

**Numerical Problems related to LMTD** (errors and omissions are expected)

**Logarithmic mean temperature difference (LMTD):**

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

$$\Delta T_1 = T_{h1} - T_{h2}$$

$$\Delta T_2 = T_{h2} - T_{c2}$$

**Arithmetic mean temperature difference:**

$$\Delta T_a = \frac{\Delta T_1 + \Delta T_2}{2}$$

$$\Delta T_1 = T_{h1} - T_{h2}$$

$$\Delta T_2 = T_{h2} - T_{c2}$$

**Energy balances:**

For hot fluid:  $q_h = G_h \times c_{ph} \times \Delta T_h$

For cold fluid:  $q_c = G_c \times c_{pc} \times \Delta T_c$

**Heat exchanger design equation:**

$$q = U_o \cdot A_o \cdot \Delta T_{lm}$$

**Equality of heat energies:**

$$q = q_h = q_c$$

**Problem1 [3, p. 90]** A hot fluid enters a concentric pipe apparatus at a temperature of 300 °F and is to be cooled to 200 °F by a cold fluid entering at 100 °F and heated to 150 °F. Shall they be directed in parallel flow or counterflow?

Hint: Calculate LMTD for both cocurrent and countercurrent flow cases. The case with higher LMTD value will exchange more heat.

Remember cocurrent flow is also called parallel flow.

**Problem 2 [5, p. 91]:** A hot fluid enters a concentric pipe apparatus at 300 °F and is to be cooled to 200 °F by a cold fluid entering at 150 °F and heated to 200 °F. Calculate LMTD for both countercurrent and cocurrent flow processes.

**Problem 3 [5, p. 91]** While a hot fluid is cooled from 300 °F to 200 °F in counterflow, a cold fluid is heated from 100 °F to 275 °F. Calculate the LMTD. What about LMTD for cocurrent flow process?

**Problem 4 [5, p. 92]:** Calculate the LMTD for both cocurrent and countercurrent when cold fluid is heated from 100 °F to 275 °F by steam at 300 °F.

**Problem 5 [5, p. 100]:** Calculate the LMTD for countercurrent flow in the following cases in which the hot fluid is cooled from 200 to 100 °F and the cold fluid is heated from a) 90 to 140 °F; b) 80 to 130 °F, and c) 60 to 110 °F. Observe the nature of the deviation of the LMTD from the arithmetic mean of the two terminal differences in each case.

**Problem 5 [ p. 343]:** A heat exchanger is required to cool 20 kg/s of water from 360 K to 340 K by means of 25 kg/s water entering at 300 K. If the overall heat transfer coefficient is constant at 2.0 kW/m<sup>2</sup>·K, calculate the surface area required in

- a) a countercurrent concentric tube exchanger, and
- b) a cocurrent flow concentric tube exchanger.

Use specific heat capacity of water as 4.18 kJ/kg·K.

**Note that there is no degree sign (°) with K.**

**Problem 6:** 9000 lb/h of hot aniline is to be cooled from 200 to 150 °F in a double pipe heat exchanger by 8000 lb/h of toluene. The inlet temperature of toluene is 100 °F and the heat transfer area of the exchanger is 50 ft<sup>2</sup>. For countercurrent flow, what are LMTD and the overall heat transfer coefficient? Use specific heat capacities of toluene and aniline as 0.45 Btu/lb·°F and 0.51 Btu/lb·°F, respectively.

Hint: Trial-and-error solution.

**Problem 7 [5, p. 100]:** Aniline is to be cooled from 200 to 150 °F in a concentric pipe apparatus having 70 ft<sup>2</sup> of external pipe surface by 8600 lb/h of the toluene entering at 100 °F. A value of  $U = 75 \text{ BTU/h}\cdot\text{ft}^2\cdot\text{°F}$  may be anticipated. How much hot aniline can be cooled in counterflow? Use specific heat capacities of toluene and aniline as 0.45 Btu/lb·°F and 0.51 Btu/lb·°F, respectively.

Hint: Trial-and-error solution.

**Problem 8 [5, p. 100]:** Benzene is to be heated in a concentric pipe apparatus having a  $1\frac{1}{4}$  in (Nominal Pipe Size) inner pipe from 100 to 140 °F by 8000 lb/h of nitrobenzene

having an initial temperature of 180 °F. A value of  $U = 100 \text{ Btu/lb}\cdot\text{°F}\cdot\text{ft}^2$  may be expected based on the outside surface of the pipe. How much cold benzene can be heated in 160 linear ft of concentric pipe in a countercurrent flow process? Take mean specific heat capacities as 0.43 and 0.39 Btu/lb·°F, respectively.

Hint: Use Trial-and-error solution.

For outer diameter of the inner pipe to calculate the total heat transfer surface area, See Dimensions of Standard Steel Pipe on p. 996 of the text [1].

**Problem 9:** A double pipe heat exchanger is available to cool 0.40 kg/s of dilute aqueous solution from 70 °C to 50 °C. Cooling water at the rate of 0.25 kg/s is at 25 °C will be used to perform the heat duty. If the overall heat transfer coefficient is 1500 W/m<sup>2</sup>·K and the inner tube outside diameter is 0.025 m, calculate the length of the tube required:

- (a) for a countercurrent operation
- (b) for cocurrent operation

Hint: For the dilute aqueous solution use specific heat capacity as that of water.

**Problem 10:** 100 g/s of hot oil enters the inner pipe (3/4 in nominal size with Schedule no. of 80) of a double pipe cooler at 400 K and it is to be cooled to 310 K. An equal amount of cooling water available at 293 K is flowing countercurrent to oil flow through the annular space formed between the inner and outer pipes. If the inside and outside heat transfer coefficients are 1.5 and 4.0 kW/m<sup>2</sup>·K, respectively what length of the heat transfer surface is required? Take specific heat capacity of oil and water as 1.5 and 4.2 kJ/kg·K, respectively and thermal conductivity of pipe wall as 45 W/m·K.

**Problem 11:** A multitube hair-pin exchanger (a special type of double pipe) as shown in the figure below contains 60 number of tubes. Each tube is of 5/8 in outside diameter and 14 BWG (Birmingham wire gauge). An aqueous solution (mean specific heat capacity as 4.30 kJ/kg·K) to be cooled enters the shell side of the exchanger at the rate of 8000 kg/s. The inlet and outlet temperatures of the aqueous solution are 150 °C and 50 °C. The tube side coolant enters at 20 °C and leaves at 60 °C. The inside and outside heat transfer coefficients are 2000 and 5000 W/m<sup>2</sup>·K, respectively. Neglecting the resistance of the wall of the exchanger, find out the length of each tube.

Hint: Unlike pipes, tubes are specified on the basis of outside diameter and BWG (Birmingham wire gauge). Although not required for this question, but gauge determines the thickness of the tube wall and specifies the internal diameter of a tube. Higher gauge means less thick wall and vice-versa. Please see p. 997 of the text [1].

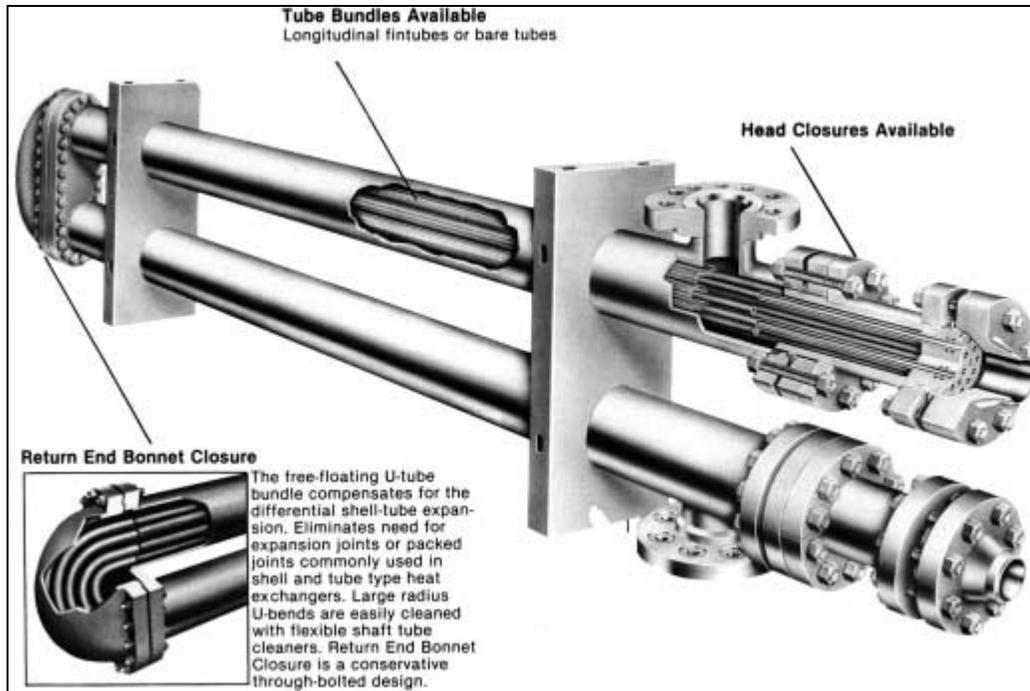


Figure 1: Multitube hair-pin heat exchanger [Taken from Ludwig, E.E. (2001). Applied process design: for chemical and petrochemical plants. vol. 3. 3<sup>rd</sup>.ed. Gulf Professional Publishing, Boston. p. 10].

If sp. heat capacities of gases or liquids are not given in the examination, you may use nomographs given in the appendix of the text [1].

**Reference:**

- [1] Geankoplis, C.J. (2003). Transport processes and separation process principles: includes unit operations. 4th ed. Prentice-Hall International, Inc.