Dimensional Expression for Heat Transfer Coefficient:

The heat transfer coefficient, *h*, from a hot closed conduit (pipe) to the surrounding fluid has been found to be influenced by velocity of fluid, a linear dimension of the surface (pipe diameter), viscosity of fluid, specific heat capacity of fluid, density of fluid, thermal conductivity of fluid, temperature difference existing between fluid and wall of the conduit, coefficient of thermal expansion of fluid, and acceleration due to gravity.

Variable	Symbol	Dimension
Heat transfer coefficient	$h = \frac{q}{A \cdot \Delta T}$	$\frac{\mathrm{H}}{\theta \cdot \mathrm{L}^2 \cdot \mathrm{T}}$
Velocity of fluid	u	$\frac{\mathrm{L}}{\theta}$
Linear dimension (Characteristic length)	l	L
Viscosity of fluid	μ	$\frac{\mathrm{M}}{\mathrm{L} \cdot \theta}$
Specific heat capacity of fluid	c_p	$\frac{\mathrm{H}}{\mathrm{M} \cdot \mathrm{T}}$
Density of fluid	ρ	$\frac{\mathrm{M}}{\mathrm{L}^3}$
Thermal conductivity of fluid	k	$\frac{\mathrm{H}}{\theta \cdot \mathrm{L} \cdot \mathrm{T}}$
Temperature difference between fluid and wall of the conduit	ΔT	T
Coefficient of thermal expansion of fluid	β	$\frac{1}{\mathrm{T}}$
Acceleration due to gravity	g	$\frac{\mathrm{L}}{\theta^2}$
Dimensionless constant used when mechanical energy is converted into heat or vice-versa	J	$\frac{\mathrm{M} \cdot \mathrm{L}^2}{\mathrm{H} \cdot \theta^2}$

No. of variables = *m* = 11

However for the convenience, ΔT, β, and g can be combined in one variable, therefore

$$\Delta T \cdot \beta \cdot g = \mathrm{T} \cdot \frac{1}{\mathrm{T}} \cdot \frac{\mathrm{L}}{\theta^2} = \frac{\mathrm{L}}{\theta^2}$$

No. of variables are now = m = 9

No. of primary dimensions = r = 5 (i.e. H, θ, T, M, and L)

At the end of analysis, no. of dimensionless groups = 9 – 5 = 4

No. of unrestricted exponents = $m - r - 1 = 9 - 5 - 1 = 3$

The basic equation relating the variables is

$$h = K \cdot (u)^a \cdot (l)^b \cdot (\mu)^c \cdot (\rho)^d \cdot (k)^e \cdot (c_p)^i \cdot (\Delta T \cdot \beta \cdot g)^m \cdot (J)^n \qquad \text{(I)}$$

Where, K is a dimensionless constant.

The corresponding dimensionless equation is

$$\frac{\mathrm{H}}{\theta \cdot \mathrm{L}^2 \cdot \mathrm{T}} = \left(\frac{\mathrm{L}}{\theta}\right)^a \cdot (\mathrm{L})^b \cdot \left(\frac{\mathrm{M}}{\mathrm{L} \cdot \theta}\right)^c \cdot \left(\frac{\mathrm{M}}{\mathrm{L}^3}\right)^d \cdot \left(\frac{\mathrm{H}}{\theta \cdot \mathrm{L} \cdot \mathrm{T}}\right)^e \cdot \left(\frac{\mathrm{H}}{\mathrm{M} \cdot \mathrm{T}}\right)^i \cdot \left(\frac{\mathrm{L}}{\theta^2}\right)^m \cdot \left(\frac{\mathrm{M} \cdot \mathrm{L}^2}{\theta^2 \cdot \mathrm{H}}\right)^n \qquad \text{(II)}$$

By applying condition of dimensional homogeneity, we can show that

$$\sum \mathrm{H} = 0 \Rightarrow 1 = e + i - n \qquad (1)$$

$$\sum \mathrm{L} = 0 \Rightarrow -2 = a + b - c - 3d - e + m + 2n \qquad (2)$$

$$\sum \theta = 0 \Rightarrow -1 = -a - c - e - 2m - 2n \qquad (3)$$

$$\sum \mathrm{T} = 0 \Rightarrow -1 = -e - i \qquad (4)$$

$$\sum \mathrm{M} = 0 \Rightarrow 0 = c + d - i + n \qquad (5)$$

Let a, i, and m as the un-restricted exponent constants.

From Eq. 4: $e + i = 1$ (6)

Using Eq. 6 with Eq. 1, it is shown that

$1 = n - 1$ or $n = 0$ (showing J = 1)
From Eq. 1: $e = 1 - i$ (7)

From Eq. 3: $c = 1 - a - e - 2m$

$$= 1 - a - 1 + i - 2m$$

$$= -a + i - 2m \tag{8}$$

From Eq. 5: $d = -c + i$

$$= a - i + 2m + i$$

$$= a + 2m$$

From Eq. 2: $b = -2 - a + c + 3d + e - m$

$$= -2 - a - a + i - 2m + 3a = 6m + 1 - i - m$$

$$= a + 3m - 1$$

Inserting exponents' relations in Eq. I, it may be shown that

$$h = K \cdot (u)^a \cdot (l)^{a+3m-1} \cdot (\mu)^{-a+i-2m} \cdot (\rho)^{a+2m} \cdot (k)^{1-i} \cdot (c_p)^i \cdot (\Delta T \cdot \beta \cdot g)^m \cdot (J)^0 \tag{9}$$

or

$$\frac{h \cdot l}{k} = K \cdot \left(\frac{l \cdot u \cdot \rho}{\mu}\right)^a \cdot \left(\frac{c_p \cdot \mu}{k}\right)^i \cdot \left(\frac{\beta \cdot g \cdot \Delta T \cdot l^3 \cdot \rho^2}{\mu^2}\right)^m \tag{10}$$

$$\frac{h \cdot l}{k} = \text{Nusselt number} = Nu \tag{11}$$

$$\frac{c_p \cdot \mu}{k} = \text{Prandtl number} = Pr \tag{12}$$

$$\frac{l \cdot u \cdot \rho}{\mu} = \text{ Reynolds number} = Re \tag{13}$$

$$\frac{\beta \cdot g \cdot \Delta T \cdot l^3 \cdot \rho^2}{\mu^2} = \text{Grashof number for heat transfer} = Gr \tag{14}$$

Therefore, Eq. 10 may also be written as

$$Nu = K \cdot (Re)^a \cdot (Pr)^i \cdot (Gr)^m \tag{15}$$

i.e. *Nu* is a function of *Re*, *Pr*, and *Gr* or

$$Nu = f(Re, Pr, Gr) \tag{16}$$

For free convection, the heat transfer rate is dependent predominately on the buoyancy effects, represented by Grashof number (*Gr*) and Reynolds number (*Re*) is omitted. Therefore for free convection heat transfer, we may write

$$Nu = f(Pr, Gr) \tag{17}$$

i.e. Nusselt number is a function of Grashof number and Prandtl number.

For forced convection, the effect of natural convection is considered negligible and Grashof number (*Gr*) may be omitted, so forced convection heat transfer, we may write

$$Nu = f(Re, Pr) \tag{18}$$

i.e. Nusselt number is a function of Reynolds number and Prandtl number.

Group	Symbol	Proportional to the ratio	Formula	Applications
Nusselt number	Nu	$\frac{heat\,transfer\,by\,convection}{heat\,trasnfer\,by\,conduction}$ or $\frac{conductive\,thermal\,resistance}{convective\,thermal\,resistance}$ Note: Convective heat transfer means total heat transfer by both molecular and bulk flow mechanisms	$\frac{h \cdot l}{k}$	Heat transfer in fluids in motion
Reynolds number	Re	$\frac{inertial\ forces}{viscous\ forces}$ $\frac{total\,momentum\,transfer}{viscous\,momentum\,transfer}$	$\frac{l \cdot u \cdot \rho}{\mu}$	Criteria for type of flow behavior Forced convection heat and mass transfer
Prandtl number	Pr	$\frac{molecular\,diffusivity\ of\ momentum}{molecular\,diffusvity\,of\ heat}$ Note: Molecular diffusivity of momentum is also called kinematic viscosity	$\frac{\nu}{\alpha} = \frac{\mu / \rho}{k/(\rho \cdot c_p)}$ $= \frac{c_p \cdot \mu}{k}$	Simultaneous heat and momentum transfer
Grashof number	Gr	$\frac{bouyant\ forces}{viscous\ forces}$	$\frac{\beta \cdot g \cdot \Delta T \cdot l^3 \cdot \rho^2}{\mu^2}$	Free convection heat and mass transfer

Group	Symbol	Definition	Formula	Applications
Prandtl number	Pr	$\frac{molecular\,diffusivity\ of\ momentum}{molecular\,diffusivity\ of\ heat}$ Note: Molecular diffusivity of momentum is also called as kinematic viscosity and molecular diffusivity of heat is also called as thermal diffusivity.	$\frac{\nu}{\alpha} = \frac{\mu / \rho}{k/(\rho \cdot c_p)}$ $= \frac{c_p \cdot \mu}{k}$	Simultaneous heat and momentum transfer
Schmidt number	Sc	$\frac{molecular\,diffusivity\ of\ momentum}{molecular\,diffusivity\ of\ mass}$ Note: Molecular diffusivity of mass is also called as mass diffusivity or diffusion coefficient.	$\frac{\nu}{D_{ij}} = \frac{\mu / \rho}{D_{ij}}$ $= \frac{\mu}{\rho \cdot D_{ij}}$	Simultaneous mass and momentum transfer
Lewis number	Le	$\frac{molecular\,diffusivity\ of\ heat}{molecular\,diffusivity\ of\ mass}$	$\frac{\alpha}{D_{ij}} = \frac{k/(\rho \cdot c_p)}{D_{ij}}$ $= \frac{k}{\rho \cdot c_p \cdot D_{ij}}$	Simultaneous heat and mass transfer