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Effect of Metacognition on Mathematical Deductive Reasoning among Secondary School Students

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Abstract

The objective of this study was to examine the effect of metacognition on mathematical deductive reasoning among secondary school students. Participants of the study were the grade 9 students of a government school in Lahore. The nature of the study was experimental, and a quasi-experimental design with pre-test, post-test, non-equivalent control group was adopted. A valid and reliable mathematical reasoning test was developed. After validation by the experts, the test was piloted on 600 students. There were 37 grade 9 students in the control group and 34 students in the experimental group. Independent samples t-test was applied to assess the effect of metacognition on mathematical deductive reasoning by comparing the mean score of both groups. A significant difference between mean scores of the experimental and control group in post-test was found. Participants of the experimental group got a higher score than the control group participants' score. The findings of the study did not support the null hypothesis. Therefore, it was rejected. It was concluded that metacognition has significant effect on mathematical deductive reasoning. It was recommended for secondary school teachers and students to use metacognitive strategies in the teaching-learning process of mathematics.

Keywords: Metacognition, Deductive reasoning, Mathematics, Secondary school students

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Introduction

Metacognition has gained much importance in mathematics education research and practices (Kramarski, 2017; Kuet al., 2010). It is an ability to know about knowing. It can also be seen as the human mind's ability to monitor and control (Dunlosky & Jacoby, 2011). The metacognition research supports the effect of metacognition on the construction of new knowledge. Metacognitive strategies facilitates secondary school students for the development of mathematical reasoning (Ball & Bass, 2003; Herbst, 2014; Mevarech & Fridkin, 2006).

Mathematical reasoning is a critical skill that enables students to use other mathematical skills. It enables students to analyze the mathematical situation and construct logical arguments. Mathematical deductive reasoning is an important aspect of mathematical reasoning. It is the process of reaching a specific conclusion from unknown mathematical rules (Harel & Weber, 2020; Rohana, 2015). The development of mathematical deductive reasoning ensures students' better academic performance in mathematics and other subjects (Briggs, 2014). Rote memorization is not useful for developing mathematical deductive reasoning among students. Mathematical deductive reasoning can be developed by effective teaching and learning strategies like metacognition (Ponte & Quaresma, 2016).

National Council of Teachers of Mathematics (2014) emphasized that secondary school mathematics curriculum should include diverse experiences to develop mathematical deductive reasoning. There is a dire need to shift mathematics instruction from rote memorization to conceptual understanding to develop mathematical deductive reasoning. The government of Pakistan also stresses developing mathematical deductive reasoning among secondary school students (Government of Pakistan, 2017). Therefore, the researcher intended to conduct this study, and the objective of the study was to examine the effect of metacognition on mathematical deductive reasoning among secondary school students. A null hypothesis that there is no significant mean difference in the scores of mathematical deductive reasoning between the control and the experimental groups. was framed.

Literature Review

Metacognition is often referred to as thinking about thinking (Kramarski, 2008; Polya, 2007). It is a regulatory system that helps people understand and control their cognitive performance (Bray & Schatz, 2013; Brehmer et al., 2016). Metacognitive strategy is one of the instructional interventions that can be explained as memorable plans or approaches the students to use to solve mathematical problems (Sperlinget al., 2002). The literature reveals that think aloud, planning, monitoring, and evaluation are effective metacognitive strategies (Lan, 2005; Schneider, 2008).

Think aloud is a brainstorming strategy, which enables students to plan, monitor and evaluate their own learning process (Tok, 2013). The teacher models how to use the think-aloud strategy and verbalizes his/her thought processes while solving the mathematical problems. After that, students solve some mathematics problems by using a think-aloud strategy (Gan & Hong, 2010; Swanson et al., 2014). Students become more skilled by using metacognitive strategies. Moreover, they gain confidence and become more independent learners (Kotsopoulos, 2010).

Planning, monitoring, and evaluation are useful metacognitive strategies that help students become reasoned persons. Self-planning helps students prepare initial activities based on their strategic ideas (Haji & Ilham, 2015; Saldana, 2015; Zakaria et al., 2010). Self-monitoring enables students to regulate their cognitive processes to solve mathematical problems. It also allows students to solve mathematical problems independently and control their thought processes (Hudesman et al., 2013). Self-evaluation allows students to have mathematical sense and justify mathematical problems' solutions (Clements & Sarma, 2011; Ponte & Quaresma, 2016).

In planning strategy, an individual plans how to accomplish the task. In monitoring strategy, the individual checks the progress on the tasks, and finally, the effectiveness of the strategies is evaluated (Harris & Brown, 2013; Kani & Shahril, 2015; Smithson, 2012). These metacognitive strategies enrich students with higher-order thinking skills. In higher-order thinking skills, mathematical reasoning is an important skill. Mathematical reasoning is a process based on assumed mathematical premises (Napitupulu et al., 2016; Yazici, 2014). It enables students to understand and make logical sense of mathematical concepts to conclude (Adams, 2007; Brodie, 2010).

There are two structural aspects of mathematical reasoning; deductive reasoning and inductive reasoning. Mathematical deductive reasoning includes the chains of the statements connected logically (Cowan, 2014). It is included in mathematical proofs. Therefore, it is important for learning of mathematics (Ayalon & Even, 2008; Kilpatrick, 2009; Sidenvall et al., 2015). Mathematical deductive reasoning is used to validate a conjecture or hypotheses (Kuhn, 2013). In mathematical deductive process, someone arrives at a specific result from general principles. In the present study, mathematical deductive reasoning is taken as the process of using general mathematical facts to valid less general or specific mathematical facts

In the 19th century, mathematics teachers and researchers believed that there were general deduction rules in mathematics, which formal teaching strategies can teach. Piaget and other researchers claimed that mathematical deductive reasoning is developed naturally, and traditional teaching has no significant influence on its development (Inhelder & Piaget, 1958). In contrast to Piaget, other researchers like Vygotsky (1962) claimed that deductive reasoning in mathematics does not occur naturally; teaching is a necessary tool for its development (Karpov, 2013; Vygotsky, 1962; Stylinanides & Stylianides, 2008).

A famous American mathematics educator Fawcett (1938), experimented with developing mathematical deductive reasoning among school students. The experiment lasted for two years. At the end of the experiment, Fawcett observed that mathematical deductive reasoning had developed among students. Moreover, students and their parents also claimed that mathematical deductive reasoning was developed due to the intervention of experimental study (Heit & Rotello, 2010; Gonzalez & Herbst, 2006; Nisbett, 2009).

The application of metacognitive strategies effectively enhances mathematical deductive reasoning regardless of classroom organization (Biryukov, 2004; Cowan, 2014; Flavell, 1979; Schoenfeld, 2007). The level of mathematical deductive reasoning of the students exposed to the metacognition strategies becomes high compared to students who are not exposed to metacognitive strategies (Churchet al., 2013; Mevarech & Amrany, 2008; Pillow & Pearson, 2012). Kramarski and Mevarech (2003) conducted a study to assess the effect of metacognition on the development of mathematical deductive reasoning among elementary school students. They found that the students who used metacognitive strategies showed a better level of mathematical deductive reasoning than those who did not use metacognitive strategies.

Theoretical Framework

The study got insight from the theory of constructivism. Constructivism supports that students construct their own knowledge based on four tenets (Pirie, 1992; Rsmussen & Marrongelle, 2006; Simmons, 1999). According to the first tenet, all the knowledge is constructed with the help of previous knowledge. According to the second tenet, knowledge is socially constructed. The third tenet focuses on a dialogue-based social learning atmosphere for knowledge construction. The fourth and the last tenet focus on contextual environment for knowledge construction (Dewi & Harap, 2016; George, 1995; Prawat, 1992; Thompson, 2013).

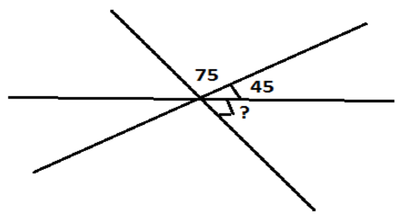
Theory of Constructivism supports the development of mathematical deductive reasoning among secondary school students through metacognition (Kramarski & Mevarech, 2003; Safitri & Arnawa, 2019). The present study was conducted in social constructive classroom settings, and participants of the experimental group were taught through metacognitive strategies. Participants were divided into small groups and were required to solve mathematical problems in collaboration. Participants of the experimental group shared their ideas with their peer students by using a think-aloud strategy. Moreover, they were assigned creative activities of solving mathematical problems in small groups.

Methodology and Design of the Study

The study was quasi-experimental, with the pretest-posttest nonequivalent control group design. The study participants were grade 9 students of the science and computer science groups, studying mathematics as a compulsory subject in a government school of Lahore city. The researcher used a quasi-experimental design because randomization was not allowed in regular classroom settings. Two intact groups of grade 9 students were selected as experimental and control group. The intact groups of students were randomly assigned as a control group and experimental group. There were 37 students in the control group and 34 students in the experimental group. All the control group and experimental group participants were of the same characteristics.

The researcher developed a mathematical reasoning test (MRT) in the extended form of multiple choice questions. A table of the specification was prepared in which two questions were selected from arithmetic, two questions from algebra, and two questions from the geometry portion. Mathematics content is mainly divided into three portions i.e., arithmetic, algebra, and geometry (Mustafa, 2011; Sidhu, 2018). Therefore, it was decided to select the two questions from each portion. The test was sent to the experts for validation. The experts were the teachers of secondary school mathematics and the research experts in mathematics education. The experts suggested altering two test items. The test was improved in light of experts' comments. A sample question of the test is as:

Question: Find the value of unknown angle



After validation by the experts, the test was piloted on 600 students. Item analysis was done to check test items' difficulty level and discrimination power. The item difficulty level was found between .38 and .43. All values were as .42, .39, .43, 48, .41 and .47. The values of all items showed their acceptance in a large-scale study. The discrimination power of all items were found as; .42, .48, .47, .51, 54 and .51. The individual item reliability for each item was calculated statistically. The individual item reliability was found as .89, .88, .89, 89, .87, .88 and .88. The value of Cronbach's Alpha coefficient was 0.89. The validated and reliable test was applied in pretest and post-test to assess mathematical deductive reasoning among secondary school students.

The researcher administered a pre-test in both experimental and control groups before starting the intervention. The researcher taught mathematics to the experimental group through metacognition strategies, prepared as an intervention of the study. While for the control group, the researcher used traditional mathematics teaching strategies. The intervention lasted for 16 weeks. Think aloud, planning, monitoring, and evaluation strategies were used as an intervention. The researcher first modeled these strategies in teaching mathematics and then guided the experimental group participants to solve mathematical problems. The researcher tried to control the threats to internal and external validity. Since intact groups were selected for this quasi-experimental study. Therefore, the threats of reactive arrangement were controlled.

Participants of intact groups did not know the purpose of their selection for the experiment. The study was conducted in regular school settings, where students of 9th class were required to attend the classes regularly. Therefore, the threat of mortality was also controlled. Participants of a control and experimental groups were of the same characteristics; therefore, the threat of regression was also controlled. Moreover, the results of data analysis of the pretest also confirmed that participants of a control group and experimental group were at a same academic level. After completing the intervention, a post-test was held for participants of a control group and an experimental group. For the scoring of MRT, a scoring rubric was developed. For giving the correct reasons of an option, participants were awarded 2 marks, and for no or false reasons, 0 mark was awarded. Each test item was awarded a maximum of 8 marks. The data collected through pretest and post-test were analyzed statistically.

Findings

Independent samples-test was applied on pre-test and post-test scores of a control group and experimental group. The purpose of using the test was to assess the effect of metacognition on mathematical deductive reasoning by comparing the mean score of a control group and experimental groups. Independent samples t-test is used to compare the means of two groups if these groups are independent of each other (Rovai et al., 2014).

Table 1
Independent Samples t-test for Comparison of Average Pre-test Score of Control Group and Experimental Group

Variable	Group	*N*	*M*	*df*	*SD*	*t*	*P(2-tailed)*
Deductive	Control	37	3.28	69	1.02	-.41	.666
Reasoning	Experimental	34	3.37		1.91		

Table 1 shows the comparison of the average score of the control and experimental groups in the pre-test. The score of control group (*M*= 3.28, *SD*= 1.02) and score of experimental group (*M*= 3.37, *SD*= 1.91) for *t*(69)= -.41 and *P*= .666 (2-tailed) indicated that there was no significant difference between average score of control group and experimental group. It was found that the scores of participants of a control group and experimental group before starting the intervention were nearly equal to each other. No group performed significantly better than the other in a pre-test.

Table 2
Independent Samples t-test for Comparison of Average Post-test Score of Control Group and Experimental Group

Variable	Group	*N*	*M*	*df*	*SD*	*t*	*P(2-tailed)*
Deductive	Control	37	4.86	69	.56	-23.89	.000
Reasoning	Experimental	34	7.59		.38		

Table 2 shows the comparison between the average score of a control group and an experimental group in post-test. The score of control group (*M*= 4.86, *SD*= .56) and score of experimental group (*M*= 7.59, *SD*= .38) for *t*(-23.89) and *P*= .000 (2-tailed) indicated a significant difference between average score of control group and experimental group. Participants of the experimental got more scores than the score of the control group participants. The study's findings did not support the null hypothesis; therefore, the null hypothesis was rejected. There was a significant positive effect of metacognition on mathematical deductive reasoning among secondary school students.

Discussion

The study revealed a positive effect of metacognitive strategies on the development of mathematical deductive reasoning among secondary school students. The study confirmed that the students exposed to metacognitive training attained a higher level of mathematical deductive reasoning. The present study has confirmed that students have been provided with the opportunities of metacognitive strategies in a collaborative setting. They performed better and show a higher level of mathematical deductive reasoning. Kramarski and Mevarch (2003) conducted a study to find the effect of metacognitive training on mathematical deductive reasoning among elementary school students. They found that the students exposed to metacognitive strategies in collaborative settings show a higher level of mathematical deductive reasoning. The study also confirmed the findings of Lestari and Jailani (2018).

Metacognitive strategies were used in collaborative learning settings. In collaborative settings, students were involved in creative activities. They shared their mathematical ideas with each other. In this way, they came to comprehend mathematical concepts. This comprehension helped them to be reasoned. After the discussion, it is concluded that the present study has confirmed the findings of previous studies. Most of the previous studies have confirmed the theory of the development of mathematical deductive reasoning among secondary school students through metacognition. The present study has also confirmed the theory of the development of mathematical deductive reasoning among secondary school students through metacognition in the Pakistani context.

Conclusions

It is concluded from the study findings that metacognitive strategies positively affected mathematical deductive reasoning among secondary school students. The study provides insights into how mathematical deductive reasoning is enhanced in collaborative settings. The development of mathematical deductive reasoning is caused due to the intervention in the form of metacognitive strategies given by the researcher. The intervention used in the present study constituted an environment in which students got opportunities to share their mathematical ideas with their peer students and teachers. Moreover, the students used self-planning, self-monitoring, and self-valuation to solve mathematical problems. These activities enhanced the ability of mathematical deductive reasoning among secondary school students.

Recommendations

- Since metacognitive strategies enable students to be critical thinkers and reasoned persons, it seems strong to say that concerned authorities may include the content in the mathematical curriculum that supports students in acquiring mathematical deductive reasoning.
- Teachers play an important role in enhancing secondary school students' mathematical deductive reasoning. Therefore, it is recommended that secondary school teachers may adopt metacognitive teaching strategies instead of traditional teaching strategies.
- Students learn better in a collaborative learning atmosphere as they are involved in creative activities. They share their mathematical ideas by using metacognitive strategies in collaborative settings. In this way, they come to comprehend mathematical concepts. The comprehension of mathematical concepts helps the students to be reasoned. Therefore, competitive classroom settings may be changed into collaborative settings.

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