

Teachers' Knowledge about the Nature of Mathematics: A Survey of Secondary School Teachers in Karachi, Pakistan

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Abstract

This study presents the findings from a study which explored patterns in teachers' knowledge about the nature of mathematics. A survey questionnaire was developed and distributed to 200 secondary school mathematics teachers teaching in public and private schools in Karachi, Pakistan. Exploratory factor analysis was performed which showed patterns in teachers' view about the nature of mathematics. The analysis illustrates that teachers hold contradicting views about the nature of mathematics i.e. mathematics, both as discovered as well as invented body of knowledge. Moreover, teachers irrespective of their professional qualification, considered mathematical knowledge as 'truth', where mathematical rules can never be proved wrong. On the other hand, teachers expressed the progressive view of mathematics such as; they considered that mathematical knowledge is useful for scientific invention and for addressing societal issues. Based on the survey findings some key issues and questions have emerged which offered insight into mathematics teacher education programmes in the context of Pakistan as well as raising new questions for the second phase of this study.

Key Terms: mathematics knowledge, nature of mathematics, discovered mathematics, invented mathematics

Introduction

Almost all the countries including Pakistan make an effort to improve the quality of education through introduction of appropriate education reform. A variety of initiatives like curriculum and resource development are initiated and through pre-service and in-service teacher education programmes teachers are encouraged to use variety of teaching strategies and resources to bring change in their teaching to provide better learning opportunities to students. Likewise, in Pakistan the Ministry of Education in the federal government with the coordination of the provincial government has undertaken curriculum development process. Currently, the newly developed curriculum is almost ready to be formally introduced in schools for implementation. New mathematics curriculum emphasizes on learning mathematics for conceptual understanding and to promote logical

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reasoning and problem solving skills. Therefore, teachers need to change their current mathematics teaching that emphasizes knowledge acquisition, drill and practice (Amirali 2000; Halai 2008; Halai, Rizvi & Rodrigues, 2007). Furthermore, need to bring change in current mathematics practices has been highlighted in the National Curriculum for Mathematics (NCM) 2006, which emphasizes that “the teachers’ role has been rerouted from ‘dispensing information’ to planning investigative tasks, managing cooperative learning environment and supporting students’ creativity in developing rational understanding of the concepts of mathematics” (p. 6). Hence, this curriculum reform demands paradigmatic shifts for many teachers, including changes in teachers’ beliefs, attitudes, and knowledge about the nature of mathematics, mathematics teaching and learning (Susan, Swars, Smith, Smith, & Hart, 2009).

Thompson (1984) and Ernest (1988) based on their wider experience of working in the field of mathematics education claim that any attempt in improving the quality of mathematics teaching and learning must begin with an understanding of the conceptions held by teachers. Lerman (1990) supports this view and asserts that unless teachers’ knowledge about mathematics, mathematics teaching and learning are examined, “little will be achieved in terms of development and change in the mathematics classroom (p. 54). One of the key reasons is that knowledge, beliefs and conceptions teachers’ hold play significant role in shaping their thinking and behaviors which influence their teaching practices (Barkatsas & Malone, 2005; Iqbal, Azam & Rana, 2009; Leder, Pehkonen, & Torner, 2002; Lerman, 1990; Pajares, 1992; Swars, Smith, Smith, & Hart, 2009). Furthermore, it is also believed that the views teachers hold about the subject, if unchallenged usually leads to the failure of curriculum reforms (Goldin, Rosken & Torner, 2009). Therefore, Hersh (1979) concludes that “the issue then, is not, what is the best way to teach, but what is mathematics really all about controversies about high school teaching cannot be resolved without confronting problems about the nature of mathematics” (p.33). Thus, implementing new curriculum along with facilitating teachers to challenge their views about the nature of mathematics, mathematics teaching and learning will contribute in bringing change in teachers’ thinking and teaching practices.

Hence, this study is a ground breaking research where first of all the baseline of Pakistani teachers’ knowledge about the nature of mathematics, mathematics teaching and learning is explored, and in phase two of the study teachers will be supported to challenge their views about mathematics in order to bring change in their thinking and teaching practices. This paper attempts to answer *‘what do Pakistani mathematics teachers know about the nature of mathematics?’*

Theoretical Underpinnings

This section explains the key term ‘knowledge’ used in this study and the theoretical frameworks which discuss the nature of mathematics.

Knowledge

In educational research literature the terms knowledge, conceptions and beliefs are often used interchangeably. Nevertheless, researchers have tried to define these terms in a variety of ways. For instance, Ponte and Chapman (2006) draw distinction between these terms and refers to ‘knowledge’ as a wide network of concepts, images, and intelligent abilities possessed by human beings; ‘beliefs’ as incontrovertible personal ‘truth’ held by everyone, drawing from experiences and fantasy; and ‘conceptions’ as the underlying organizing frames of concepts, having essentially a cognitive nature. Thus ‘knowledge’ is considered as a broader category, including conceptions, beliefs, insights, mental images and understandings that people possess either derived from formal or practical experiences. Thompson (1992) refers to conceptions as conscious or subconscious beliefs, understanding, meaning, mental images, and preferences. In this study ‘knowledge’ is considered as a broader concept which includes teachers’ conscious and unconscious beliefs, meanings, mental images, understandings, preferences, beliefs and conceptions constructed or developed through enculturation, education and schooling.

Nature of Mathematics

In the history of mathematics education different philosophical perspectives exist pertaining to the nature of mathematics. At one extreme, mathematics is seen as static, fixed and either discovered or waiting to be discovered i.e. ‘absolutist view of mathematics’ and at the other extreme mathematics is seen and interpreted as socially constructed phenomena i.e. ‘fallibilist view of mathematics’.

Lakatos (1976) suggested that the apparent multiplicity of philosophies of mathematics can be identified as two competing aspects that he calls Euclidean and Quasi-empirical. Proponents of Euclidean base of mathematics forms universal absolute foundation, whereas quasi-empiricist sees the growth of mathematical knowledge as a process of conjectures, proofs and refutations, and accept the uncertainty of mathematical knowledge as part of the nature of mathematics. Similarly, Dossey (1992) draws on this discussion of the nature of mathematics as far back as the fourth century BC, with Plato and Aristotle as two main contributors to these broader views of mathematics. Plato took the position that the objects of mathematics had an existence of their own beyond the mind in the external

world and Aristotle's view of mathematics was based on "experienced reality, where knowledge is obtained from experimentation, observation and abstraction" (p.40). Moreover, Lerman (1990) discusses two contrasting views of mathematical knowledge i.e. absolutism and fallibilism. Therefore, in the mathematics education literature even today, whether mathematical knowledge is discovered or invented, is part of an ongoing debate.

Furthermore, Ernest (1991) states three different philosophical views about mathematics i.e. instrumentalist, Platonist and problem solving view of mathematics and their implication on teachers' teaching practices. He elaborates that those who hold the 'instrumentalists view consider mathematics as an accumulation of facts, rules and skills to be used in the pursuance of some external end. This means that mathematics is considered as a set of tools and knowing mathematics is to know what tools you have and how to use them. Thus, mathematics is seen as a set of unrelated but utilitarian rules and facts. Mathematics teachers holding instrumentalist views will consider themselves as masters having and imparting mathematical knowledge. Platonists consider mathematics as a "static, but unified body of knowledge, a crystalline realm of interconnecting structures and truth, bound together by filaments of logic and meaning. Thus, mathematics is a monolithic, a static immutable product" (p.132). This means that Platonists focus more on the holistic approach, knowing how various tools work together and what makes them work. Mathematics teachers holding the Platonist view would try to find linkages among the mathematical concepts rather than considering them as unrelated rules and facts. The problem solving view of mathematics encompasses mathematics as a dynamic, continually expanding field of human creation and an ever changing field with inventions generating patterns and then distilled into knowledge. These three philosophies of mathematics can be seen as forming a hierarchy; Instrumentalism at the lowest level (involving knowledge of mathematical facts, rules and methods as separate entities), followed by Platonist view (involving a global understanding of mathematics as a consistent, connected and objective structure) and problem solving view at the highest (seeing mathematics as a dynamically organized structure located in a social context). Thus, these different philosophical views about the nature of mathematics enabled researchers to develop survey questionnaire in order to explore mathematics teachers' views about the nature of mathematics.

Research Design

In social science research depending on the research purpose both the quantitative and qualitative research methods are adopted. Brewer and Hunter (2006) discuss one of the possible reasons for using both the methods

in a research study that “individual methods might be flawed, but fortunately the flaws in each are not identical” (p.4). Furthermore, elaborate that this diversity of imperfection allows us to combine methods, not only to gain their individual strengths, but also to compensate for their particular faults and limitations. Likewise, based on the research purpose i.e. first to explore Pakistani teachers’ knowledge about the nature of mathematics, teaching and learning of mathematics and then to build on what mathematics teachers know about mathematics by engaging them in a process of exploring alternative views of mathematics for the development of their thinking process and teaching practices, researchers opted for multi method research. Hence, the research design included two phases of data collection and analysis, starting with a quantitative method and then expanding to the qualitative method. In this paper, researchers reported only the results of quantitative data obtained through a survey questionnaire.

Development of the Questionnaire

The process of developing the survey questionnaire began with a review of conceptual and theoretical literature and adapting from the available survey questionnaire with prior permission. Some of the questionnaires studied were, ‘Mathematics belief scale’ (Margaret, 2001); ‘Attitudes and beliefs about the nature and the teaching of mathematics and science’ (McGinnis et al, 1998); ‘Attitude towards mathematics inventory’ (Tapia & Marsh, 2004); ‘Perception of mathematics and mathematics education’ (Lerman, 1990); and ‘Conception and attitude towards mathematics’ (Amirali, 2007). The survey questionnaire designed for the study includes 37 items under the following three major subscales along with three open-ended questions to explore who supported the teachers in learning mathematics as well as in learning to teach mathematics. The three major subscales of the questionnaire are as follows:

- a) *Perceptions of the Nature of Mathematics (11 items)*: In this subscale the teachers’ orientation towards mathematics such as absolutist, Platonic, instrumentalist, problem solving and fallibilist view of mathematics were explored.
- b) *Teaching and Learning Mathematics (12 items)*: In this subscale the teachers’ perceptions about mathematics learning and teaching were explored for example whether teacher considers mathematics learning as a solitary process or being socially constructed.
- c) *Teaching Practices (14 items)*: In this subscale the frequency of different teaching strategies and resources teachers reported using in their teaching practice were explored.

Validity and Reliability

In order to assess the tool's validity and reliability, first of all the developed questionnaire was distributed to the content and methodological experts for their 'expert review'. Secondly, the questionnaire was translated into the Urdu language to make it more reader friendly. A 'forward translation' of the English version of the questionnaire was performed in Urdu which was then independently 'back translated' to English. Finally, the tool was 'pilot tested'. Furthermore, to indicate the extent to which the responses on the items within a measure are consistent, internal consistency using Cronbach's alpha (α) being the most widely used reliability measure was used (Rubio, 2005). Test of reliability produced an overall alpha value of 0.8, which is considered good for social science research (Field, 2005; Rizvi & Elliot, 2005).

Research Sample

There are 18 Towns in the city of Karachi. A list of all government and private secondary schools in these towns was accessed from Sindh Education Management Information System –SEMIS, Government of Sindh and the Director School Division Department - Private schools, Government of Sindh. As Karachi is a large city spread over a wide geographical area therefore a decision was made to sample eight towns from eighteen towns and the criteria used was to try and obtain the maximum numbers of secondary schools in the towns of Karachi. According to the information provided by the data management officers the total number of public secondary schools listed were 531 and the private schools listed were 1575 in these identified towns. This was contrary to the general perception that the number of secondary schools in the public sector is higher than those in the private sector. Actually in the government sector, the schools are listed as elementary and secondary / higher secondary schools. Whereas in the private schools typically schools are listed as secondary schools and the elementary schools are subsumed within. However, for the purpose of this study, the list provided through the official sources was used to draw the sample. Each school was assigned a number and through the stratified random sampling procedure schools were identified for the survey study. Finally, 46 schools (20 public schools and 26 private schools) showed their willingness to participate in the survey study. All the mathematics teachers teaching in these schools were included in the study.

The questionnaire was distributed to two hundred secondary school teachers teaching mathematics to grades VI to X in public and private schools in Karachi. One of the key reasons for identifying secondary school teachers as a research sample is due to the fact that in Pakistani context in middle and high schools particularly in the public school system, teachers

are appointed on the basis of the subject specification and therefore, their identities as subject teacher for instance ‘mathematics teacher’ is developed. Whereas this is not the case with primary school teachers as they are obliged to teach almost all subjects.

Out of 200 secondary school mathematics teachers 174 completed the questionnaire. The following table gives the account of the public and private schools (male and female) mathematics teachers who participated in the survey study. The return rate from the public sector is 100% and the private sector is 80% with overall return rate of 87%. One of the possible reasons for the 100% return rate from the public sector was due to getting an opportunity to talk to teachers personally and handing over the questionnaire to them rather than their head teacher / principal handing over to them.

Table 1
Mathematics teachers’ participation in the study

	Male	Female	Total
Government	11	53	64
Private	30	80	110
Total	41	133	174

The table illustrates that in this study the ratio between male and female teachers is almost 1:3. However, this is quite different from the ratio presented in the National Professional Standards for Teachers in Pakistan in Sindh province. According to National Education Census 2005 ratio between male and female teachers is equal in Sindh province i.e. 50.1% male and 49.9% female. In Karachi being a metropolitan city both men and women are working and moreover more females are in teaching profession as compared to the females teaching in rural context of Sindh, therefore there is a difference in the male and female ratio in the study.

Data Management

The survey was conducted using a five-point Likert scale ranging from ‘strongly agree’ through ‘neutral’ to ‘strongly disagree’. First of all, teachers’ responses to the 5-point Likert scales were converted into a numerical scale. Strongly agree was coded as ‘5’ while ‘1’ was for strongly disagree. These numbers do not have interval value, i.e. they are not measures; they can be used to indicate trends and differences among the data. Teachers’ responses to the survey questionnaire were compiled in SPSS data file sheets and analyzed using SPSS software.

Data Analysis

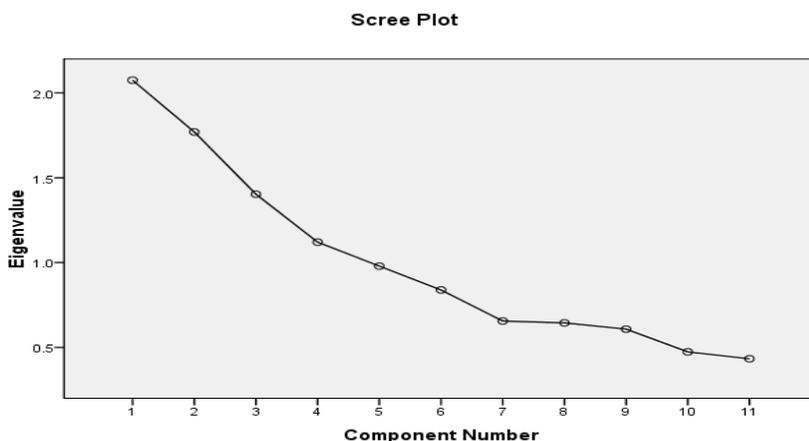
Exploratory factor analysis (EFA) was performed to reduce the data set to few factors (Field, 2005) and to explore patterns (Cohen et al., 2000) pertaining to teachers' knowledge about mathematics.

Prior to performing EFA the suitability of data for factor analysis was determined. Foster (2001) suggests that, "for factor analysis the number of respondents should not be less than 100, and there should be at least twice as many respondents as variables" (p.231). Thus, the study meets the criteria to perform factor analysis as the number of respondents is more than 100 and also has more than twice as many respondents as variable. Therefore, the decision to perform factor analysis was justified.

Next the commonly used 'Kaiser-Meyer-Olkin' (KMO) measure of sampling adequacy and 'Bartlett's Test' of sphericity was run to see whether the sample size is appropriate for factor analysis and the strength of the relationship among the variables is significant (Blaikie, 2003). For the data set KMO was 0.631 and Bartlett's test was significant [$\chi^2(666) = 1357.789$, $p < 0.001$] supporting the suitability of the data for factor analysis. Once it was assured that factor analysis is possible, eigenvalues and scree plot was used to retain the factors. To elaborate items holding eigenvalues greater than one were considered significant and using Cattell's (1966b cited in Field, 2005) argument that the cut-off point for selecting factors on the scree plot was at the point of inflexion of the curve. While extracting factors another decision was taken with respect to at what degree variables load onto these factors, i.e. 'factor loading'. Varimax factor rotation method with suppression of loadings less than 0.4 was used as it tries to load a smaller number of variables highly onto each factor resulting in more interpretable clusters of factors. Field (2005) suggests that the suppression of loadings less than 0.4 and ordering variables by loading size makes interpretation considerably easier. In case items were loaded to more than one factor, the decision was taken to assign the item to a factor for which they had the highest loading, provided that the item contributed to the meaning of the factor. In other words conceptual meaning was prioritized over technical results. After factorization mean score and standard deviation values were also calculated and were used for further interpretation of the teachers' responses to the questionnaire.

Results

Using the criteria mentioned earlier and by observing the following scree plot it was decided to retain four factors under this section. The four factor solution explained 57.12% of the variance, with Factor 1 contributing 18.55%, Factor 2 contributing 16%, Factor 3 contributing 12.41%, and Factor 4 contributing 10.16%.



Looking at the items grouped under four factors (see Table 2), Factor 1 was titled ‘Absolutist views about mathematical knowledge’ because the items under this factor describe mathematics as a static discipline; Factor 2 ‘Source and uses of mathematics knowledge’ because it explains different usage of mathematical knowledge in the real life; Factor 3 ‘Components and connections within mathematics’ and Factor 4 ‘Human beings as mathematical knowledge constructors’. Table 2 presents the results using factor loadings, mean score and standard deviation values to explain the patterns in teachers’ knowledge about the nature of mathematics as reflected in the extracted factors.

The analysis implies that these four factors explain teachers’ knowledge about the nature of mathematics. The loading column in Table 2 shows that the items correlate strongly (greater than 0.4) with the factors. Though there is a variation in the mean scores of the items in the four factors, mean scores are still high and the standard deviation represents variation in the teachers’ responses in terms of their views pertaining to the nature of mathematics.

Factor 1 comprises items which indicate teachers’ views about mathematics that it is the uniform body of knowledge hence it is same throughout the world. Also teachers consider mathematical knowledge as ‘truth’. Further analysis of these responses given by professionally and non-professionally qualified teachers it was evident that both have responded similarly i.e. 41% of the professionally qualified teachers and 47% of the non-professionally qualified teachers show their level of agreement to the statement that mathematical rules can never be proved wrong. Likewise, the analysis in terms of novice teachers and teachers having teaching experience of more than 10 years are quite similar i.e. 53% of the novice and 50% of the experienced teachers consider that mathematical rules can never be proved wrong.

Table 2
Item loadings in the factor analysis for nature of mathematics section of the questionnaire

Factors	Loadings	Mean	SD
Factor 1: Absolutist views about mathematical knowledge			
3. Mathematical rules can never be proved wrong.	0.752	3.44	1.068
5. Current mathematical knowledge will remain same in the future.	0.685	2.93	1.161
9. Mathematical knowledge is same throughout the world.	0.677	3.62	1.115
10. Study of mathematics is suited mostly to males.	0.536	2.32	1.213
Factor 2: Source and uses of mathematics knowledge			
2. Mathematics contributes to scientific inventions.	0.676	4.38	0.652
6. Mathematics existed in the world even before human creation.	0.796	4.10	0.897
11. Mathematical knowledge can contribute in addressing societal issues (for e.g. inequality, environmental issues)	0.694	3.76	0.881
Factor 3: Components and connections within mathematics			
1. Mathematics comprises of only formulae, symbols and rules.	-0.674	3.48	1.231
4. Mathematical knowledge consists of several concepts which have connections among them.	0.788	4.12	0.854
Factor 4: Human being as mathematical knowledge constructor.			
7. Mathematics is a creative subject like arts / music.	0.665	3.96	1.026
8. Human beings create mathematical knowledge.	0.788	3.85	0.986

The low mean score and high SD of the item 10 (Study of mathematics is suited mostly to males) indicates that most of the teachers (70% respondents) show their level of disagreement and consider mathematics as a subject which is suitable for all irrespective of their gender. This shows that any underlying gender issue is not highlighted in this item.

The Factor 2 loading column in Table 2 shows that the items correlate strongly (being greater than 0.6) with the factors. The high mean scores and the low SD values show that most of the teachers have scored high and have agreed with these items. The high factor loadings of the items in Factor 2 illustrate that teachers consider mathematical body of knowledge as a discovered body of knowledge. This shows that teachers strongly believe that mathematics existed in the world before human beings were created. The further analysis showed the level of agreement towards mathematics as a discovered body of knowledge is stronger among professionally qualified teachers and teachers having more than 10 years of teaching experiences (82%, 79%) compared to novice teachers having 2 years or less teaching experiences (47%). This difference in teachers' responses needs to be further

explored to understand the reasons for their response. The mean scores of item 2 and 6 in Factor 2 are very high. This brings to light that teachers consider mathematics as an important subject insofar as it contributes to utilitarian purposes. For example they consider that mathematical knowledge contributes in scientific invention and also addresses societal issues.

Mean scores for both the items in Factor 3 (see Table 2) are very high. This highlights that most of the teachers strongly viewed mathematical knowledge as the collection of rules, formulae and symbols with connections among them. This shows a strong tendency towards an instrumental view of mathematics where mathematical rules, formula and symbols are considered as key to mathematical knowledge.

The high factor loadings of the items in Factor 4 illustrate that teachers considered mathematics as an invented body of knowledge where human beings play a major in knowledge construction. Consequently, teachers considered mathematics as a creative subject. This shows that teachers value the human contribution which could lead them to engage students in mathematical knowledge construction rather than dictating them the procedure to solve mathematical tasks. This shows a strong tendency towards a fallibilist view of mathematics along with an absolutist view highlighted in the findings of Factor 1.

Table 3
Cross tabulation of item 6 and 8 of Nature of Mathematics

		Human beings create mathematical knowledge				
		Strongly Disagree	Disagree	Neutral	Agree	Strongly agree
Mathematics existed in the world even before human creation.	Strongly Disagree	0	0	0	0	2
	Disagree	0	0	2	5	1
	Neutral	1	2	4	13	2
	Agree	1	6	6	40	13
	Strongly agree	4	5	5	28	18

The overall analysis of section one - 'nature of mathematics' - indicates that teachers give more priority to the utility of mathematics and consider mathematics as a tool for scientific invention as well as a tool to address societal issues and 45% of the teachers consider that current mathematical knowledge will change in future. The other key finding is teachers' agreement to both, 'mathematics as discovered and invented', i.e. to both absolutist and fallibilist views of mathematics (see Table 3). For instance 63% of teachers strongly agreed that mathematical knowledge is discovered and is 'truth' where mathematical rules can never be proved wrong, but they

also consider mathematics as a creative subject where human beings create knowledge which therefore might not be perfect and can change with further inventions.

Also teachers consider mathematics as fixed body of knowledge and hence view mathematics as collections of rules, formula and symbols which students have to learn from them and practice to solve the questions given in the textbooks to develop their mathematical understanding.

Findings and Discussions

In this section the discussion on key findings illustrates Pakistani teachers' knowledge about mathematics, which is presented with reference to the relevant literature. Also key questions emerging from the survey findings are highlighted in order to be addressed in Phase two of the study.

Contradicting Views about the Nature of Mathematics

Teachers hold contradicting views about the nature of mathematics i.e. they consider mathematics as both discovered and invented body of knowledge. In my view one of the possible reasons for agreeing to both aspect of mathematics knowledge particularly in the Islamic society like Pakistan could be due to the teachers' faith in Almighty as 'knowledge creator', and also based on their experience that human beings contribute to the generation of mathematical knowledge. To further elaborate, teachers might view that mathematicians discover mathematical concepts or relationships present in the nature which is created by Allah and then they invent the mathematics representation to communicate mathematical concepts or relationships. Thus they subscribe to both absolutist and fallibilist view of mathematics.

This finding questions some of the models of the nature of mathematics recommended in the literature. For example it is recognized that there exist 'absolutism view of mathematics' at one extreme and 'fallibilism view of mathematics' on other extreme (Lerman, 1990). Lerman (1990) further explained that absolutists consider mathematics as an absolute, certain, infallible and objective body of knowledge where human experience has no place in creating mathematics. In contrast, fallibilists consider mathematics as fallible, and it is developed through conjectures, proofs, and refutations, where uncertainty is accepted as inherent in the discipline (Thompson, 1992). Lerman (1990) concludes that a fallibilist view is associated with a preference for a non-directive and open-teaching style, while the reverse is true for an absolutist view. Ernest (1991) also elaborates on these perspectives that "the absolutist view that mathematical truth is absolutely certain, that mathematics is the one and perhaps the only realm of certain, unquestionable and objective knowledge [whereas] fallibilist view that

mathematical truth is corrigible, and can never be regarded as being above revision and correction” (Ernest, 1991, p. 3). In this study, as teachers hold both the views simultaneously it challenge the philosophical models presented in the literature for the nature of mathematics. It seems necessary to understand what meaning teachers attach when they consider mathematics as discovered or invented as this influences their teaching practices (Andrews & Hatch, 1999; Ernest, 1991; Lerman, 1990) rather categorizing them under absolutist or fallibilist view of mathematics.

Perception about Mathematical Knowledge is Static and ‘Truth’

Most of the teachers (63%) irrespective of whether they are professionally qualified or not or whether they are novice or experienced teachers they considered mathematical knowledge as ‘truth’ where mathematical rules can never be proved wrong. This view influences teachers to accept that mathematical knowledge is a static body of knowledge which will remain same in future and this view in turn influences teachers’ teaching practice. For instance, if a teacher believes that mathematical knowledge can never be proved wrong then this view might not allow them to give space to students to analyze mathematical knowledge critically and think differently.

One of the possible reasons for teachers to accept mathematical knowledge as static and truth could be based on their personal experience and experience with schooling and instruction (Liljedahl, Rolka & Rösken, 2007; Richardson, 2003). Richardson elaborates that experience with schooling and instruction is the most important formal source for teachers’ beliefs since they had been students in formal schools for many years. Liljedahl, Rolka and Rösken (2007) elaborate that if teachers consider teaching mathematics is ‘all about telling how to do it’ and learning mathematics is ‘all about being told how to do it’, this may have come from personal experiences as a learner of mathematics.

However, in this study most of the teachers mentioned that they found mathematics interesting when they were in schools. Therefore, it raises few questions about some of the key mathematical learning experiences acquired in their childhood which shaped their views about mathematical knowledge. What aspect of their childhood mathematics learning experiences did they consider interesting? Is it when their teachers explain everything clearly and then ask them to practice the mathematical questions or it is something else? In some cases teachers mentioned that they found mathematics difficult, so what made them feel so and how are they teaching differently not to make their students experience the same?

Thus, this finding raises some major questions to be taken up further to understand in phase two of the study. First of all, overall what experiences have led teachers to believe that mathematical knowledge is ‘truth’ which

can never be proved wrong even though they consider mathematics as a creative subject where human beings create mathematics knowledge and therefore might not be perfect and can change with further inventions? Do the professional development programmes teachers have undertaken are making any difference in teachers' views about mathematics? It is declared in the National Professional Standards of Teachers for Pakistan (2009) that most of the teacher preparation programmes in Pakistan neither provide broader general education to foster effective communication skills and critical thinking nor promote in-depth understanding of the concepts to be dealt at school level. Moreover, it also states that pedagogical skills taught in teacher education programmes foster rote learning, unquestioning textual materials and passive preparation for the test. If this is the case then what contribution do in-service and pre-service professional development programmes make in improving teachers' practices? In other words do professional development programmes in Pakistani context matter?

Progressive views about Mathematics

The findings also demonstrate some of the progressive views of mathematics which could prove to be supportive in developing teachers' thinking as well as teaching and learning of mathematics. For example, on one hand teachers considered that mathematical knowledge is useful for scientific invention and for addressing the societal issues and on other hand they consider mathematical knowledge being a human creation. I think this view towards mathematics if nurtured teachers would realize that mathematical knowledge is an ever growing knowledge which enables scientific innovation and allow addressing emerging societal issues. One of the questions raised that when teachers consider mathematics being a human creation then why don't they let their students experience the same exciting moments as mathematicians experienced? Is it because the teacher's job is to pass on what other historical people have defined mathematics to be? Or they replicate their own teachers who taught them mathematics when they were in schools? Nevertheless, I think that if teachers' progressive view towards mathematics if developed it could lead them to engage students in developing reasoning and problem solving skills in order to prepare them to address the societal issues and contribute in scientific inventions.

Conclusion

The mathematics curriculum reforms in most parts of the world including Pakistan strongly recommend problems solving approaches to school mathematics. Such curricular reform depends to a larger extent on individual teachers changing their approaches to the teaching of mathematics. Ernest (1989) concludes that, "teaching reforms cannot take

place unless teachers' deeply held beliefs about mathematics and its teaching and learning change (p. 99). Hence, understandings teachers' view about the nature of mathematics is important in order to challenge them and support teachers to improve their thinking and teaching practice.

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