

## **M.Sc. Programme**

2 years M.Sc. Mathematics programme consists of two parts namely Part-I and Part II. The regulation, Syllabi and Courses of Reading for the M.Sc. (Mathematics) Part-I and Part-II (Regular Scheme) are given below.

### **Regulations**

The following regulations will be observed by M.Sc. (Mathematics) regular students

- i. There are a total of 1200 marks for M.Sc. (Mathematics) for regular students as is the case with other M.Sc. subjects.
- ii. There are five papers in Part-I and six papers in Part-II. Each paper carries 100 marks.
- iii. There is a Viva Voce Examination of M.Sc. Part II. The topics of Viva Voce Examination shall be:
  - a) Analysis (Real, Complex and Functional)
  - b) Algebra and Topology
  - c) Mechanics

### **M.Sc. Part-I**

The following five papers shall be studied in M.Sc. Part-I:

|           |  |
|-----------|--|
| Paper I   | Real Analysis                              |
| Paper II  | Algebra                                    |
| Paper III | Complex Analysis and Differential Geometry |
| Paper IV  | Mechanics                                  |
| Paper V   | Topology and Functional Analysis           |

**Note:** All the papers of M.Sc. Part-I given above are compulsory.

### **M.Sc. Part-II**

In M.Sc. Part-II examinations, there are six written papers. The following three papers are compulsory. Each paper carries 100 marks.

|           |                                 |
|-----------|---------------------------------|
| Paper I   | Advanced Analysis               |
| Paper II  | Methods of Mathematical Physics |
| Paper III | Numerical Analysis              |

## Optional Papers

A student may select any three of the following optional courses:

|                           |   |
|---------------------------|---|
| Paper IV-VI option (i)    | Mathematical Statistics                               |
| Paper IV-VI option (ii)   | Computer Applications                                 |
| Paper IV-VI option (iii)  | Group Theory  |
| Paper IV-VI option (iv)   | Rings and Modules                                     |
| Paper IV-VI option (v)    | Number Theory   |
| Paper IV-VI option (vi)   | Fluid Mechanics                                       |
| Paper IV-VI option (vii)  | Quantum Mechanics                                     |
| Paper IV-VI option (viii) | Special Theory of Relativity and Analytical Mechanics |
| Paper IV-VI option (ix)   | Electromagnetic Theory                                |
| Paper IV-VI option (x)    | Operations Research                                   |
| Paper IV-VI option (xi)   | Theory of Approximation and Splines                   |
| Paper IV-VI option (xii)  | Advanced Functional Analysis                          |
| Paper IV-VI option (xiii) | Solid Mechanics                                       |
| Paper IV-VI option (xiv)  | Theory of Optimization                                |

**Note:** The students who opt for Computer Applications paper shall have to pass in both the theory and practical parts of the examinations.

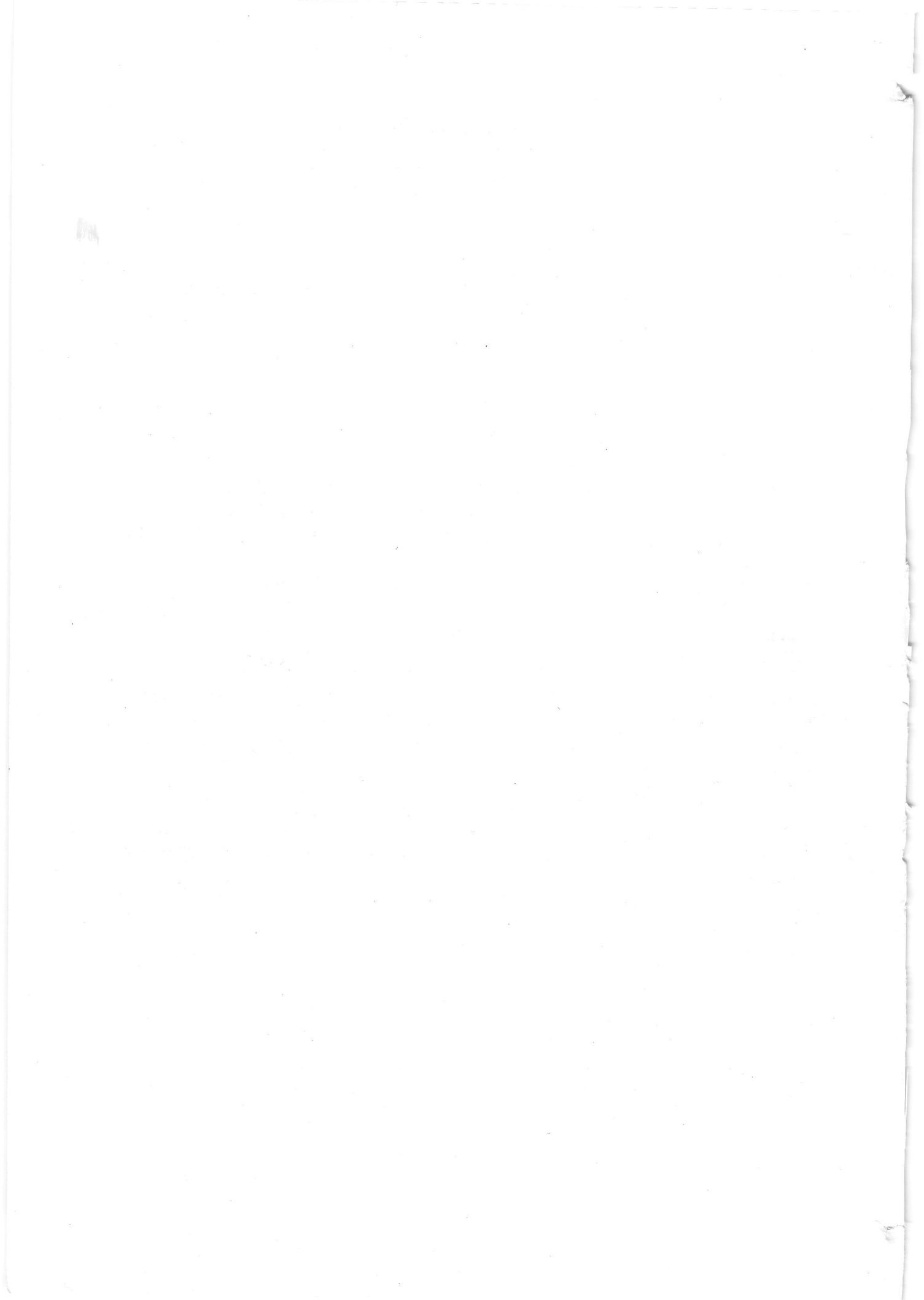
# P R O S P E C T U S

M.Sc Mathematics



## DEPARTMENT OF MATHEMATICS

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## Detailed Outline of Courses

### M.Sc. Part I Papers

#### Paper I: Real Analysis

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

##### Section-I (5/9)

##### Real Number System

- Ordered sets, Fields, Completeness property of real numbers
- The extended real number system, Euclidean spaces

##### Sequences and Series

- Sequences, Subsequences, Convergent sequences, Cauchy sequences
- Monotone and bounded sequences, Bolzano Weierstrass theorem
- Series, Convergence of series, Series of non-negative terms, Cauchy condensation test
- Partial sums, The root and ratio tests, Integral test, Comparison test
- Absolute and conditional convergence

##### Limit and Continuity

- The limit of a function, Continuous functions, Types of discontinuity
- Uniform continuity, Monotone functions

##### Differentiation

- The derivative of a function
- Mean value theorem, Continuity of derivatives
- Properties of differentiable functions.

##### Functions of Several Variables

- Partial derivatives and differentiability, Derivatives and differentials of composite functions
- Change in the order of partial derivative, Implicit functions, Inverse functions, Jacobians
- Maxima and minima, Lagrange multipliers

##### Section-II (4/9)

##### The Riemann-Stieltjes Integrals

- Definition and existence of integrals, Properties of integrals
- Fundamental theorem of calculus and its applications
- Change of variable theorem
- Integration by parts



### Functions of Bounded Variation

- Definition and examples
- Properties of functions of bounded variation

### Improper Integrals

- Types of improper integrals
- Tests for convergence of improper integrals
- Beta and gamma functions
- Absolute and conditional convergence of improper integrals

### Sequences and Series of Functions

- Definition of point-wise and uniform convergence
- Uniform convergence and continuity
- Uniform convergence and integration
- Uniform convergence and differentiation

### Recommended Books

1. W. Rudin, *Principles of Mathematical Analysis*, (McGraw Hill, 1976)
2. R. G. Bartle, *Introduction to Real Analysis*, (John Wiley and Sons, 2000)
3. T. M. Apostol, *Mathematical Analysis*, (Addison-Wesley Publishing Company, 1974)
4. A. J. Kosmala, *Introductory Mathematical Analysis*, (WCB Company, 1995)
5. W. R. Parzynski and P. W. Zipse, *Introduction to Mathematical Analysis*, (McGraw Hill Company, 1982)
6. H. S. Gaskill and P. P. Narayanaswami, *Elements of Real Analysis*, (Printice Hall, 1988)

### Paper II: Algebra (Group Theory and Linear Algebra)

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

#### Section-I (5/9)

##### Groups

- Definition and examples of groups
- Subgroups lattice, Lagrange's theorem
- Cyclic groups
- Groups and symmetries, Cayley's theorem

##### Complexes in Groups

- Complexes and coset decomposition of groups
- Centre of a group
- Normalizer in a group
- Centralizer in a group
- Conjugacy classes and congruence relation in a group

##### Normal Subgroups

- Normal subgroups



- (2)
- Proper and improper normal subgroups
  - Factor groups
  - Isomorphism theorems
  - Automorphism group of a group
  - Commutator subgroups of a group

### **Permutation Groups**

- Symmetric or permutation group
- Transpositions
- Generators of the symmetric and alternating group
- Cyclic permutations and orbits, The alternating group
- Generators of the symmetric and alternating groups

### **Sylow Theorems**

- Double cosets
- Cauchy's theorem for Abelian and non-Abelian group
- Sylow theorems (with proofs)
- Applications of Sylow theory
- Classification of groups with at most 7 elements

## **Section-II (4/9)**

### **Ring Theory**

- Definition and examples of rings
- Special classes of rings
- Fields
- Ideals and quotient rings
- Ring Homomorphisms
- Prime and maximal ideals
- Field of quotients

### **Linear Algebra**

- Vector spaces, Subspaces
- Linear combinations, Linearly independent vectors
- Spanning set
- Bases and dimension of a vector space
- Homomorphism of vector spaces
- Quotient spaces

### **Linear Mappings**

- Mappings, Linear mappings
- Rank and nullity
- Linear mappings and system of linear equations
- Algebra of linear operators
- Space  $L(X, Y)$  of all linear transformations

### **Matrices and Linear Operators**

- Matrix representation of a linear operator

- Change of basis
- Similar matrices
- Matrix and linear transformations
- Orthogonal matrices and orthogonal transformations
- Orthonormal basis and Gram Schmidt process

### **Eigen Values and Eigen Vectors**

- Polynomials of matrices and linear operators
- Characteristic polynomial
- Diagonalization of matrices

### **Recommended Books**

1. J. Rose, *A Course on Group Theory*, (Cambridge University Press, 1978)
2. I. N. Herstein, *Topics in Algebra*, (Xerox Publishing Company, 1964)
3. G. Birkhoff and S. MacLane, *A Survey of Modern Algebra*, (Macmillan, 1964)
4. Seymour Lipschutz, *Linear Algebra*, (McGraw Hill Book Company, 2001)
5. Humphreys, John F. *A Course on Group Theory*, (Oxford University Press, 2004)
6. P. M. Cohn, *Algebra*, (John Wiley and Sons, 1974)
7. J. B. Fraleigh, *A First Course in Abstract Algebra*, (Pearson Education, 2002)

## **Paper III: Complex Analysis and Differential Geometry**

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

### **Section-I (5/9)**

#### **The Concept of Analytic Functions**

- Complex numbers, Complex planes, Complex functions
- Analytic functions
- Entire functions
- Harmonic functions
- Elementary functions: Trigonometric, Complex exponential, Logarithmic and hyperbolic functions

#### **Infinite Series**

- Power series, Derived series, Radius of convergence
- Taylor series and Laurent series

#### **Conformal Representation**

- Transformation, conformal transformation
- Linear transformation
- Möbius transformations

#### **Complex Integration**

- Complex integrals
- Cauchy-Goursat theorem



- Cauchy's integral formula and their consequences
- Liouville's theorem
- Morera's theorem
- Derivative of an analytic function

### **Singularity and Poles**

- Review of Laurent series
- Zeros, Singularities
- Poles and residues
- Cauchy's residue theorem
- Contour Integration

### **Expansion of Functions and Analytic Continuation**

- Mittag-Leffler theorem
- Weierstrass's factorization theorem
- Analytic continuation

## **Section-II (4/9)**

### **Theory of Space Curves**

- Introduction, Index notation and summation convention
- Space curves, Arc length, Tangent, Normal and binormal
- Osculating, Normal and rectifying planes
- Curvature and torsion
- The Frenet-Serret theorem
- Natural equation of a curve
- Involute and evolute, Helices
- Fundamental existence theorem of space curves

### **Theory of Surfaces**

- Coordinate transformation
- Tangent plane and surface normal
- The first fundamental form and the metric tensor
- The second fundamental form
- Principal, Gaussian, Mean, Geodesic and normal curvatures
- Gauss and Weingarten equations
- Gauss and Codazzi equations

### **Recommended Books**

1. H. S. Kasana, *Complex Variables: Theory and Applications*, (Prentice Hall, 2005)
2. M. R. Spiegel, *Complex Variables*, (McGraw Hill Book Company, 1974)
3. J. W. Brown, R. V. Churchill, *Complex Variables and Applications*, (McGraw Hill, 2009)
4. Louis L. Pennisi, *Elements of Complex Variables*, (Holt, Linehart and Winston, 1976)
5. W. Kaplan, *Introduction to Analytic Functions*, (Addison-Wesley, 1966)

6. R. S. Millman and G.D. Parker, *Elements of Differential Geometry*, (Prentice-Hall, 1977)
7. E. Kreyzig, *Differential Geometry*, (Dover Publications, 1991)
8. M. M. Lipschutz, *Schaum's Outline of Differential Geometry*, (McGraw Hill, 1969)
9. D. Somasundaram, *Differential Geometry*, (Narosa Publishing House, 2005)

## **Paper IV: Mechanics**

**NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.**

### **Section-I (5/9)**

#### **Vector Integration**

- Line integrals
- Surface area and surface integrals
- Volume integrals

#### **Integral Theorems**

- Green's theorem
- Gauss divergence theorem
- Stoke's theorem

#### **Curvilinear Coordinates**

- Orthogonal coordinates
- Unit vectors in curvilinear systems
- Arc length and volume elements
- The gradient, Divergence and curl
- Special orthogonal coordinate systems

#### **Tensor Analysis**

- Coordinate transformations
- Einstein summation convention
- Tensors of different ranks
- Contravariant, Covariant and mixed tensors
- Symmetric and skew symmetric tensors
- Addition, Subtraction, Inner and outer products of tensors
- Contraction theorem, Quotient law
- The line element and metric tensor
- Christoffel symbols

### **Section-II (4/9)**

#### **Non Inertial Reference Systems**

- Accelerated coordinate systems and inertial forces
- Rotating coordinate systems
- Velocity and acceleration in moving system: Coriolis, Centripetal and transverse acceleration
- Dynamics of a particle in a rotating coordinate system

#### **Planar Motion of Rigid Bodies**



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- Introduction to rigid and elastic bodies, Degrees of freedom, Translations, Rotations, instantaneous axis and center of rotation, Motion of the center of mass
- Euler's theorem and Chasle's theorem
- Rotation of a rigid body about a fixed axis: Moments and products of inertia of various bodies including hoop or cylindrical shell, circular cylinder, spherical shell
- Parallel and perpendicular axis theorem
- Radius of gyration of various bodies

#### **Motion of Rigid Bodies in Three Dimensions**

- General motion of rigid bodies in space: Moments and products of inertia, Inertia matrix
- The momental ellipsoid and equimomental systems
- Angular momentum vector and rotational kinetic energy
- Principal axes and principal moments of inertia
- Determination of principal axes by diagonalizing the inertia matrix

#### **Euler Equations of Motion of a Rigid Body**

- Force free motion
- Free rotation of a rigid body with an axis of symmetry
- Free rotation of a rigid body with three different principal moments
- Euler's Equations
- The Eulerian angles, Angular velocity and kinetic energy in terms of Euler angles, Space cone
- Motion of a spinning top and gyroscopes- steady precession, Sleeping top

#### **Recommended Books**

1. G. E. Hay, *Vector and Tensor Analysis*, (Dover Publications, Inc., 1979)
2. G. R. Fowles and G. L. Cassiday, *Analytical Mechanics*, (Thomson Brooks/Cole, 2005)
3. H. Goldstein, C. P. Poole and J. L. Safko, *Classical Mechanics*, (Addison-Wesley Publishing Co., 2001)
4. M. R. Spiegel, *Theoretical Mechanics*, (McGraw Hill Book Company, 1980)
5. M. R. Spiegel, *Vector Analysis*, (McGraw Hill Book Company, 1981)
6. D. C. Kay, *Tensor Calculus*, (McGraw Hill Book Company, 1988)
7. E. C. Young, *Vector and Tensor Analysis*, (Marcel Dekker, Inc., 1993)
8. L. N. Hand and J. D. Finch, *Analytical Mechanics*, (Cambridge University Press, 1998)

### **Paper V: Topology & Functional Analysis**

**NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.**

#### **Section-I (4/9)**

##### **Topology**

- Definition and examples
- Open and closed sets

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- Subspaces
  - Neighborhoods
  - Limit points, Closure of a set
  - Interior, Exterior and boundary of a set

### **Bases and Sub-bases**

- Base and sub bases
- Neighborhood bases
- First and second axioms of countability
- Separable spaces, Lindelöf spaces
- Continuous functions and homeomorphism
- Weak topologies, Finite product spaces

### **Separation Axioms**

- Separation axioms
- Regular spaces
- Completely regular spaces
- Normal spaces

### **Compact Spaces**

- Compact topological spaces
- Countably compact spaces
- Sequentially compact spaces

### **Connectedness**

- Connected spaces, Disconnected spaces
- Totally disconnected spaces
- Components of topological spaces

## **Section-II (5/9)**

### **Metric Space**

- Review of metric spaces
- Convergence in metric spaces
- Complete metric spaces
- Completeness proofs
- Dense sets and separable spaces
- No-where dense sets
- Baire category theorem

### **Normed Spaces**

- Normed linear spaces
- Banach spaces
- Convex sets
- Quotient spaces
- Equivalent norms
- Linear operators
- Linear functionals
- Finite dimensional normed spaces



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- Continuous or bounded linear operators
- Dual spaces

### **Inner Product Spaces**

- Definition and examples
- Orthonormal sets and bases
- Annihilators, Projections
- Hilbert space
- Linear functionals on Hilbert spaces
- Reflexivity of Hilbert spaces

### **Recommended Books**

1. J. Dugundji, *Topology*, (Allyn and Bacon Inc., 1966)
2. G. F. Simmon, *Introduction to Topology and Modern Analysis*, (McGraw Hill Book Company, 1963)
3. Stephen Willard, *General Topology*, (Addison-Wesley Publishing Co., 1970)
4. Seymour Lipschutz, *General Topology*, (Schaum's Outline Series, McGraw Hill Book Company, 2004)
5. E. Kreyszig, *Introduction to Functional Analysis with Applications*, (John Wiley and Sons, 2006)
6. A. L. Brown and A. Page, *Elements of Functional Analysis*, (Van Nostrand Reinhold, 1970)
7. G. Bachman and L. Narici, *Functional Analysis*, (Academic Press, 1966)
8. F. Riesz and B. Sz. Nagay, *Functional Analysis*, (Dover Publications, Inc., 1965)

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## M.Sc. Part II Papers

### Paper I: Advanced Analysis

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

#### Section-I (4/9)

##### Advanced Set Theory

- Equivalent Sets
- Countable and Uncountable Sets
- The concept of a cardinal number
- The cardinals  $\aleph_0$  and  $c$
- Addition and multiplication of cardinals
- Cartesian product, Axiom of Choice, Multiplication of cardinal numbers
- Order relation and order types, Well ordered sets, Transfinite induction
- Addition and multiplication of ordinals
- Statements of Zorn's lemma, Maximality principle and their simple implications

#### Section-II (5/9)

##### Measure Theory

- Outer measure, Lebesgue Measure, Measureable Sets and Lebesgue measure, Non measurable sets, Measureable functions

##### The Lebesgue Integral

- The Rieman Integral, The Lebesgue integral of a bounded function
- The general Lebesgue integral

##### General Measure and Integration

- Measure spaces, Measureable functions, Integration, General convergence theorems
- Signed measures, The  $L_p$ -spaces, Outer measure and measurability
- The extension theorem
- The Lebesgue Stieltjes integral, Product measures

##### Recommended Books

1. D. Smith, M. Eggen and R. ST. Andre, *A transition to Advanced Mathematics*, (Brooks Cole, 2004)
2. Seymour Lipschutz, *Set Theory and Related Topics*, (McGraw Hill, 1964)
3. Frankel, A. *Abstract Set theory*, (North Holland Publishing Co., 1961)
4. Royden, H. L. *Real Analysis*, (Prentice Hall, 1988)
5. Suppes, P. *Axiomatic Set Theory*, (Dover Publications Inc., May 1973)
6. Halmos, P. R. *Naive Set Theory*, (Springer, 1974)
7. Halmos, P. R. *Measure Theory*, (Springer, 1974)
8. Rudin, W. *Real and Complex Analysis*, (McGraw-Hill Higher Education, 1987)



## Paper II: Methods of Mathematical Physics

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

### Section-I (5/9)

#### Sturm Liouville Systems

- Some properties of Sturm-Liouville equations
- Regular, Periodic and singular Sturm-Liouville systems and its applications

#### Series Solutions of Second Order Linear Differential Equations

- Series solution near an ordinary point
- Series solution near regular singular points

#### Series Solution of Some Special Differential Equations

- Hypergeometric function  $F(a, b, c; x)$  and its evaluation
- Series solution of Bessel equation
- Expression for  $J_n(X)$  when  $n$  is half odd integer, Recurrence formulas for  $J_n(X)$
- Orthogonality of Bessel functions
- Series solution of Legendre equation

#### Introduction to PDEs

- Review of ordinary differential equation in more than one variables
- Linear partial differential equations (PDEs) of the first order
- Cauchy's problem for quasi-linear first order PDEs

#### PDEs of Second Order

- PDEs of second order in two independent variables with variable coefficients
- Cauchy's problem for second order PDEs in two independent variables

#### Boundary Value Problems

- Laplace equation and its solution in Cartesian, Cylindrical and spherical polar coordinates
- Dirichlet problem for a circle
- Poisson's integral for a circle
- Wave equation
- Heat equation

### Section-II (4/9)

#### Fourier Methods

- The Fourier transform
- Fourier analysis of generalized functions
- The Laplace transform

#### Green's Functions and Transform Methods

- Expansion for Green's functions
- Transform methods
- Closed form of Green's functions

#### Variational Methods

- Euler-Lagrange equations
- Integrand involving one, two, three and  $n$  variables
- Necessary conditions for existence of an extremum of a functional

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- Constrained maxima and minima

### **Recommended Books**

1. D.G. Zill and M.R. Cullen, *Advanced Engineering Mathematics*, (Jones and Bartlett Publishers, 2006)
2. W.E. Boyce and R. C. DiPrima, *Elementary Differential Equations and Boundary Value Problems*, (John Wiley & Sons, 2005)
3. E.T. Whittaker, and G. N. Watson, *A Course of Modern Analysis*, (Cambridge University Press, 1962)
4. I.N. Sneddon, *Elements of Partial Differential Equations*, (Dover Publishing, Inc., 2006)
5. R. Dennemyer, *Introduction to Partial Differential Equations and Boundary Value Problems*, (McGraw Hill Book Company, 1968)
6. D.L. Powers, *Boundary Value Problems and Partial Differential Equations*, (Academic Press, 2005)
7. W.E. Boyce, *Elementary Differential Equations*, (John Wiley & Sons, 2008)
8. M.L. Krasnov, G.I. Makarenko and A.I. Kiselev, *Problems and Exercises in the Calculus of Variations*, (Imported Publications, Inc., 1985)
9. J. Brown and R. Churchill, *Fourier Series and Boundary Value Problems*, (McGraw Hill, 2006)

### **Paper III: Numerical Analysis**

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

#### **Section-I (5/9)**

##### **Error Analysis**

- Errors, Absolute errors, Rounding errors, Truncation errors
- Inherent Errors, Major and Minor approximations in numbers

##### **The Solution of Linear Systems**

- Gaussian elimination method with pivoting, LU Decomposition methods, Algorithm and convergence of Jacobi iterative Method, Algorithm and convergence of Gauss Seidel Method
- Eigenvalue and eigenvector, Power method

##### **The Solution of Non-Linear Equation**

- Bisection Method, Fixed point iterative method, Newton Raphson method, Secant method, Method of false position, Algorithms and convergence of these methods

##### **Difference Operators**

- Shift operators
- Forward difference operators
- Backward difference operators
- Average and central difference operators

##### **Ordinary Differential Equations**

- Euler's, Improved Euler's, Modified Euler's methods with error analysis



- Runge-Kutta methods with error analysis
- Predictor-corrector methods for solving initial value problems
- Finite Difference, Collocation and variational methods for boundary value problems

## **Section-II (4/9)**

### **Interpolation**

- Lagrange's interpolation
- Newton's divided difference interpolation
- Newton's forward and backward difference interpolation, Central difference interpolation
- Hermit interpolation
- Spline interpolation
- Errors and algorithms of these interpolations

### **Numerical Differentiation**

- Newton's Forward, Backward and central formulae for numerical differentiation

### **Numerical Integration**

- Rectangular rule
- Trapezoidal rule
- Simpson rule
- Boole's rule
- Weddle's rule
- Gaussian quadrature formulae
- Errors in quadrature formulae
- Newton-Cotes formulae

### **Difference Equations**

- Linear homogeneous and non-homogeneous difference equations with constant coefficients

### **Recommended Books**

1. Curtis F. Gerald and Patrick O. Wheatley, *Applied Numerical Analysis*, (Addison-Wesley Publishing Co. Pearson Education, 2003)
2. Richard L. Burden and J. Douglas Faires, *Numerical Analysis*, (Brooks/Cole Publishing Company, 1997)
3. John H. Mathews, *Numerical Methods for Mathematics, Science and Engineering*, (Prentice Hall International, 2003)
4. Steven C. Chapra and Raymond P. Canale, *Numerical Methods for Engineers*, (McGraw Hill International Edition, 1998)

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**Paper (IV-VI) option (i):**

**Mathematical Statistics**

**NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.**

**Section-I (4/9)**

**Probability Distributions**

- The postulates of probability
- Some elementary theorems
- Addition and multiplication rules
- Baye's rule and future Baye's theorem
- Random variables and probability functions

**Discrete Probability Distributions**

- Uniform, Bernoulli and binomial distribution
- Hypergeometric and geometric distribution
- Negative binomial and Poisson distribution

**Continuous Probability Distributions**

- Uniform and exponential distribution
- Gamma and beta distributions
- Normal distribution

**Mathematical Expectations**

- Moments and moment generating functions
- Moments of binomial, Hypergeometric, Poisson, Gamma, Beta and normal distributions

**Section-II (5/9)**

**Functions of Random Variables**

- Distribution function technique
- Transformation technique: One variable, Several variables
- Moment-generating function technique

**Sampling Distributions**

- The distribution of mean and variance
- The distribution of differences of means and variances
- The Chi-Square distribution
- The  $t$  distribution
- The  $F$  distribution

**Regression and Correlation**

- Linear regression



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- The methods of least squares
- Normal regression analysis
- Normal correlation analysis
- Multiple linear regression (along with matrix notation)



### **Recommended Books**

1. J. E. Freund, *Mathematical Statistics*, (Prentice Hall Inc., 1992)
2. Hogg and Craig, *Introduction to Mathematical Statistics*, (Collier Macmillan, 1958)
3. Mood, Greyill and Boes, *Introduction to the Theory of Statistics*, (McGraw Hill)
4. R. E. Walpole, *Introduction to Statistics*, (Macmillan Publishing Company, 1982)
5. M. R. Spiegel and L. J. Stephens, *Statistics*, (McGraw Hill Book Company, 1984)

### **Paper (IV-VI) option (ii): Computer Applications**

#### **NOTE:**

The evaluation of this paper will consist of two parts:

1. Written examination: 50 marks
2. Practical examination: 50 marks

The practical examination includes 10 marks for the note book containing details of work done in the computer laboratory and 10 marks for the oral examination. It will involve writing and running programs on computational projects. Practical examination will be of three hours duration in which one or more computational projects will be examined.

**NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.**

#### **Section-I (4/9)**

##### **Fortran Programming**

- Constants
- Variables
- Implicit declaration
- Intrinsic functions
- Arithmetic operations
- Arithmetic expressions
- Assignment statements
- Relational operators
- Format statements
- The block if structure
- The block do loop structure

- 2 ✓
- Count controlled do loop structure
  - Logical constants, variables and expressions
  - The case statement
  - Function subprograms
  - Subroutines
  - One dimensional and two dimensional Arrays
  - Implementation of Fortran in terms of short programs

## **Section-II (5/9)**

### **Implementation of Fortran to the numerical methods**

#### **System of linear equations**

- Gaussian elimination method with pivoting
- LU Decomposition methods
- Jacobi's iterative method
- Gauss-Seidel method

#### **Solutions of non-linear equations**

- Bisection method
- Newton-Raphson method
- Secant method
- Regula Falsi method

#### **Interpolation**

- Lagrange interpolation
- Newton's divided and forward difference interpolation

#### **Numerical integration**

- Rectangular rule
- Trapezoidal rule
- Simpson's rule
- Boole's rule
- Weddle's rule

#### **Differential equations**

- Euler's methods
- Runge- Kutta methods
- Predictor-corrector methods



#### **Mathematica**

- Syntax of Fortran in Mathematica
- Symbolic representation
- Algebraic calculations
- Graphs

#### **Recommended Books**

1. Michel Metcalf, John Reid and Malcolm Cohen, *FORTTRAN 95/2003 Explained*, (Oxford University Press, 2004)



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2. V. Rajaraman, *Computer Programming in Fortran 90 and 95*, (Prentice Hall of India, 1999)
  3. John H. Mathews, *Numerical Methods for Mathematics*, Science and Engineering, (Prentice Hall International, 2003)
  4. Steven C. Chapra and Raymond P. Canale, *Numerical Methods for Engineers*, (McGraw Hill International Edition, 1998)
  5. Stephen Wolfram, *The Mathematica Book*, (Cambridge University Press, 1996)
  6. Roman E. Maeder, *Computer Science with Mathematica*, (Cambridge University Press, 2000)
  7. William T. Shaw and Jason Tigg, *Applied Mathematica*, (Addison-Wesley Publishing Company, 1994)
  8. Cameron Smith and Nancy Blachman, *The Mathematica Graphics Guidebook*, (Addison-Wesley Publishing Company, 1995)

**Paper (IV-VI) option (iii): Advanced Group Theory**

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

**Section-I (4/9)**

**The Orbit Stabilizer Theorem**

- Stabilizer, Orbit, A group with  $p^2$  elements
- Simplicity of  $A_n$ ,  $n \geq 5$
- Classification of Groups with at most 8 elements

**Sylow Theorems**

- Sylow theorems (with proofs)
- Applications of Sylow Theory

**Products in Groups**

- Direct Products
- Classification of Finite Abelian Groups
- Characteristic and fully invariant subgroups
- Normal products of groups
- Holomorph of a group

**Section-II (5/9)**

**Series in Groups**

- Series in groups
- Zassenhaus lemma
- Normal series and their refinements
- Composition series
- The Jordan Holder Theorem

**Solvable Groups**

- Solvable groups, Definition and examples
- Theorems on solvable groups

**Nilpotent Groups**

- Characterisation of finite nilpotent groups
- Frattini subgroups

### Extensions

- Central extensions
- Cyclic extensions
- Groups with at most 31 elements

### Linear Groups

- Linear groups, types of linear groups
- Representation of linear groups
- The projective special linear groups

### Recommended Books

1. J. Rotman, *The Theory of Groups*, (Allyn and Bacon, London, 1978)
2. J. B. Fraleigh, *A First Course in Abstract Algebra*, (Addison-Wesley Publishing Co., 2003)
3. H. Marshall, *The Theory of Groups*, (Macmillan, 1967)
4. J. A. Gallian, *Contemporary Abstract Algebra*, (Narosa 1998)
5. I.N. Herstein, *Topics in Algebra*, (Xerox Publishing Company Mass, 1972)
6. J. S. Rose, *A Course on Group Theory*, (Dover Publications, 1994)
7. Humphreys, John F. *A Course on Group Theory*, (Oxford University Press, 2004)
8. K. Hoffman, *Linear Algebra*, (Prentice Hall, 1971)
9. I.D. Macdonald, *The Theory of Groups*, (Oxford, Clarendon Press, 1975)

### Paper (IV-VI) option (iv): Rings and Modules

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

#### Section-I (5/9)

##### Ring Theory

- Construction of new rings
- Direct sums, Polynomial rings
- Matrix rings
- Divisors, units and associates
- Unique factorisation domains
- Principal ideal domains and Euclidean domains

##### Field Extensions

- Algebraic and transcendental elements
- Degree of extension



- Algebraic extensions
- Reducible and irreducible polynomials
- Roots of polynomials

## Section-II (4/9)

### Modules

- Definition and examples
- Submodules
- Homomorphisms
- Quotient modules
- Direct sums of modules
- Finitely generated modules
- Torsion modules
- Free modules
- Basis, Rank and endomorphisms of free modules
- Matrices over rings and their connection with the basis of a free module
- A module as the direct sum of a free and a torsion module

### Recommended Books

1. I. N. Herstein, *Topics in Algebra*, (Xerox Publishing Company Mass, 1972)
2. B. Hartley and T. O. Hauvkes, *Rings, Modules and Linear Algebra*, (Chapmann and Hall Ltd., 1970)
3. R. B. Allenly, *Rings, Fields and Groups: An Introduction to Abstract Algebra*, (Edward Arnold, 1985)
4. J. Rose, *A Course on Rings Theory*, (Cambridge University Press, 1978)
5. G. Birkhoff and S. MacLane, *A Survey of Modern Algebra*, (Macmillan, 1964)
6. J. B. Fraleigh, *A First Course in Abstract Algebra*, (Addison-Wesley Publishing Co., 2003)
7. J. A. Gallian, *Contemporary Abstract Algebra*, (Narosa Publishing House, 1998)
8. K. Hoffman, *Linear Algebra*, (Prentice Hall, 1971)

## Paper (IV-VI) option (v): Number Theory

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

### Section- I (5/9)

#### Congruences

- Elementary properties of prime numbers
- Residue classes and Euler's function
- Linear congruences and congruences of higher degree
- Congruences with prime moduli
- The theorems of Fermat, Euler and Wilson

#### Number-Theoretic Functions

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- Möbius function
  - The function  $[x]$ , The symbols  $O$  and their basic properties

### **Primitive roots and indices**

- Integers belonging to a given exponent  $(\text{mod } p)$
- Primitive roots and composite moduli
- Determination of integers having primitive roots
- Indices, Solutions of Higher Congruences by Indices

### **Diophantine Equations**

- Equations and Fermat's conjecture for  $n = 2, n = 4$

## **Section-II (4/9)**

### **Quadratic Residues**

- Composite moduli, Legendre symbol
- Law of quadratic reciprocity
- The Jacobi symbol

### **Algebraic Number Theory**

- Polynomials over a field
- Divisibility properties of polynomials
- Gauss's lemma
- The Eisenstein's irreducibility criterion
- Symmetric polynomials
- Extensions of a field
- Algebraic and transcendental numbers
- Bases and finite extensions, Properties of finite extensions
- Conjugates and discriminants
- Algebraic integers in a quadratic field, Integral bases
- Units and primes in a quadratic field
- Ideals, Arithmetic of ideals in an algebraic number field
- The norm of an ideal, Prime ideals

### **Recommended Books**

1. W. J. Leveque, *Topics in Number Theory*, (Vols. I and II, Addison-Wesley Publishing Co., 1961, 1965)
2. Tom M. Apostol, *Introduction to Analytic Number Theory*, (Springer International, 1998)
3. David M. Burton, *Elementary Number Theory*, (McGraw Hill Company, 2007)
4. A. Andrew, *The Theory of Numbers*, (Jones and Barlett Publishers, 1995)
5. Harry Pollard, *The Theory of Algebraic Numbers*, (The Mathematical Association of America, 1975)



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**Paper (IV-VI) option (vi): Fluid Mechanics**

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

**Section-I (5/9)**

**Conservation of Matter**

- Introduction
- Fields and continuum concepts
- Lagrangian and Eulerian specifications
- Local, Convective and total rates of change
- Conservation of mass
- Equation of continuity
- Boundary conditions

**Nature of Forces and Fluid Flow**

- Surface and body forces
- Stress at a point
- Viscosity and Newton's viscosity law
- Viscous and inviscid flows
- Laminar and turbulent flows
- Compressible and incompressible flows

**Irrotational Fluid Motion**

- Velocity potential from an irrotational velocity field
- Streamlines
- Vortex lines and vortex sheets
- Kelvin's minimum energy theorem
- Conservation of linear momentum
- Bernoulli's theorem and its applications
- Circulation, Rate of change of circulation (Kelvin's theorem)
- Axially symmetric motion
- Stokes's stream function

**Two-dimensional Motion**

- Stream function
- Complex potential and complex velocity, Uniform flows
- Sources, Sinks and vortex flows
- Flow in a sector
- Flow around a sharp edge
- Flow due to a doublet

**Section-II (4/9)**

**Two and Three-Dimensional Potential Flows**

- Circular cylinder without circulation
- Circular cylinder with circulation

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- Blasius theorem
  - Kutta condition and the flat-plate airfoil
  - Joukowski airfoil
  - Vortex motion
  - Karman's vortex street
  - Method of images
  - Velocity potential
  - Stoke's stream function
  - Solution of the Potential equation
  - Uniform flow
  - Source and sink
  - Flow due to a doublet

### **Viscous Flows of Incompressible Fluids**

- Constitutive equations
- Navier-Stokes equations and their exact solutions
- Steady unidirectional flow
- Poiseuille flow
- Couette flow
- Flow between rotating cylinders
- Stokes' first problem
- Stokes' second problem

### **Approach to Fluid Flow Problems**

- Similarity from a differential equation
- Dimensional analysis
- One dimensional, Steady compressible flow

### ***Recommended Book***

1. T. Allen and I. L. Ditsworth: *Fluid Mechanics*, (McGraw Hill, 1972)
2. I. G. Currie: *Fundamentals of Mechanics of Fluids*, (CRC, 2002)
3. Chia-Shun Yeh: *Fluid Mechanics: An Introduction to the Theory*, (McGraw Hill, 1974)
4. F. M. White: *Fluid Mechanics*, (McGraw Hill, 2003)
5. R. W. Fox, A. T. McDonald and P. J. Pritchard: *Introduction to Fluid Mechanics*, (John Wiley and Sons Pte. Ltd., 2003)



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**Paper (IV-VI) option (vii): Quantum Mechanics**

**NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.**

**Section-I (4/9)**

**Inadequacy of Classical Mechanics**

- Black body radiation
- Photoelectric effect
- Compton effect
- Bohr's theory of atomic structure
- Wave-particle duality
- The de Broglie postulate
- Heisenberg uncertainty principle

**The Postulates of Quantum Mechanics: Operators, Eigenfunctions and Eigenvalues**

- Observables and operators
- Measurement in quantum mechanics
- The state function and expectation values
- Time development of the state function (Schrödinger wave equation)
- Solution to the initial-value problem in quantum mechanics
- Parity operators

**Preparatory Concepts: Function Spaces and Hermitian Operators**

- Particle in a box
- Dirac notation
- Hilbert space
- Hermitian operators
- Properties of Hermitian operators

**Additional One-Dimensional Problems: Bound and Unbound States**

- General properties of the 1-dimensional Schrodinger equation
- Unbound states
- One-dimensional barrier problems
- The rectangular barrier: Tunneling

**Section-II (5/9)**

**Harmonic Oscillator and Problems in Three-Dimensions**

- The harmonic oscillator
- Eigenfunctions of the harmonic oscillator
- The harmonic oscillator in momentum space
- Motion in three dimensions
- Spherically symmetric potential and the hydrogen atom

**Angular Momentum**

- Basic properties

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- Eigenvalues of the angular momentum operators
  - Eigenfunctions of the orbital angular momentum operators  $L^2$  and  $L_z$
  - Commutation relations between components of angular momentum and their representation in spherical polar coordinates

### Scattering Theory

- The scattering cross-section
- Scattering amplitude
- Scattering equation
- Born approximation
- Partial wave analysis

### Perturbation Theory

- Time independent perturbation of non-degenerate and degenerate cases
- Time-dependent perturbations

### Identical Particles

- Symmetric and antisymmetric eigenfunctions
- The Pauli exclusion principle

### Recommended Book

1. R. L. Liboff, *Introductory Quantum Mechanics*, (Addison-Wesley Publishing Co., 2003)
2. V. K. Thankappan, *Quantum Mechanics*, (New Age Publishers, 1993)
3. B. C. Reed: *Quantum Mechanics*, (Jones and Bartlett Pub. Inc., 1998)
4. C. Phillips, *Introduction to Quantum Mechanics*, (John Wiley and Sons Ltd., 2003)
5. J. C. Taylor, *Quantum Mechanics: An Introduction*, (George Allen & Lenwin Ltd., Birkenhead, 1970)

### Paper (IV-VI) optional (viii): **Special Relativity and Analytical Dynamics**

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

#### Section-I (5/9)

#### Derivation of Special Relativity

- Fundamental concepts
- Einstein's formulation of special relativity
- The Lorentz transformations
- Length contraction, Time dilation and simultaneity
- The velocity addition formulae
- Three dimensional Lorentz transformations

#### The Four-Vector Formulation of Special Relativity



- The four-vector formalism
- The Lorentz transformations in 4-vectors
- The Lorentz and Poincare groups
- The null cone structure
- Proper time

### **Applications of Special Relativity**

- Relativistic kinematics
- The Doppler shift in relativity
- The Compton effect
- Particle scattering
- Binding energy, Particle production and particle decay

### **Electromagnetism in Special Relativity**

- Review of electromagnetism
- The electric and magnetic field intensities
- The electric current
- Maxwell's equations and electromagnetic waves
- The four-vector formulation of Maxwell's equations

## **Section-II (4/9)**

### **Lagrange's Theory of Holonomic and Non-Holonomic Systems**

- Generalized coordinates
- Holonomic and non-holonomic systems
- D'Alembert's principle, D-delta rule
- Lagrange equations
- Generalization of Lagrange equations
- Quasi-coordinates
- Lagrange equations in quasi-coordinates
- First integrals of Lagrange equations of motion
- Energy integral
- Lagrange equations for non-holonomic systems with and without Lagrange multipliers
- Hamilton's Principle for non-holonomic systems

### **Hamilton's Theory**

- Hamilton's principle
- Generalized momenta and phase space
- Hamilton's equations
- Ignorable coordinates, Routhian function
- Derivation of Hamilton's equations from a variational principle
- The principle of least action

### **Canonical Transformations**

- The equations of canonical transformations

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- Examples of canonical transformations
  - The Lagrange and Poisson brackets
  - Equations of motion, Infinitesimal canonical transformations and conservation theorems in the Poisson bracket formulation

### Hamilton-Jacobi Theory

- The Hamilton-Jacobi equation for Hamilton's principal function
- The harmonic oscillator problem as an example of the Hamilton-Jacobi method
- The Hamilton-Jacobi equation for Hamilton's characteristic function
- Separation of variables in the Hamilton-Jacobi equation

### Recommended Books

1. A. Qadir, *An Introduction to Special Theory of Relativity*, (World Scientific, 1989)
2. M. Saleem and M. Rafique, *Special Relativity: Applications to Particle and the Classical Theory of Fields*, (Prentice Hall, 1993)
3. J. Freund, *Special Relativity for Beginners*, (World Scientific, 2008)
4. W. Rindler, *Introduction to Special Relativity*, (Oxford University Press, 1991)
5. H. Goldstein, C.P. Poole and J.L. Safko, *Classical Mechanics*, (Addison-Wesley Publishing Co., 2003)
6. W. Greiner, *Classical Mechanics – Systems of Particles and Hamiltonian Dynamics*, (Springer-Verlag, 2004)
7. E.J. Saletan and J.V. Jose, *Classical Dynamics: A Contemporary Approach*, (Cambridge University Press, 1998)
8. S.T. Thornton and J.B. Marion, *Classical Dynamics of Particles and Systems*, (Brooks Cole, 2003)

### Paper (IV-VI) option (ix): **Electromagnetic Theory**

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

#### Section-I (5/9)

##### Electrostatic Fields

- Coulomb's law, The electric field intensity and potential
- Gauss's law and deductions, Poisson and Laplace equations
- Conductors and condensers
- Dipoles, The linear quadrupole
- Potential energy of a charge distribution, Dielectrics
- The polarization and the displacement vectors

##### Magnetostatic Fields

- The Magnetostatic law of force
- The magnetic induction
- The Lorentz force on a point charge moving in a magnetic field
- The divergence of the magnetic field



- The vector potential
- The conservation of charge and the equation of continuity
- The Lorentz condition
- The curl of the magnetic field
- Ampere's law and the scalar potential

### **Steady and Slowly Varying Currents**

- Electric current, Linear conductors
- Conductivity, Resistance
- Kirchhoff's laws
- Current density vector
- Magnetic field of straight and circular current
- Magnetic flux, Vector potential
- Forces on a circuit in magnetic field
- The Faraday induction law
- Induced electromotive force in a moving system
- Inductance and induced electromotive force
- Energy stored in a magnetic field

### **Section-II (4/9)**

#### **The Equations of Electromagnetism**

- Maxwell's equations in free space and material media
- Solution of Maxwell's equations

#### **Electromagnetic Waves**

- Plane electromagnetic waves in homogeneous and isotropic media
- The Poynting vector in free space
- Propagation of plane electromagnetic waves in non-conductors
- Propagation of plane electromagnetic waves in conducting media
- Reflection and refraction of plane waves

#### **Guided Waves**

- Guided waves, Coaxial line, Hollow rectangular wave guide
- Radiation of electromagnetic waves
- Electromagnetic field of a moving charge

#### **Recommended Books**

1. D. Corson and P. Lorrison, *Introduction to Electromagnetic Fields and Waves*, (W.H. Freeman and Company, 1962)
2. J. R. Reitz, F. J. Milford and R. W. Christy, *Foundations of Electromagnetic Theory*, (Addison-Wesley Publishing Co., 1993)
3. D. J. Griffiths, *Introduction to Electrodynamics*, (Prentice-Hall, 1999)
4. J. D. Jackson, *Classical Electrodynamics*, (John Wiley & Sons, Inc., 1999)
5. G. E. Owen, *Introduction to Electromagnetic Theory*, (Dover Publications, 2003)

  
**Paper (IV-VI) option (x):**

**Operations Research**

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

**Section-I (5/9)**

**Linear Programming**

- Linear programming, Formulations and graphical solution
- Simplex method
- M-Technique and two-phase technique
- Special cases

**Duality and Sensitivity Analysis**

- The dual problem, Primal-dual relationships
- Dual simplex method
- Sensitivity and postoptimal analysis

**Transportation Models**

- North-West corner
- Least-Cost and Vogel's approximations methods
- The method of multipliers
- The assignment model
- The transshipment model
- Hungarian method

**Section-II (4/9)**

**Net work Minimization and Integer Programming**

- Network minimization
- Shortest-Route algorithms for acyclic networks
- Maximal-flow problem
- Matrix definition of LP problem
- Revised simplex method, Bounded variables
- Decomposition algorithm
- Parametric linear programming
- Applications of integer programming
- Cutting-plane algorithms
- Branch-and-bound method
- Elements of dynamic programming
- Programmes by dynamic programming

**Recommended Books**

1. Hamdy A. Taha, *Operations Research - An Introduction*, (Macmillan Publishing Company Inc., 2006)
2. B. E. Gillett, *Introduction to Operations Research*, (Tata McGraw Hill Publishing Company Ltd., 2006)



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3. F. S. Hillier and G. J. Lieberman, *Operations Research*, (CBS Publishers and Distributors, 2005)
  4. W. M. Harvey, *Operations Research*, (North Holland, 2001)

**Paper (IV-VI) option (xi):**

**Theory of Approximation and Splines**

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

**Section-I (4/9)**

**Euclidean Geometry**

- Basic concepts of Euclidean geometry
- Scalar and vector functions
- Barycentric coordinates
- Convex hull
- Affine maps: Translation, Rotation, Scaling, Reflection and shear

**Approximation using Polynomials**

- Curve Fitting: Least squares line fitting, Least squares power fit, Data linearization method for exponential functions, Nonlinear least-squares method for exponential functions, Transformations for data linearization, Linear least squares, Polynomial fitting
- Chebyshev polynomials, Padé approximations


**Section-II (5/9)**

**Parametric Curves (Scalar and Vector Case)**

- Cubic algebraic form
- Cubic Hermite form
- Cubic control point form
- Bernstein Bezier cubic form
- Bernstein Bezier general form
- Uniform B-Spline cubic form
- Matrix forms of parametric curves
- Rational quadratic form
- Rational cubic form
- Tensor product surface, Bernstein Bezier cubic patch, Quadratic by cubic Bernstein Bezier patch, Bernstein Bezier quartic patch
- Properties of Bernstein Bezier form: Convex hull property, Affine invariance property, Variation diminishing property
- Algorithms to compute Bernstein Bezier form
- Derivation of Uniform B-Spline form

**Spline Functions**

- Introduction to splines
- Cubic Hermite splines
- End conditions of cubic splines: Clamped conditions, Natural conditions, 2<sup>nd</sup> Derivative conditions, Periodic conditions, Not a knot conditions

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- General Splines: Natural splines, Periodic splines
  - Truncated power function, Representation of spline in terms of truncated power functions, examples

### **Recommended Books**

1. David A. Brannan, *Geometry*, (Cambridge University Press, 1999).
2. Gerald Farin, *Curves and Surfaces for Computer Aided Geometric Design: A Practical Guide*, (Academic Press. Inc., 2002)
3. John H. Mathews, *Numerical Methods for Mathematics, Science and Engineering*, (Prentice-Hall International Editions, 1992)
4. Steven C. Chapra and Raymond P. Canale, *Numerical Methods for Engineers*, (McGraw Hill International Edition, 1998)
5. Richard H. Bartels, John C. Beatty, and John C. Beatty, *An Introduction to Spline for use in Computer Graphics and Geometric Modeling*, (Morgan Kaufmann Publisher 2006)
6. I. D. Faux, *Computational Geometry for Design and Manufacture*, (Ellis Horwood, 1979)
7. Carl de Boor, *A Practical Guide to Splines*, (Springer Verlag, 2001)
8. Larry L. Schumaker, *Spline Functions: Basic Theory*, (John Wiley and Sons, 1993)

### **Paper (IV-VI) option (xii): Advanced Functional Analysis**

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

#### **Section-I (4/9)**

##### **Compact Normed Spaces**

- Completion of metric spaces
- Completion of normed spaces
- Compactification
- Nowhere and everywhere dense sets and category
- Generated subspaces and closed subspaces
- Factor Spaces
- Completeness in the factor spaces

##### **Complete Orthonormal set**

- Complete orthonormal sets
- Total orthonormal sets
- Parseval's identity
- Bessel's inequality

##### **The Specific geometry of Hilbert Spaces**

- Hilbert spaces
- Bases of Hilbert spaces
- Cardinality of Hilbert spaces
- Linear manifolds and subspaces
- Orthogonal subspaces of Hilbert spaces



- Polynomial bases in  $L^2$  spaces

## Section-II (5/9)

### Fundamental Theorems

- Hahn Banach theorems
- Open mapping and closed graph theorems
- Banach Steinhaus theorem

### Semi-norms

- Semi norms, Locally convex spaces
- Quasi normed linear spaces
- Bounded linear functionals
- Hahn Banach theorem

### Dual or Conjugate spaces

- First and second dual spaces
- Second conjugate space of  $l_p$
- The Riesz representation theorem for linear functionals on a Hilbert spaces
- Conjugate space of  $C[a, b]$
- A representation theorem for bounded linear functionals on  $C[a, b]$

### Uniform Boundedness

- Weak convergence
- The Principle of uniform boundedness
- Consequences of the principle of uniform boundedness

### Recommended Books

1. G. Bachman and L. Narici, *Functional Analysis*, (Academic Press, New York, 1966)
2. A. E. Taylor, *Functional Analysis*, (John Wiley and Sons, Toppan, 1958)
3. G. Helmbert, *Introduction to Spectral theory in Hilbert spaces*, (N. H. Publishing Company 1969)
4. E. Kreyszig, *Introduction to Functional Analysis with Applications*, (John Wiley and Sons, 2004)
5. F. Riesz and B. Sz. Nagay, *Functional Analysis*, (Dover Publications, New York, Ungar, 1965)

## Paper (IV-VI) option (xiii): Solid Mechanics

Five questions to be attempted, selecting at least two questions from each section.

### Section-I (4/9)

#### Elasticity

- Analysis of stress and strain
- Generalized Hook's law
- Differential equations of equilibrium in terms of stress and in terms of displacements
- Boundary conditions

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- Compatibility equations
  - Plane stress, Plane strain, Stress functions
  - Two-dimensional problems in rectangular and polar co-ordinates
  - Torsion problems

## **Section-II (5/9)**

### **Elastodynamics**

- Equations of wave propagation in elastic solids
- Primary and secondary waves
- Reflection and transmission at plane boundaries
- Surface wave: Love waves and Raleigh waves
- Dispersion relations
- Geophysical applications

### **Recommended Books**

1. S. P. Timoshenko, and J. N. Goodier, *Theory of Elasticity*, (McGraw-Hill Higher Education, 1970)
2. I. S. Sokolnikoff, *Mathematical Theory of Elasticity*, (McGraw Hill Book Company, 1956)
3. W. Prager, *Introduction to Mechanics of Continua*, (Dover Publications, 2004)
4. J. D. Achenbach, *Wave Propagation in Elastic Solids*, (Elsevier Science, 1984)
5. W. M. Ewing, *Elastic Waves in Layered Media*, (McGraw Hill, 1957)
6. K. F. Graff, *Wave Motion in Elastic Solids*, (Dover Publications, 1991)
7. K. E. Bullen, *An Introduction to the Theory of Seimology*, (Cambridge University Press, 1965)

## **Paper (IV-VI) optional (xiv): Theory of Optimization**

**NOTE:** Attempt any FIVE questions selecting at least TWO questions from each section.

### **Section-I (5/9)**

#### **The Mathematical Programming Problem**

- Formal statement of the problem
- Types of maxima, the Weierstrass Theorem and the Local-Global theorem
- Geometry of the problem

#### **Classical Programming**

- The unconstrained case
- The method of Lagrange multipliers
- The interpretation of the Lagrange multipliers

#### **Non-linear Programming**

- The case of no inequality constraints
- The Kuhn-Tucker conditions
- The Kuhn-Tucker theorem
- The interpretation of the Lagrange multipliers



- Solution algorithms

### **Linear Programming**

- The Dual problems of linear programming
- The Lagrangian approach; Existence, Duality and complementary slackness theorems
- The interpretation of the dual
- The simplex algorithm

## **Section-II (4/9)**

### **The Control Problem**

- Formal statement of the problem
- Some special cases
- Types of Control
- The Control problem as one of programming in on infinite dimensional space; The generalized Weierstrass theorem

### **Calculus of Variations**

- Euler equations
- Necessary conditions
- Transversality condition
- Constraints

### **Dynamic Programming**

- The principle of optimality and Bellman's equation
- Dynamic programming and the calculus of variations
- Dynamic programming solution of multistage optimization problems

### **Maximum Principle**

- Co-state variables, The Hamiltonian and the maximum principle
- The interpretation of the co-state variables
- The maximum principle and the calculus of variations
- The maximum principle and dynamic programming
- Examples

### **Recommended Books**

1. M.D. Intriligator, *A Mathematical Optimization and Economic Theory* (Prentice Hall, 1989)
2. B.S. Gottfried & W. Joel, *Introduction to Optimization Theory*, (Prentice Hall, 1973)
3. R.K. Sudaram, *A First Course in Optimization Theory*, (Cambridge University Press, 1996)
4. S. S. Rao, *Optimization: Theory and Application*, (John Wiley and Sons Ltd, 1978)
5. M. J. Fryer and J. V. Greenman, *Optimization Theory: Applications in Operation Research and Economics*, (Butterworth-Heinemann Ltd, 1987)
6. K. V. Mital and C. Mohan, *Optimization Methods in Operation Research and Systems Analysis*, (New Age Publications, 2005)

