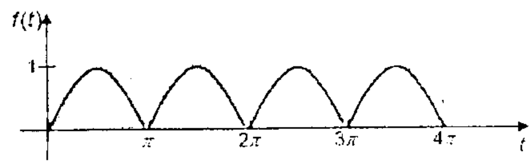
Roll No. ....

PAPER: Methods of Mathematical Physics **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-417 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Short Questions $5 \times 4 = 20$

1. Find the Laplace transform of the periodic function as shown in the figure below



2. Find the solution of the following algebraic equation

$$x^2 + \epsilon x - 1 = 0$$

up to second order expansion in ϵ.

3. Using convolution theorem, find inverse laplace transform of $\frac{3}{s^2(s^2+9)}$.

4. Show that the Fourier transform of an even and real function is real.

5. From among the curves connecting the points $A(1,3)$ and $B(2,5)$, find the extremal curve of the functional

$$I[y] = \int_1^2 y'(x)\left(1 + x^2 y'(x)\right) dx$$

Subjective Questions $3 \times 10 = 30$

1. Show that the shortest distance curve, joining two points on the surface of a sphere, is an arc of a great circle.

2. Solve the following partial differential equation using Fourier transform method:

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial u(x,t)}{\partial t}; \qquad t \geq 0, -\infty < x < \infty$$

subject to the conditions:

$$u(x,0) = e^{-x^2},$$
$$\lim_{x \to \pm\infty} u(x,t) = 0; \quad \lim_{x \to \pm\infty} u_x(x,t) = 0.$$

3. Construct Green's function for the B.V.P.

$$xu''(x) + u'(x) - \frac{n^2}{x} u(x) + \lambda r(x) u(x) = 0,$$
$$u(0) \text{ is finite}; \qquad u(1) = 0.$$

UNIVERSITY OF THE PUNJAB

Roll No. ....

Eighth Semester - 2017
Examination: B.S. 4 Years Programme

PAPER: Numerical Analysis-II | **TIME ALLOWED: 30 mins.**
Course Code: MATH-418 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Note: Attempt all questions.

(Objective)

Q.1 Encircle the correct answer. **10×1=10**

1. If the data is not equally spaced, then to find the derivative, we use ________
   (a) Newton Greogry forward formula
   (b) Newton Greogry backward formula
   (c) Central difference formula
   (d) Lagrange's Formula
2. The formula
   $Df(a)=\frac{1}{h}[\Delta-\frac{\Delta^2}{2}+\frac{\Delta^3}{3}-\frac{\Delta^4}{4}...]f(a)$ is used for obtaining_________
   (a) Second derivative
   (b) Third derivative
   (c) First derivative
   (d) None of these
3. The order of the difference equation $\Delta^3 y_k+\Delta^2 y_k+\Delta y_k+y_k=0$ is _______.
   (a) 0
   (b) 1
   (c) 2
   (d) 3
4. The degree of difference equation $y_{k+3}-9y_{k+2}+9y_{k+1}+y_k=3x+2$ is _______.
   (a) 0
   (b) 1
   (c) 2
   (d) 3
5. The difference equation $y_{k+n}+a_1 y_{k+n-1}+a_2 y_{k+n-2}+...+a_{n-1}y_{k+1}+a_n y_k=0$ is _______.
   (a) Homogeneous
   (b) Non-Homogeneous
   (c) Both (a) & (b)
   (d) None of these

P.T.O.

6. The forrmula for numerical integration $\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})]$ is known as

(a) Rectangular rule
(b) Trapezoidal rule
(c) Simpson's 1/3 rule
(d) Simpson's 3/8 rule

7. The formula for numerical integration

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{3}[(y_0 + y_n) + 2(y_2 + y_4 + ... + y_{2n-2}) + 4(y_1 + y_3 + ... + y_{n-1})]$$

is known as

(a) Rectangular rule
(b) Trapezoidal rule
(c) Simpson's 1/3 rule
(d) Simpson's 3/8 rule

8. $\int_a^b f(x)dx = \frac{b-a}{2}\left[f\left(\frac{1}{\sqrt{3}}\right) + f\left(-\frac{1}{\sqrt{3}}\right)\right]$ is known as Gauss quadrature formula for -------

(a) One point
(b) Two points
(c) Three points
(d) Four points

9. The formula $y_{k+1} = y_k + hf(x_k, y_k)$ to solve differential equations is called ---------

(a) Heun's Method
(b) Taylor Series Method
(c) Euler's Method
(d) Runge-Kutta Method

10. The formula $y_{k+1} = y_k + \frac{h}{2}[f(x_k, y_k) + f(x_{k+1}, y_{k+1})$ is known as ----------

(a) Euler's Method
(b) Heun's Method
(c) Taylor Series Method
(d) Runge-Kutta Method

UNIVERSITY OF THE PUNJAB

Eighth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Numerical Analysis-II — **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-418 — **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

(Subjective)

Q2: Answer the following short questions. $4\times 5 = 20$

1. Solve the difference equation $y_{k+2} - 6y_{k+1} + 8y_k = 0$.
2. Find first two derivatives of $f(x)$ at $x = 3.5$ using Gauss's forward formula from the table

x	1	2	3	4	5	6
y	3	11	31	69	131	223

3. Use trapezoidal rule to evaluate $\int_0^1 \frac{1}{1+x^2}dx$ with $h = 0.2$.
4. Solve the following differential equation
   $\frac{dy}{dx} = x + y^2; y(1) = 0$ at $x = 1.1, 1.2$.
   by Euler's method.
5. Solve the difference equation $y_{k+2} + \frac{1}{4}y_k = 0$

Long Questions $6\times 5 = 30$

Q3: Find the polynomial passing through points (-4,1245), (-1,33), (0,5), (2,5) and (5, 1335) using Newton divided difference formula. Also find first derivative at x = -1

Q4:Solve the difference equation $y_{k+2} - 6y_{k+1} + 7y_k = 3^k$.

Q5: Using Gauss Quadrature formula for two points, evaluate $\int_0^1 \frac{\sin x}{x}dx$.

Q6: Solve $\frac{dy}{dx} = x^2 + y^2; y(0) = 1, h = 0.05$. Find $y(0.1)$ by Taylor's series method of order 2.

Q7: Use Runge-Kutta method of order four to solve the differential equation
$\frac{dy}{dx} = 1 + y^2; y(0) = 0, h = 0.2$ for $y(0.2)$ and $y(0.4)$.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Eighth Semester - 2017
Examination: B.S. 4 Years Programme

PAPER: Mathematical Statistics-II | **TIME ALLOWED: 30 mins.**
Course Code: MATH-419 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION – I

Q. 1	MCQs (1 Mark each)
(i)	If the correlation coefficient $\gamma = 0.7$ Then proportion of variation for Y explained by X is (a) 0.49 (b) 0.50 (c) 0.70 (d) $\sqrt{0.70}$
(ii)	If $F \sim F(v_1, v_2)$, then mode of F is (a) $\frac{v_2(v_1-2)}{v_1(v_2+2)}$ (b) $\frac{v_2(v_1+2)}{v_1(v_2-2)}$ (c) $\frac{v}{v-2}$ (d) None of these
(iii)	If unexplained variation between variables X and Y is 0.40 then γ^2 is (a) 0.75 (b) 0.60 (c) 0.40 (d) None of these
(iv)	The mode of chi-square distribution is (a) $V+1$ (b) $V-1$ (c) $\frac{V}{V+1}$ (d) $V-2$
(v)	The strength of linear relationship between two random variables Y and X is measured by (a) γ^2 (b) X (c) γ (d) None of these
(vi)	Which one of the following relations holds? (a) $r_{13.2} = \sqrt{b_{12.3} \times b_{21.3}}$ (b) $r_{13.2} = \sqrt{b_{13.2} \times b_{31.2}}$ (c) $r_{13.2} = \sqrt{b_{23.1} \times b_{32.1}}$ (d) All of these
(vii)	The least square regression line always passes through the point; (a) $(\overline{X}, \overline{Y})$ (b) $(\overline{X}, Y)$ (c) $(X, \overline{Y})$ (d) (X, Y)
(viii)	All odd order moments of chi-square distribution are; (a) Positive (b) Equal (c) 0 (d) Negative
(ix)	The method of least squares minimizes sum of squares of; (a) Units (b) Errors (c) Constants (d) Regressors
(x)	The parameters of the regression model are estimated by; (a) Method of least squares (b) Matrix rules (c) Integration (d) None of these

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Eighth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematical Statistics-II | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-419 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION – II

Q. 2	SHORT QUESTIONS	
(i)	If $\sum(Y-\bar{Y})^2 = 300.8$ and $\sum(\hat{Y}-\bar{Y})^2 = 162.7431$, then find the coefficient of Multiple Determination.	(4)
(ii)	Define correlation coefficient. Given $r_{xy} = -0.67$. Also given that $\mu = \frac{x-10}{5}$ and $v = \frac{y-15}{10}$. Determine $r_{\mu v}$.	(4)
(iii)	Write down the four properties of least square regression line.	(4)
(iv)	Show that coefficient of correlation is independent of change of origin and scale.	(4)
(v)	(a) State the central limit theorem. (b) Write down the assumption for *t*-distribution.	(4)

SECTION – III

	LONG QUESTIONS	
Q.3	If X_r and X_s are the rth' and sth' random variable of random sample of size *n* drawn from the finite population $\{C_1, C_2, ...C_N\}$. Then $Cov(X_r, X_s) = \frac{\sigma^2}{N-1}$	(10)
Q.4	Verify that the Chi-square(χ^2) distribution has the following density function $f(\chi^2) = \frac{1}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})}(\chi^2)^{\frac{n}{2}-1} e^{-\frac{\chi^2}{2}}, \quad 0 < \chi^2 < \infty$	(10)
Q.5	Prove that for a t-distribution with '*n*' degrees of freedom $\mu'_{2r} = \frac{n(2r-1)}{(n-2r)}\mu'_{2r-2}$ Where μ' represents moment about origin.	(10)

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Eighth Semester - 2017
Examination: B.S. 4 Years Programme

PAPER: Theory of Modules | **TIME ALLOWED: 30 mins.**
Course Code: MATH-423 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION-I

Q. 1 **MCQs (1 Mark each)**

(i) A root of a polynomial equations over the field of rational numbers is called

(a) Integer (b) Algebraic Number

(c) Rational Integer (d) Algebraic Integer

(ii) Every R- module is isomorphic to a ------------------ of a free R-module

(a) Direct summand (b) quotient module

(c)Isomorphism (d) equivalent

(iii) Every FG R-module is homomorphic image of its ....

(a) sub-module (b) FG Sub-module (c) Free R-module (d) Free sub-module

(iv) According to Dedikind Module law ....

(a) $(A\cup B)+C=(A\cap B)+C$ (b) $A\cup(B+C)=(A+B)\cup C$

(c) $A+(B\cup C)=(A+B)\cup C$ (d) $A+(B\cap C)=(A+B)\cap C$

(v) Degree of zero polynomial is

(a) 1 (b) 0 (c) Not defined (d) 2

(vi) If $x\neq 0$, $y\neq 0$ are elements of a ring R such that $xy=0$. Then x and y are called

(a) Multiplicative inverse (b) Zero Divisor

(c) Additive Inverse (d) Identity

(vii) The degree of the polynomial $3+6x+75x^2+2x^3+4x^5$ is

(a) 1 (b) 5 (c) 3 (d) 4

(viii) A root is polynomial equations over the field of rational numbers is called

(a) Integer (b) Algebraic Number

(c) Rational Integer (d) Algebraic Integer

(ix) A module homomorphism f is injective if and only if

(a) $im f=\{0\}$ (b) $im f=\ker f$

(c) $\ker f=0$ (d) None of These

(x) If *K* and *L* are sub-modules of an R-module *M*, then ----------

(a) $(K+L)/K\cong L/(L\cap K)$ (b) $(K-L)/K\cong L/(L\cap K)$

(c) $(K+L)/K\cong L/(L\cup K)$ (d) $(KL)/K\cong L/(L\cap K)$

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Eighth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Theory of Modules | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-423 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Q. 2

SECTION-II

(i) If M is an irreducible R-modole prove that either M is cyclic or that for every $m \in M$ and $r \in R$, $rm = 0$ (4)

(ii) Let T be a module homomorphism and $K(T) = \{x \in M : Tx = 0\}$ then show that K is an isomorphism iff $K(T) = 0$ (4)

(iii) Let R be a Euclidean ring then show that any finitely generated R- module M is the direct sum of a finite number of cyclic submodule. (4)

(iv) It T is a homorphism of M on to N with $K(T) = A$, Prove that N is isomorphic to M/A. (4)

(v) Show that every vector space over a field F is torsion free. (4)

SECTION-III

Q.3 Show that an irreducible right R-module is cyclic. (6)

Q.4 If A and B are submodule of a module C, then prove that A+B is a sub-module of C. (6)

Q.5 Prove that a module M satisfies the ascending chain condition for submodule if and only if every submodule of M is finitely generated (6)

Q.6 Show that there exists a free R-module on any set S. (6)

Q.7 Let A, B and C be submodules of an R-Module M and $A \subseteq B$, then show that (6)

$$A \cap (B + C) = B + (A \cap C)$$

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Eighth Semester - 2017
Examination: B.S. 4 Years Programme

PAPER: Number Theory-II | **TIME ALLOWED: 30 mins.**
Course Code: MATH-424 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION-I

Q. 1	MCQs (Marks =10)
(i)	Product of a quadratic non-residues and a quadratic residue of a prime number is a a) Quadratic residue b) Quadratic non-residue c) Both(a&b) d) neither
(ii)	The Prime number 997 has ___________ Quadratic residues a) 997 b)450 c)996 d)449
(iii)	$\left(\frac{7}{43}\right) =$ a) -1 b) 0 c) 1 d) neither
(iv)	$\left(\frac{60}{23}\right) =$ a) -1 b) 0 c) 1 d) neither
(v)	Exactly one of the numbers below is a prime number. Which one is it? a) 511 b) 517 c) 519 d) 521
(vi)	The number of solutions of the Diophantine equation $x^3 + y^3 = z^3$ a) 6 b) 3 c) 1 d) 0
(vii)	The product of two primitive polynomials is a) Reduced Polynomial b)Non-primitive c)Primitive d) Neither
(viii)	The vectors (1,2,3), (2,3,4), and (0,0,0) are always a) Independent b) Dependent c) Both (a&b) d) neither
(ix)	The zero vector space has __________ as a basis a) no b) Empty set c) Infinite set d) {0}
(x)	Degree of the polynomial $ax^2 + bx + c = 0$, $a, b, c \in F$ where F is a number field, is? a) 2 b) at least 2 c)0 d) at most 2

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Eighth Semester - 2017
Examination: B.S. 4 Years Programme

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PAPER: Number Theory-II — **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-424 — **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q. 2	Short Questions (4x5 = 20 Marks)
(i)	Apply Quadratic Resciprocity law to evaluate $\left(\frac{701}{997}\right)$.
(ii)	Prove that Product of two quadratic residues of a prime number is again a quadratic residue.
(iii)	Prove F_5 is composite, where F_5 is a Fermat's number.
(iv)	Prove or disprove that the set of algebraic numbers is countable.
(v)	Define negative least residue and Jacobi numbers with examples.

Section-III

	Long Questions (6x5 = 30 Marks)
Q.3	Let p be an odd prime and a any integer co-prime to p. If m denote the number of least positive integers in the set $\{a, 2a, \ldots, \frac{p-1}{2}a\}$ that exceed $\frac{p}{2}$. Then $\left(\frac{a}{p}\right) = (-1)^m$.
Q.4	Prove that if θ is algebraic over R then every element $\alpha \in R(\theta)$ is algebraic over R. Explain why $11^{\sqrt{11}}$ not an algebraic number is?
Q.5	Prove the existence of the transcendental numbers.
Q.6	If K is finite extension over F, E over K, then show that E is finite extension over F.
Q.7	State and Prove Gauss lemma for primitive polynomials.

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Eighth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Operations Research-II | **TIME ALLOWED: 30 mins.**
Course Code: MATH-428 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION – I

Q1. MCQs (Marks=10)	
(i)	A connected graph containing no cycles is called a) loop b) tree c) forest d) none
(ii)	Floyd's algorithm is used to find a) Shortest route b) spanning tree c) longest route d) none
(iii)	The z-coefficient of non-basic in revised simplex method are calculated as a) $C_B B^{-1} P_j - c_j$ b) $C_B B^{-1} b$ c) $B^{-1} P_j - c_j$ (d) none
(iv)	In the bounded variable algorithm the basic variable θ_2 is computed as a) $\min_i \left\{ \frac{(B^{-1}b)_i}{(B^{-1}P_j)_i} \mid B^{-1}P_j < 0 \right\}$ b) $\min_i \left\{ \frac{(B^{-1}b)_i - (U_B)_i}{(B^{-1}P_j)_i} \mid B^{-1}P_j > 0 \right\}$ c) $\min_i \left\{ \frac{(B^{-1}b)_i - (U_B)_i}{(B^{-1}P_j)_i} \mid B^{-1}P_j < 0 \right\}$
(v)	In parametric linear programming the point t_1 for which the solution at $t = 0$ remains optimal and feasible for the interval $0 \le t \le t_1$ is called a) Starting point b)critical point c) end point d) zero
(vi)	In branch and bound method, the subproblem LP_i is said to be ___________ if LP_i may not yield any better solution and no further branching is required. a) optimal b) feasible c) fathomed d) none
(vii)	The ___________ can be used as source row in fractional cut algorithm. a) Objective row only b) *constraint row* only c) any row
(viii)	In _____________ the original LP problem is decomposed into different stages a) Dynamic programming b) fractional cut algorithm c) revised simplex d) none
(ix)	In revised simple LP model is represented by a) Simplex tableau b) matrix form c) none
(x)	If objective function is parallel to one of the constraint then solution is a) degenerate b) infeasible c) alternate optima exist d) none

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Eighth Semester - 2017
Examination: B.S. 4 Years Programme

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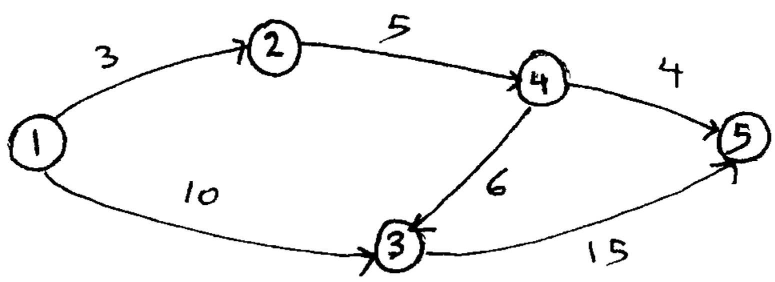
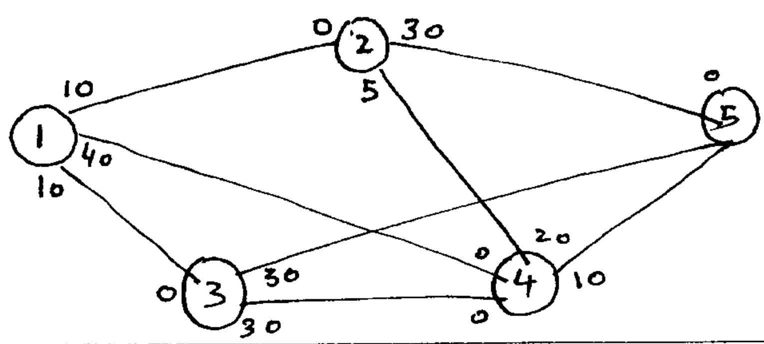
PAPER: Operations Research-II **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-428 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION – II

Q.2	**Short Questions (5 × 4 = 20 Marks)**
(i)	Write the algorithm of revised simplex method.
(ii)	Write a brief note on mixed integer programming
(iii)	Find the shortest route between nodes 1 & 5 by using Floyd's Algorithm. 
(iv)	Find maximum flow for the following network. 

P.T.O.

Q.3	**Long Questions (10 × 3 = 30 Marks)**
(i)	Solve by using revised simplex method Maximize z = 6x - 2y + 3w Subject to $2x - y + 2w \le 2$ $x + 4w \le 4$ $x, y, w \ge 0$
(ii)	Solve the following bounded variable problem. Maximize $z = 3x_1 + 5x_2 + 3x_3$ Subject to $x_1 + 2x_2 + 2x_3 \le 14$ $2x_1 + 4x_2 + 3x_3 \le 23$ $0 \le x_1 \le 4, 0 \le x_2 \le 5, 0 \le x_3 \le 3$
(iii)	Solve the following by using Integer Linear Programming. Maximize $z = 4x_1 + 6x_2 + 2x_3$ Subject to $4x_1 - 4x_2 \le 5$ $-x_1 + 6x_2 \le 5$ $-x_1 + x_2 + x_3 \le 5$ $x_1, x_2, x_3 \ge 0$ & *inegers*

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Eighth Semester - 2017

Examination: B.S. 4 Years Programme

PAPER: Theory of Approximation and Splines-II **TIME ALLOWED: 30 mins.**
Course Code: MATH-429 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE

Q1. Fill in the following blanks (1x10=10)

1. Bernstein Bezier form is ....of cubic Hermite form.
   a) Particular case b) Approximate form c) Both d) None of these
2. Control polygon is related to ....
   a) Control point form b) Bernstein Bezier form c) Cubic Hermite form
   d) Both a) and b)
3. Symbolic representation of Bernstein Bezier form is ....
   a) $f(\theta)=[\theta E+(1-\theta)I]^n b_0$ b) $f(\theta)=[\theta E+(1-\theta)I]^n b_i$
   c) $f(\theta)=[\theta' E+(1-\theta)I]b_1$ d) $f(\theta)=[\theta E+(1+\theta)I]^n b_0$
4. The positive difference of a polynomial of *nth* degree is ....
   a) Zero b) constant c) polynomial of degree *n-1* d) polynomial of degree 1
5. In general $\sum_{i\in Z} N_i^k(t)=$.....
   a) Positive b) 0 c) 1 d) negative
6. In control point form if we put *k=3*, then we get ....
   a) Ball form b) Bernstein Bezier form c) Timer form
   d) Normal form
7. In plus function magnitude of jump discontinuity at x_i is.......................
   a) a! b) 0 c) 1 d) n!
8. Variation diminishing property satisfied by...................
   a) Cubic Hermite form b) Bernstein Bezier form c) Polar form
   d) None of these
9. Bernstein Bezier polynomial of degree *n* is...............................
   a) $B_i^n(\theta)=\binom{n}{i}(\theta-1)^{n-i}\theta^{i-1}$ b) $B_i^n(\theta)=\binom{n}{i}(1-\theta)^{n-i}\theta^{i}$
   c) $B_i^n(\theta)=\binom{n}{i}(\theta-1)^{n+i}\theta^{i-1}$ d) $B_i^n(\theta)=\binom{n}{i}(\theta-1)^{n-i}\theta^{i+1}$
10. In $N_i^k(t)=\int_{t-1}^{t} N_i^{k-1}(\hat{t})\,d\hat{t}$ degree of spline is.....................
    a) k b) k+1 c) k-1 d) k-2

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Eighth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Theory of Approximation and Splines-II **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-429 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE

Q2. Solve the following short questions (4x5=20)

1. Determine $N_{-5}^{3}(t)$ using Basic Principal.

2. Determine a function $P:[0,2]\to R$ such that $P(0)=2,\ P(1)=3,\ P(2)=2$, and P is linear over $[0,1[$ and quadratic over $[1,2]$

3. Write the matrix form of Bernstein Bezier cubic form.

4. Find new control points for B.B cubic form from B.B quadratic form.

Q3. Solve the following **Long Questions.**

1. Determine whether the following function is a spline? Determine the degree if it is spline.

$$f(t)=\begin{cases}3t^2+2, & t\in[-2,-1[\\ -6t-1, & t\in[-1,1[\\ 6t^2-18t+5, & t\in[1,2[\\ 3t^2-6t-7, & t\in[2,3[\end{cases}$$ (08)

Can *f(t)* be expressed as truncated power function representation? If Yes then determine, *f(t)* in truncated power function representation.

2. Let S be a cubic spline on [-1,2[with knots at the points -1, 1, 2. Find S so that $S(-1)=-1,\ S(1)=11,\ S(2)=29,\ S'(-1)=5,\ S'(2)=-7$.

(Construct a system of equations only) (07)

3. Consider the following cubic Hermiteinterpolatory function

$$S(t)=(1-\theta)^2\ (1+2\theta)f_i+(1-\theta)^2\theta\, h_i\, d_i+\theta^2(3-2\theta)f_{i+1}-\theta^2(1-\theta)h_{i+1}\,d_{i+1},$$
$$t_i\le t\le t_{i+1},\ i=0,1,2,3,...,n-1$$ (07)

Apply second derivative continuity at knots and derive tridiagonal system of *(n-1)* equations in *(n+1)* unknowns $d_i s$.

4. Determine clamped cubic Hermite spline that passes through $(0,0),\ (1,0.5),\ (2,2),\ (3,1.5)$ with the boundary conditions $S'(0)=0.4\ ,\ S'(3)=-3$. (08)

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Eighth Semester - 2017
Examination: B.S. 4 Years Programme

PAPER: Fluid Mechanics-II | **TIME ALLOWED: 30 mins.**
Course Code: MATH-431 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION – I

Q. 1	MCQs (1 Mark each)
(i)	The dimension of Reynolds number is given by a) m/s b) Kg c) m/g d) None of these
(ii)	The equation of continuity for two dimensional flow is a) $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ b) $\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}=0$ c) $\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}=0$ d) $\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=0$ where u and v are components of velocity.
(iii)	The profile of aero-plane wing which lifts it up is called a) wing shaped b) curved profile c) aero foil profile d) none of these
(iv)	Turbulent flow usually occurs at speeds a) low b) high c) very high d) sometimes high or low
(v)	The Blasius theorem is applied on the flows which are a) Incompressible b) Irrotational c) both (a) and (b) d) inviscid

P.T.O.

(vi)	For an incompressible fluid, continuity equation is		
		a) $\nabla . \boldsymbol{u} = 0$	b) $\nabla \times \boldsymbol{u} = \boldsymbol{0}$
		c) $\Delta . \boldsymbol{u} = 0$	d) none of these
(vii)	Drag force is given by		
		a) Newton's law	b) Pascal's law
		c) Gauss's law	d) Stokes law
(viii)	Laminar flow usually occurs at speeds		
		a) low	b) high
		c) very high	d) sometimes high or low
(ix)	Flow past through a circular cylinder is an example of		
		a) Laminar flow	b) Turbulent flow
		c) Internal flow	d) External flow
(x)	For an Irrotational fluid,		
		a) $\nabla . \boldsymbol{u} = 0$	b) $\nabla \times \boldsymbol{u} = \boldsymbol{0}$
		c) $\Delta . \boldsymbol{u} = 0$	d) none of these

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Eighth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Fluid Mechanics-II
Course Code: MATH-431

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SECTION – II

Q. 2	SHORT QUESTIONS	
(i)	What is Reynolds number? Explain the principle of dynamical similarity.	(3+3)
(ii)	State and prove Blasius theorem.	(2+5)
(iii)	Derive the Navier Stokes equations for an incompressible viscous fluid.	(7)

SECTION – III

	LONG QUESTIONS	
Q.3	If a cylinder of an aerofoil shape is placed in a uniform stream of speed U, with circulation Γ around the cylinder, then the lift per unit length of the cylinder is of magnitude $\rho U\Gamma$ in the direction perpendicular to the direction of the stream.	(10)
Q.4	Derive the Navier Stokes equations for an incompressible viscous fluid. Show that in the case of Couette flow velocity of the fluid is given by $$u = \frac{U}{h}y - \frac{h^2}{2\mu}\frac{dp}{dx}\frac{y}{h}(1 - \frac{y}{h})$$	(10)
Q.5	Define mean motion and fluctuations in a turbulent flow and prove that the mean value of a fluctuating quantity is always zero.	(10)