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Learning Mathematics in English Medium Classrooms in Pakistan: Implications for Policy and Practice

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Abstract

This is an in-depth qualitative study of students learning mathematics in classrooms where the medium of instruction was English which was the second or third language of the learners i.e. they were multilingual classrooms. Moreover, to promote reasoning and learning with understanding, the teachers had introduced group work, mathematical discussion, and mathematics tasks employing everyday contexts and language.

Focus of the paper is the complex relationship between the language of instruction and the process of learning mathematics. The study demonstrates that learning mathematics in multilingual classrooms brings added complexity because students moved back and forth from the language of instruction to their own language. This movement involved a demonstrated need on part of the learners to understand the language structures, grammar and vocabulary of the language of instruction, and it involved translation which is a nuanced and complex process. This in turn led to further issues for mathematics learning because students may or may not translate appropriately. Additionally, the discourse of mathematics classrooms which promote mathematical discussion and reasoning, and where the mathematics texts are encoded in the "everyday use phrases" in the language of instruction, required students to understand the mathematics intent embedded in the "every day use phrases" and at the same time understand the language of instruction. .

The current wave of Education reform in Pakistan has led to a review of the national curriculum, a new scheme of studies and new policies in language of instruction in mathematics. This paper raises significant issues for policy and practice of mathematics teaching in the current reform and policy renewal context in Pakistan.

Introduction

Pakistan is a linguistically diverse country with over 300 dialects and approximately 57 languages spoken throughout the country's four major provinces (Khan, 2002). While Urdu is the country's national language it is the primary language of less than 10 percent of the population (Laporte, 1998). English is the preferred language of education and is most often recognized as the language of the elite and the ruling class. Pakistani schools that use English as the medium of instruction are called English medium schools. They are found in both urban and rural areas. Students in Pakistan's

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English medium schools learn their subject matter content and the English language simultaneously and are expected to become proficient in both. According to Khan (2002) and Haque (1993), almost all Pakistanis prefer to study in English medium schools because it is seen as a language that opens the doors to professional and academic opportunities.

In English medium mathematics classrooms, such as the one reported here, the process of learning becomes more complex because the socio-cultural tools that learners use often includes their first language which is different from their language of instruction. For example, students moved from the language of instruction to their own language in the course of mathematics learning. This movement across languages has been the focus of many studies mainly in the field of linguistics and socio-linguistics and is often referred to as code-switching (Boztepe, 2003). While the term code-switching itself has been used in multifarious ways, for my purposes I take it as an overarching term which covers the phenomena of moving between two languages. Adler's (2001) seminal work in multilingual mathematics classrooms has looked at dilemmas emerging from code switching but these are dilemmas for teachers and teaching. In subsequent work, Setati and Adler (2001) state that, "Code switching is a practice that enables learners to harness their main language as a learning resource. As a mechanism for learning and access, code switching has almost become a taken for granted "good thing". However, this study suggests that this phenomenon of code switching as a "good thing" needs to be problematised because most research into bilingual classrooms has been from the perspective of teachers. There is very little research that looks at students' experience of multilingual classrooms. Hence, there is a need to understand *why* students move across languages and what are the issues involved in it. Clarkson (2002) raises a similar question in a study of bi-lingual children learning mathematics, "Why did they swap languages when doing mathematics" (p. 2). Cleghorn, Mtetwa, Dube, & Munesti (1998) identify a similar question for further research, "Under what circumstance is language switching the best strategy to foster understanding of the topic being taught? (p.474). In these questions there is a recognition that 'language switching' and 'language swapping' is something that occurs in multilingual classrooms and could be a potential key to understanding the process of learning in these classrooms. But, as Sierpinska (2002) says, research in multilingual classrooms provides little if any understanding of students' experience of learning. Even in studies where students appear in the discussion they are not the main focus of the research (e.g. Gorgorio & Planas, 2001). Hence, insight into students' side of the story does not develop.

Furthermore, what appears to be missing from these deliberations emerging from multilingual classrooms is recognition of the significant role that translation plays in the course of moving from one language to another. Translation raises the issue of knowing and/or finding word and phrases that communicate the essence of the meaning of what is being translated. Issues of accurate, let alone nuanced translation are legion (Strauss & Corbin, 1998). I would argue that translation requires a careful selection of words and phrases that express the appropriate meaning conveyed in the original text. Switching to own language would be a resource for learners if language was translated appropriately to the discourse of mathematics classrooms.

Context and Setting

The study was based in two classrooms (henceforth, classroom A and B) in Karachi, Pakistan. The fieldwork spread over the course of one academic year during which two small groups of students were observed, one in each class. One, was a group of three boys from class seven, (age 11-12 years), namely Faizullah, Mansoor and Saleem. The other was a group of four girls, Maheen Naima, Samina and Shabnum, and was from class six (10-11 years). In addition to the observations there were stimulated recall interviews with students and post-observation conversations with the teachers. Data was analysed using the grounded theory procedures (Strauss & Corbin, 1998). This involved reading and re-reading data and coding it line by line. The codes emerged from the ongoing analysis, and were clustered together in broad categories. Moreover, my emphasis on meanings emerging from my interaction with data meant that in coding and naming categories I tried to capture the experience or the issue being represented by the category.

Both schools that were my research sites were English medium schools. This meant that the official medium of instruction including the textbooks, other instructional material used and the tasks set in the class were in the English language. The teacher's formal introduction of the lesson was never in Urdu. Similarly, the groups when presenting their work to the whole class always spoke in English but during group work students spoke in a mixture of Urdu and English. The teachers also reverted to Urdu when they went up to the various groups. At times students *took permission* from the teacher and reported entirely in Urdu. However, the mathematical terms used in this mixture were invariably in English because these terms came from the textbook, which was in the English language. Teachers and students were both not fluent in the use of English language.

Both teachers were graduates of an innovative teacher education programme at a local university. As a result the teachers had introduced teaching strategies like co-operative learning strategies (Johnson, Johnson & Houlbec, 1993). And they had developed and introduced curriculum materials that linked mathematics to everyday language and experience. Their goal was to promote mathematical discussion and reasoning among the students. Now, mathematics classrooms in Pakistan are characterised by a focus on memorisation of rules and on the producing “one right answer” to mathematics problems (Halai, 1998; 2006; Warick & Reimers, 1996).

Hence, this change introduced by the teachers was a major shift in the classroom practices and as the lesson episodes illustrate the teachers had succeeded in creating a discursive environment in the classroom. However, several issues emerge due to the complex interaction of language of instruction and the processes of learning mathematics.

Data and Analysis

Provided below are illustrative samples from classroom data and analysis.

Lesson Episode I

In this lesson on ratios, the students in class VII worked on the task named ‘Anisa’s Drink’. The task involved ratio comparisons when the drink, mixing *Rooh Afza* and water was prepared by Anisa. *(Rooh Af*za is a local drink prepared by mixing *Rooh Afza* liquid and water in varying quantities). In the data item I, shared below students are working from the worksheet (see Appendix A) using a combination of Urdu and English.

Data Item I
0 Saleem: (reads) Which of these sentences are true and which are not? Give reasons for your answers.
Some mumbles and a pause
1Mansoor: (reads question 3, statement a) ‘’The Rooh Afza drink in C is the strongest.’’
2 Saleem: Ab dakho C ka Strongest kyun hay?
Now look why is C strongest?
3 Mansoor: Hann strongest hay.
Yes is strongest
4 Saleem: Kyoon.
Why?
5Mansoor Iss mann **ziada** hay naa.
It has more that is why
6 Saleem Pahley sab ke ratio likh laitay hain.
Let us first write the ratios of all (Saleem writes all ratios)
7 Saleem: Aab dekho(points to the writing) konsay (ratios) barabar hain?
Now look (points to the writing) which (ratios) are equal?
8Mansoor: Koi barabar nahi hai
None are equal
9Faizullah: Saab saay strongest kya hai?
Which is the strongest of all
10Mansoor: C hai. C is the strongest. Vo strongest hai. Iss mein

ziada hai na
C is. C is the strongest. It is the strongest it has more that is why
Slight pause
11Faizullah: Drink in C is the strongest. Han na tau hai na
Drink in C is the strongest. Yes so it is
12Mansoor: **Ziada** hai na. Iss (drink A) mein dau hai tau iss mein kaisa ho ga. Yeh
(drink C) ziada log piyain gaay tau vo strong ho ga
It (drink C) has more. It (drink A) has two so how can this be? This (drink
C) more people will drink so this will be strong
Laughter
13 Saleem: Yeh (drink B) bhi tau char hain aur iss mein panch cup hain
This (drink B) is also four, and it has five jugs
14 Faizullah: (laughs)
15 Saleem: Hain botlain barabar hain. Magar iss mein (refers to drink B) pani kam
hai, **ziada** tez tau yeh hi hua na (compares B and C)
Yes the bottles are the same but in this (refers to drink B) water is less so
this will be more strong.
16 Mansoor: Yeh kyun hua?
Why is this so?
17 Faizullah: Kya hua bhai
Why is this so brother?
18 Saleem: Char botlain iss mein bhi.
Four bottles in this one (refers to B)
19 Mansoor: Haan
Yes
20 Saleem: Iss mein bhi char botlain. Iss mein bas 8 cup hain iss mein--
In this (refers to C) also four bottles. In this (refers to C) there are eight Cups---
21 Mansoor: In this (refers to B) there are five
22 Saleem: Five jugs so why are you looking at five jugs?
23 Mansoor: Okay okay
24 Faizullah: In this (drink B) there is more Rooh Afza silly, almost one bottle (laughs and looks at Mansoor).
25 Saleem: C is not strongest
26 Faizullah: Yeah

In Urdu language the word "*ziada*" is mainly used to mean "more of something". Moreover, in the Urdu language degrees of comparison are shown in one of two ways. One, the drink could have been *tez* drink (strong drink), *tez taar* drink (stronger drink), or *tez tareen* drink (strongest drink). In this case the word *tez* stands for strong. Alternatively, the degree of comparison could have been shown by pre-fixing the word *ziada* (more) to the attribute, which is supposed to be compared. Hence a *tez* drink (a strong) would be *ziada tez* drink (more strong drink). The issue of students' usage of the word *ziada* becomes compounded because the students are not translating the word "strong drink" rather they are using a mixture of Urdu and English.

There is a tension in the way the word "*ziada*" was being used in this excerpt. The one usage was "more in volume" the other was "more in relative concentration". Mansoor, started by stating the *Rooh Afza* drink in C was the strongest (line 1). His reason (in lines 5 and 10) that drink C was strongest was that 'the drink C has more'. It was not clear what more, water or *Rooh Afza*? It seems he referred to the total quantity of the drink being more because in line 12 he said that more people would be able to drink it. It appeared that Mansoor was using the word "*ziada*" in the sense of more in volume because in line 12 he explains "*ziada log piyain gaay* i.e. more people will drink, suggesting that the greater volume of the drink will be enough for a greater number of people. An interpretation is that Mansoor regarded the word 'strong' to mean more so that his interpretation of 'strongest drink' was bounded by the context of the problem task that is 'strongest drink means more drink. Saleem's explanation (line 15) uses the word *ziada* in the sense of ratio. Saleem appeared to recognise that there was a particular usage of this phrase. So, he tried to enable Mansoor and Faizullah to interpret the phrase in the sense of its usage in the problem task i.e. most *Rooh Afza* drink per unit quantity of water. Saleem, in lines 13 and 15 encouraged Faizullah and Mansoor to focus on the *Rooh Afza* and water in drinks B and C. These focusing statements by Saleem appear to have led Faizullah to change his thinking. He questioned Saleem's statement in line 15 by asking the question in line 17. However, Faizullah's question was directed to no one in particular. It appeared to me that he asked this question of himself. The question seemed more for the purpose of Faizullah's own reflection. Saleem continued with his conversation with them until Faizullah apparently suddenly declared in line 24, that, "In this (drink B) there is more Rooh Afza silly, almost one bottle". Hence, in this case insight into ratio comparison seemed to have occurred for Faizullah when he decided that drink B was the strongest because it had **almost one bottle** of *Rooh Afza* for a jug of water.

Subsequent to the task above, students were asked to identify from four different combinations, mixtures of *Rooh Afza* to water which have the same strength and asked them to provide reasons for their answers. But, this subsequent work on ratio tasks showed that the students including Saleem used additive reasoning. The incorrect strategy used by them was focusing on the difference. Although the use of additive reasoning is consistent with research findings elsewhere (Sowder, Amstrong, Lamon, Simon, Sowder & Thompson, 1998; Hart, 1981; Noelting, 1980) I was surprised because Saleem and Faizullah seemed to have used multiplicative reasoning previously in the task on Anisa's Drink.

My findings regarding students' learning of ratio confirm the claim by Sowder et al (1998) that "There is some consensus that additive reasoning develops quite naturally and intuitively through encounters with many situations that are primarily additive in nature. Multiplicative reasoning does not develop so naturally; schooling is required to develop a deep understanding of multiplicative situations and appropriate responses in these situations"(p.120-129). Hence, one interpretation of the students' inability to perform on ratio tasks could be that the concept of ratio and quantitative reasoning is inherently complex and therefore difficult to learn. Indeed, students' varied performance on the ratio tasks suggests that they were at a developmental stage in their understanding of quantitative reasoning. Another interpretation is that language patterns and discursive practices in the classroom are supposed to enable students to abstract the mathematical concepts and relationships. But, in this setting language use itself was problematic and causing confusions so that issues in learning ratio reasoning became entwined with issues of multiple language use. I believe that the second interpretation bears some weight because there were similar examples from subsequent lessons in the same class. For example in a lesson on proportions the teacher asked the students to find the "fair share" in the profit when two friends had set up a business, investing capital in a ratio of 2:3. Now Mansoor translated the phrase "fair share" as *barabar hisaay* i.e. equal shares. Hence, their calculation of shares in profit was equal and not proportional which would be the mathematical essence of a fair share.

Lesson Episode III

There were two lessons based on "relations between quadrilaterals". In the first lesson the teacher had asked students in their groups to draw a number of different quadrilaterals on a dotted sheet with which they were provided. Then students were asked to classify those quadrilaterals as parallelograms, squares, rectangles, rhombuses. In the second lesson, which this vignette draws upon, the teacher asked the question, "Can you give other names to the quadrilaterals you had classified on the dotted sheet?" She provided one example to the class that another name of a square could be a rectangle. The teacher then asked students to work in their groups to find the other names for the quadrilaterals, which they had earlier drawn and classified. The conversation is with reference to the quadrilaterals already drawn on the dotted sheet. The groups were expected to go up to the blackboard and explain their work to the whole class. The conversation starts when Saleem is making sense of the purpose of the task.

Data Item III

1 Saleem: No we don't have to do anything to them (refers to the quadrilaterals they had drawn on the dotted paper). Have we made any mistake? We have not made any mistake. If, for them (again refers to the quadrilaterals) you can give some name so give. For a square think of a new name. There isn't one. Did you think of any other name for a square? Come on another name.

2 Mansoor: What do we have to do when we go to the black board?

3 Faizullah: Name for square (spells out) S, Q, U, A, R, E

4 Mansoor: What do we have to do on the black board? What do we have to do on the black board?

5 Saleem: Yes so what will you do? Are we just going to sit?

6 Mansoor: What do we have to do on the blackboard?

7 Saleem: What we will have to do is, if you think of another name, Friend, give another name, think.

8 Faizullah: *Yar*[1], I do not understand anything

9 Mansoor: Let us see, let us make
(Saleem and Faizullah talk about cricket)

10 Mansoor: Speed up *yar* work fast. What are we going to do there?

The data above shows that in their attempts to interpret the phrase 'other names' the students were at a loss as to how to go ahead. Saleem's opening statements in line 1 implied that he was interpreting the teacher's direction to look for 'other names' as an indication that the names that they had already assigned in classifying the quadrilaterals on the dotted sheet were wrong. So, he looked at the quadrilaterals they had drawn, checked to see if there were any mistakes, and ruled out the possibility of those names being wrong. Thus his statement in line 1, "have we made any mistake? We have not made any mistake". Then, Saleem (lines 1 & 7) invited the other two students to think of other names for a square. However, the students did not appear to know where the other names for the quadrilaterals were to come from, or on what grounds to give another name. Faizullah, jokingly (as he specified to me in the interview subsequently) spelled out S, Q, U, A, R E

[1] Yar is a commonly used word in the Urdu language. It means friend or mate.

as another name for a square.

To resolve the impasse arising out of no one knowing how to go about the work they beckoned the teacher to ask her what to do. The teacher came and gave another example, that, “a rectangle can also be called a parallelogram”. Once the teacher left, Saleem repeated the teacher’s example and wrote it down. After this example from the teacher, they still did not know how to proceed with finding other names for the various quadrilaterals that they had drawn earlier on the dotted sheet. In the segment above Faizullah (line 8) said that he did not understand anything. When the others got diverted into talking about cricket Mansoor (line 10) tried to bring them back to task.

Van Hiele (1984) proposed four levels of geometric thought to indicate the depth of students’ understanding.

> Level 0 (Visualisation)
> At the base level of geometry, figures are judged by their appearance as a whole.
>
> Level 1 (Analysis)
> At the first level of geometry students are alert to the various properties of the figures
>
> Level 2 (Informal deduction)
> At the second level students begin to see relationships among figures and even among properties of figures.
>
> Level 3 (Deduction)
> At the third level students understand deductive reasoning
>
> Level 4 (Rigour)
> At level four the thought is most rigorous - the depth of which is similar to that of a mathematician. Van Hiele (1984, p.245-246)

Considering that students had successfully drawn and classified the quadrilaterals that they had on their dotted sheet I concluded that they had reached level one of van Hiele’s geometric thought. However, the implication of using this classification is that students would be expected to use deductive reasoning, which is the third level of thought in van Hiele’s geometric thought. Students’ difficulty in seeing the inter-relationship between the various quadrilaterals suggested that they had not reached level two of the geometric thought described above. The teacher used the everyday usage phrase ‘other names’ for a specific purpose and with a specific meaning in mind. The purpose was to enable students to see the inclusion of one class of quadrilaterals in another. The meaning thus attributed to the phrase ‘other names’ was those other names that indicate the class inclusion of quadrilaterals. Students’ approach to the task suggests that they had difficulty in coming up with ‘other names’ for the quadrilaterals under consideration. The students it seems did not share with the teacher the specific purpose or the meaning that she had attributed to the phrase. Hence, students' difficulty in doing the task could be a consequence of their difficulty in understanding the discourse of the mathematics classroom, and not because they were at a lower level of geometric thought, as was suggested earlier. The significance of the particular meanings of everyday words need to be discussed and made explicit. Moreover, these phrases might be everyday usage in the language of instruction, thereby raising questions about the assumption that they would be everyday usage for learners who did not have the language of instruction as their first language.

Findings and Discussion

In what follows I discuss the findings while locating them in the classroom evidence presented above.

***Linguistic structures of the language of instruction**:* As students worked at mathematical tasks it appeared that their understanding of the statement of the problem task required interpretation at least at two levels. One was understanding the language involved and the other was to understand the mathematics involved. It is reasonable to assume that the learners in any classroom would need to understand the language of instruction before they make sense of the mathematics encoded in that language. However, in a multilingual classroom interpreting the language of instruction posed an additional challenge because the language of instruction, and hence the problem was stated in English, while the students used Urdu in thinking through the problem and doing work on it. For example, in Episode I the use of the word stronger had to be understood as "more in relative comparison". And there are specific ways of showing the degrees of comparison in Urdu which is different from the ways it is shown in English. Students' usage of the word "stronger" as *ziada* in Urdu does not reflect the second degree of comparison. It was this mathematically appropriate usage of stronger which was key to students' abstracting the essence of ratio comparison. Similarly, it was proportional share which would be regarded as a fair share in the context of a mathematics lesson on proportions. Likewise, in episode II, it was the understanding of "will" as future tense that would enable students to do the mathematics correctly. Hence, linguistic structures of the language of instruction including issues of grammar, vocabulary, and semantics take added significance in multilingual mathematics classrooms.

Translation from the language of instruction: A pattern in the classroom interactions was that the students moved from English to Urdu during the course of their work on mathematics tasks. This observation of students reverting to Urdu suggests that they were more comfortable sharing their thinking in Urdu and the use of the English language might be problematic for them. However, an alternate interpretation could be that the group interactions being in Urdu were aiding students in their effort to learn mathematics meaningfully. But it is not clear why students were shifting from one language to the other, so that Urdu was interspersed with key words from English.

An issue pertaining to translation was the suitability or not of the translation to the discourse of mathematics. This issue was further compounded because not only was there translation from Urdu to English, but translation of mathematical meaning as well. Classroom data shared in the examples illustrates that students did not translate English phrases according to the mathematical discourse and therefore were not able to perform on the mathematics tasks.

Interpreting the mathematical intent embedded in phrases of everyday usage*:* In these classrooms the discourse appeared to value students' experiences and everyday language. For example, the contexts of the mathematics tasks were everyday life situations and the teacher used words and phrases of everyday usage to communicate mathematics concepts and properties. It meant among other things that students understand the use and purpose of those everyday words in English that the teacher used to "facilitate" students' learning. Classroom interactions showed that students failed to understand the mathematical purpose of everyday words in English and could not link them to mathematics concepts or relationships. For example, stronger to mean more in proportional comparison, and other names to mean mathematically appropriate names. While this could also be an issue in monolingual mathematics classrooms, in multilingual classrooms it raises questions about the extent to which students' understanding of the language of instruction was at play.

Implications for Policy and Practice

Focus of this paper was to illustrate the complex relationship in the language of instruction and the process of learning mathematics. I established that learning mathematics in classrooms where the language of instruction is not the first or the second language of the learners makes the process of learning more complex. This complexity arises because students and teachers moved across languages in the course of teaching and learning, and students demonstrated a need to understand the linguistic structure of the language of instruction. Moreover, this movement across languages required translation from one language to the other and the process of mathematics learning appeared to be affected by the choice of words and phrases employed in translation from one language to the other. Translation was not according to the discourse of the mathematics classrooms. I maintain that movement between languages in the course of mathematics learning cannot be regarded as a straight forward resource. Rather more research is required into understanding why learners move across languages and, how does the process of translation facilitate or hinder mathematics learning. The paper raises other significant questions and issues which are rooted in the socio-political dynamics of language use in mathematics classrooms. These have been discussed in some detail in Halai (forthcoming)

Current reforms in mathematics education place communicating mathematically as a central indicator of mathematics learning (e.g. NCTM, 2000). In Pakistan the recent review & revision of the national curriculum in mathematics has led to five standards one of which is “Reasoning and Logical Thinking” (Ministry of Education, 2006). There are several implications if this curriculum reform is situated in the policy context according to which beyond 2011, English would be the language of instruction in mathematics for grade six onwards (Hasan Aly 2006, MoE, June 2006). A direct implication for mathematics teachers and teacher educators is to enable students to become proficient in communicating mathematics ideas and relationships through various oral and written approaches in English language. But, findings suggest that factors, processes and developments that apply to mathematics learners in multilingual classrooms would be different from those where learners use their first language to learn mathematics. Which would in turn imply that the support and curriculum materials for learners in a multilingual classroom require would be different from that required by learners who were learning mathematics in their first language. This then makes it imperative to develop insights through further research into students’ experience and perspective of language use in learning mathematics in multilingual classrooms. I maintain that movement between languages in the course of mathematics learning cannot be regarded as a straight forward resource. Rather more research is required into understanding why learners move across languages and, how does the process facilitate or hinder mathematics learning.

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Appendix A

Anisa's Drink

1. Here are three Rooh Afza and water drinks.

Which of these sentences are true and which are not? Give reasons for your answers.

a. The Rooh Afza drink in C is the strongest.
b. The drink in C is Rooh Afza and water in the ration 6 : 9
c. The drink in B is Rooh Afza and water in the ration 2 : 3
d. All the drinks have the same strength.
e. Drinks A is weaker than drink B because it is Rooh Afza and water in the ration 2 : 3 and the ration for B is 4 : 5
f. A drink made with Rooh Afza water 1 the ration 10 : 15 will be stronger than the drink in A.
g. If you used 10 times more Rooh Afza and 10 times more water in A the Rooh Afza drink would be 10 times stronger.

2. A Rooh Afza drink is made from Rooh Afza water in the ration 1 : 5 which of these mixtures of Rooh Afza to water have the same strength?

   a. 2 : 10 b. 2 : 7 c. 10 : 50 d. 2 : 6