Punjab University
Journal of Mathematics (ISSN 1016-2526)
Vol.47(1)(2015) pp.00.00

Super Edge-Magic Deficiency of Disjoint Union of Shrub Tree, Star and Path Graphs

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Received: 11 December, 2014 / Accepted: 04 March, 2015 / Published online: 24 April, 2015

Abstract. Let $C = (M, N)$ be a finite, undirected and simple graph with $|M(C)| = t$ and $|N(C)| = s$. The *labeling* of a particular graph is a function which maps vertices and edges of graph or both into numbers (generally $+ve$ integers).

If the domain of the given graph is the vertex-set then the labeling is described as a *vertex* labeling and if the domain of the given graph is the edge-set then the labeling is defined as an *edge* labeling. If the domain of the graph is the set of vertices and edges then the labeling defined as a *total* labeling.

A graph will be termed as *magic*, if there is an edge labeling, using the positive numbers, in such a way that the sums of the edge labels in the order of a vertex equals a constant (generally called an *index* of labeling), without considering the choice of the vertex.

An *edge magic total labeling* of a given graph comprising t vertices and s edges is a $(1-1)$ function that maps the vertices and edges onto the integers $1, 2, \ldots, t+s$, with the intention that the sums of the labels on the edges and the labels of their end vertices are always an identical number, consequently they are independent of any specific edge. To a greater extent, we can define a labeling as *super* if the t least possible labels happen at the vertices.

The Super edge-magic deficiency of a graph C, signified as $\mu_s(C)$, is the least non negative integer m' so that $C \cup m'K_1$ has a Super edge-magic total labeling or $+\infty$ if such m' does not exist.

In this paper, we will take a look at the Super edge-magic deficiencies of acyclic graphs for instance disjoint union of shrub graph with star, disjoint union of the shrub graph with two stars and disjoint union of the shrub graph with path.

MR (2000) Subject Classification : 05C78
Key Words: Super edge-magic total labeling is written as SEM total labeling, deficiency, disjoint union of acyclic graphs, shrub graph.

1. INTRODUCTION AND MAIN RESULTS

In this discussed paper, we have presumed finite, undirected and simple graphs $C = (M, N)$, for which we have supposed that $|M(C)| = m$ and $|N(C)| = n$. An edge magic labeling of a graph C is a bijection $d' : M(C) \cup N(C) \rightarrow \{1, 2, ..., m+n\}$, where there exist a constant w s.t $d'(k) + d'(kl) + d'(l) = w$, for each edge $kl \in N(C)$. An edge magic total labeling d' is termed as SEM total if $d'(M(C)) = \{1, 2, ..., m\}$.
In [11], it is originated that for some graph "C" there happens to exist an edge magic graph H' so that $H' \cong C \cup mK_1$ for some non negative integer m. This piece of information leads to the idea of edge magic deficiency of a graph C, which is the bare minimum non negative integer m such that $C \cup mK_1$ is edge magic and it is indicated as $\mu(C)$. In particular,

$$\mu(C) = min\{m \geq 0 : C \cup mK_1 \text{ is edge magic}\}$$

In the same paper, they gave an upper bound of the edge magic deficiency of a graph C using m' vertices, $\mu(C) \leq F_{m'+2} - 2 - m' - \frac{1}{2}m'(m'-1)$, where F'_m is the m'th "Fibonacci number". Motivated by Kotzig and Rosa's initiative of edge magic deficiency, In [6], they defined a similar concept for SEM total labelings. The super edge magic deficiency of a given graph C, which is signified by $\mu_s(C)$, is the bare minimum non negative integer m s.t $C \cup mK_1$ has a SEM total labeling or $+\infty$ if there does not exist such m.

$$Let\ M(C) = \{m \geq 0 : C \cup mK_1 \text{ is a SEM graph}\}, then$$
$$\mu_s(C) = \begin{cases} min\ M(C), & \text{if } M(C) \neq \phi; \\ +\infty, & \text{if } M(C) = \phi. \end{cases}$$

As a consequence of the beyond definitions on deficiencies, we have ended that for each graph C, $\mu(C) \leq \mu_s(C)$.

In [8, 6], they originate the precise values of SEM deficiencies of numerous classes of graphs, such as cycles, complete graphs, 2-regular graphs, and complete bipartite graphs $K_{2,m}$. They moreover demonstrated that every single one of the forests have finite deficiencies. In particular, they proved that

$$\mu_s(dK_2) = \begin{cases} 0, & \text{when } d \text{ is odd;} \\ 1, & \text{when } d \text{ is even.} \end{cases}$$

In [14], they proved some upper bound for the SEM deficiency of fans, double fans, and wheels. In [7], they proved $\mu_s(P_m \cup K_{1,n})$ is 1 if $m = 2$ and n is odd or $m = 3$ and $n \not\equiv 0 (\text{mod } 3)$, and 0 otherwise. In the same paper, they proved that $\mu_s(K_{1,n} \cup K_{1,m})$ is 0 if either m is a multiple of $n+1$ or n is s multiple of $m+1$ and otherwise 1. Furthermore, they conjectured that every forest with two components has deficiency ≤ 1.

In [1], they found SEM deficiency of of unicyclic graphs. In [2, 3] they provide some upper bound and exact value for the SEM deficiency of the forests created by paths, stars, comb, banana trees, and subdivisions of $K_{1,3}$.

In this paper, we will provide the deficiencies of acyclic graphs such as disjoint union of a shrub graph with a star, disjoint union of a shrub graph with two stars and disjoint union of a shrub graph with a path.

In proving the results in this paper, we frequently use the lemma below.

LEMMA 1.1. [5] *A graph G containing t vertices and s edges is SEM total iff there subsists a bijective function $\lambda : V(G) \to \{1, 2, \cdots, t\}$ such that the set $S = \{\lambda(u) + \lambda(v) | uv \in E(G)\}$ consists of s successive integers. In such a circumstance, λ directs to a super edge magic total labeling of G.*

DEFINITION 1. *A shrub $\check{S}h(c_{i,1}, c_{i,2}, \ldots, c_{i,m})$ for $1 \leq i \leq n$ is a graph obtained from a star $St(n)$ by connecting each leaf c_i to $c_{i,j}$ new vertices for $1 \leq j \leq m$ and is denoted by $\check{S}h_n$.*

In the theorem 1 we establish an upper bound for the SEM deficiency of disjoint union of a shrub graph with a star.

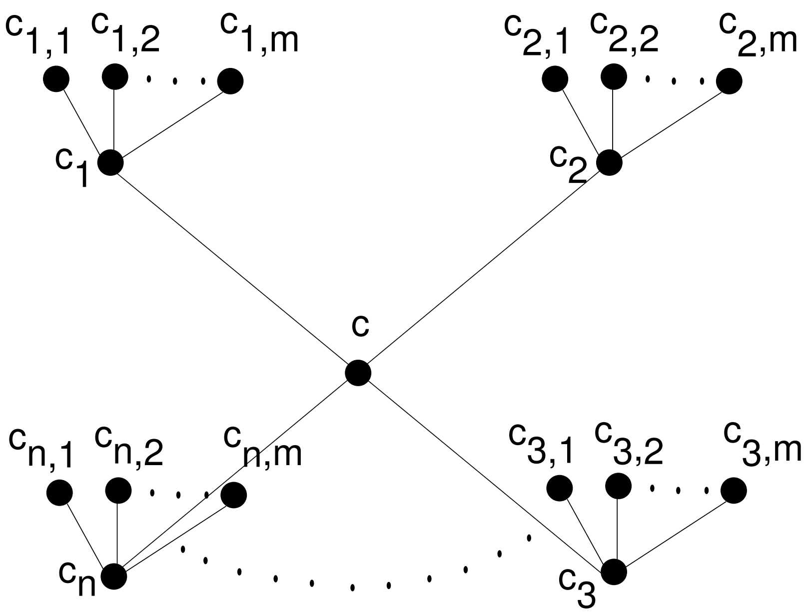

FIGURE 1. Shrub graph $\check{S}h_n$

THEOREM 1.1. *Let* $m, r \geq 3$, *then* $\mu_s(\check{S}h(r) \cup St(m)) \leq \lfloor \frac{r-2}{2} \rfloor$.

Proof. Foremost we delineate the vertex and edge sets of a shrub graph and the star in the subsequent way.

$V(\check{S}h_r) = \{c, c_i, c_{i,j}; 1 \leq i \leq r, \;\; 1 \leq j \leq m\}$, $|V(\check{S}h_r)| = 1 + r + mr$

$V(St(m)) = \{x, y_j; 1 \leq j \leq m\}$, $|V(St(m))| = 1 + m$

$E(\check{S}h_r) = \{cc_i, c_ic_{i,j}; 1 \leq i \leq r, \;\; 1 \leq j \leq m\}$, $|E(\check{S}h_r)| = r + mr$

$E(St(m)) = \{xy_j; 1 \leq j \leq m\}$, $|E(St(m))| = m$

Let $G \cong \check{S}h_r \cup St(m) \cup \lfloor \frac{r-2}{2} \rfloor K_1$, then

$V(G) = V(\check{S}h_r) \cup V(St(m)) \cup \{z_r : 1 \leq t \leq \lfloor \frac{r-2}{2} \rfloor\}$ with

$$|V(G)| = m(r+1) + r + 2 + \left\lfloor \frac{r-2}{2} \right\rfloor$$

$E(G) = E(\check{S}h_r) \cup E(St(m))$ with $|E(G)| = m(r+1) + r$.

At this instant to attest the above statement we define a labelling $\psi : V(G) \rightarrow \{1, 2, \ldots, m(r+1)+r+2+\lfloor \frac{r-2}{2} \rfloor\}$, for $1 \leq j \leq m$ in the ensuing way.

$\psi(x) = 1, \psi(c_i) = i+1$, for $1 \leq i \leq r$

- while $r \equiv 1(mod2)$

  $\psi(c) = \frac{r(2m+3)+1}{2},\ \psi(y_j) = \frac{r(2j+1)+3}{2}$,

  $\psi(z_t) = r(m+1)+1+r$ for $1 \leq t \leq \lfloor \frac{r-2}{2} \rfloor$

  $$\psi(c_{i,j}) = \begin{cases} \frac{r(2j+1)+3}{2}+i, & \text{if } 1 \leq i \leq \frac{r-1}{2} \\ \frac{r(2j-1)+3}{2}+i, & \text{if } \frac{r+1}{2} \leq i \leq r-1 \\ \frac{r(2m+3)+1}{2}+j, & \text{if } i = r \end{cases}$$

- when $r \equiv 0(mod2)$

  $\psi(c) = \frac{r(2m+3)+2m+2}{2},\ \psi(y_j) = \frac{r(2j+1)+2(j+1)}{2}$,

  $\psi(z_t) = (r+1)(m+1)+t$, for $1 \leq t \leq \lfloor \frac{r-2}{2} \rfloor$

  $$\psi(c_{i,j}) = \begin{cases} \frac{r(2j+1)+2(j+1)}{2}+i, & \text{if } 1 \leq i \leq \frac{r}{2} \\ \frac{r(2j-1)+2j}{2}+i, & \text{if } \frac{r+2}{2} \leq i \leq r \end{cases}$$

It is unproblematic to make sure that every single one of the edge sums form the set of q successive integers

$\{\frac{3r+5}{2}, \frac{3r+7}{2}, \ldots, \frac{r(2m+5)+2m+3}{2}\}$, for $r \equiv 1(mod2)$

$\{\frac{3r+6}{2}, \frac{3r+8}{2}, \ldots, \frac{r(2m+5)+2m+4}{2}\}$, for $r \equiv 0(mod2)$

Therefore by Lemma 1.1, ψ can be inclusive to a SEM total labeling. Hence, the graph G asserts a SEM total labeling. This illustrates so as to

$$\mu_s(\check{S}h(r) \cup St(m)) \leq \lfloor \frac{r-2}{2} \rfloor.$$

In the theorem 2 we originate an upper bound for the SEM deficiency of disjoint union of a shrub graph with two stars.

THEOREM 1.2. *Let $m \geq 3, n \geq 5$, then*

$$\mu_s(\check{S}h_n \cup St_n \cup St_{\lceil \frac{n}{2} \rceil}) \leq \left\lfloor \frac{n-2}{2} \right\rfloor,\ for\ n \equiv 1(mod2)$$

$$\mu_s(\check{S}h_n \cup St_n \cup St_{\frac{n}{2}}) \leq \frac{n}{2} + \left\lfloor \frac{n-1}{3} \right\rfloor,\ for\ n \equiv 0(mod2)$$

Proof. First we classify the vertex and edge sets of shrub graph and two stars in the following way.

$V(\check{S}h_n) = \{c, c_i, c_{i,j}; 1 \leq i \leq n,\ \ 1 \leq j \leq m\}$, $|V(\check{S}h_n)| = 1 + n + mn$

$E(\check{S}h_n) = \{cc_i, c_ic_{i,j}; 1 \leq i \leq n,\ \ 1 \leq j \leq m\}$, $|E(\check{S}h_n)| = n + mn$

$V(St_n) = \{x, y_s; 1 \leq s \leq n\}$, $|V(St(m))| = 1 + n$

$E(St_n) = \{xy_s; 1 \leq s \leq n\}$, $|E(St_n)| = n$

$V(St_{\lceil \frac{n}{2} \rceil}) = \{u, v_l; 1 \leq l \leq \lceil \frac{n}{2} \rceil\}$, $|V(St(m))| = 1 + \lceil \frac{n}{2} \rceil$

$E(St_{\lceil \frac{n}{2} \rceil}) = \{uv_l; 1 \leq l \leq \lceil \frac{n}{2} \rceil\}$, $|E(St_n)| = \lceil \frac{n}{2} \rceil$

- when $n \equiv 1 (mod 2)$

  Let $G \cong \check{S}h_n \cup St_n \cup St_{\lceil \frac{n}{2} \rceil} \cup \lfloor \frac{n-2}{2} \rfloor K_1$, then

  $V(G) = V(\check{S}h_n) \cup V(St_n) \cup V(St_{\lceil \frac{n}{2} \rceil}) \cup \{z_r : 1 \leq r \leq \lfloor \frac{n-2}{2} \rfloor\}$ with

  $$|V(G)| = n(m + 2) + 3 + \left\lceil \frac{n}{2} \right\rceil + \left\lfloor \frac{n-2}{2} \right\rfloor$$

  $E(G) = E(\check{S}h_n) \cup E(St_n) \cup E(St_{\lceil \frac{n}{2} \rceil})$ with $|E(G)| = n(m+2) + \lceil \frac{n}{2} \rceil$.

  Currently to provide evidence of the beyond statement we characterize a labeling $\psi : V(G) \to \{1, 2, \ldots, n(m + 2) + 3 + \lceil \frac{n}{2} \rceil + \lfloor \frac{n-2}{2} \rfloor\}$ for $1 \leq j \leq m$ in the following way.

  $\psi(x) = 1, \psi(c) = n + 2, \psi(u) = n + 3 + \lfloor \frac{n-2}{2} \rfloor$,

  $\psi(c_i) = i + 1$, for $1 \leq i \leq n$

  $\psi(z_r) = n + 2 + r$, for $1 \leq r \leq \lfloor \frac{n-2}{2} \rfloor$

  $\psi(v_l) = n(m + 1) + 3 + \lfloor \frac{n-2}{2} \rfloor + l$, for $1 \leq l \leq \lceil \frac{n}{2} \rceil$

  $\psi(y_s) = n(m + 1) + 3 + \lceil \frac{n}{2} \rceil + \lfloor \frac{n-2}{2} \rfloor + s$, for $1 \leq s \leq n$

  $$\psi(c_{i,j}) = \begin{cases} jn + 6 + \lfloor \frac{n-2}{2} \rfloor + i, & \text{if } 1 \leq i \leq n-3 \\ (j-1)n + 6 + \lfloor \frac{n-2}{2} \rfloor + i, & \text{if } n-2 \leq i \leq n \end{cases}$$

- when $n \equiv 0 (mod 2)$

  Let $H \cong \check{S}h_n \cup St_n \cup St_{\frac{n}{2}} \cup \left(\frac{n}{2} + \lfloor \frac{n-1}{3} \rfloor\right) K_1$, then

  $V(H) = V(\check{S}h_n) \cup V(St_n) \cup V(St_{\frac{n}{2}})$
  $\cup \{z^1_r : 1 \leq r \leq \lfloor \frac{n-1}{3} \rfloor\} \cup \{z^2_r : 1 \leq r \leq \frac{n-2}{2}\} \cup \{z^\star\}$ with

  $$|V(H)| = n(m + 3) + 3 + \left\lfloor \frac{n-1}{3} \right\rfloor$$

  $E(H) = E(\check{S}h_n) \cup E(St_n) \cup E(St_{\frac{n}{2}})$ with $|E(H)| = \frac{n(2m+5)}{2}$.

Now to attest the above declaration we define a labeling $\psi : V(H) \to \{1, 2, \dots, n(m+3)+3+\left\lfloor \frac{n-1}{3} \right\rfloor\}$ for $1 \leq j \leq m$ in the following way.

$\psi(x) = 1, \psi(c) = n+2, \psi(u) = n+3+\lfloor \frac{n-1}{3} \rfloor,$

$\psi(c_i) = i+1$, for $1 \leq i \leq n$

$\psi(z^{\star}) = n(m+1) - m + 4 + \lfloor \frac{n-1}{3} \rfloor$

$\psi(z_r^1) = n+2+r$, for $1 \leq r \leq \lfloor \frac{n-1}{3} \rfloor$

$\psi(z_r^2) = \frac{1}{2}[n(2m+5) - 2m] + 4 + \lfloor \frac{n-1}{3} \rfloor + r$, for $1 \leq r \leq \frac{n-2}{2}$

$\psi(v_l) = n(m+1) + 4 - m + \lfloor \frac{n-1}{3} \rfloor + l$, for $1 \leq l \leq \frac{n}{2}$

$\psi(y_s) = \frac{1}{2}[n(2m+3) - 2m] + 4 + \lfloor \frac{n-1}{3} \rfloor + s$, for $1 \leq s \leq n$

$$\psi(c_{i,j}) = \begin{cases} n(m+3) - m + 3 + \lfloor \frac{n-1}{3} \rfloor + j, & \text{if } i = 1 \\ \frac{1}{2}[n(2j+1) + 6 - 2j] + \lfloor \frac{n-1}{3} \rfloor + i, , & \text{if } 2 \leq i \leq \frac{n}{2} \\ \frac{1}{2}[n(2j-1) + 8 - 2j] + \lfloor \frac{n-1}{3} \rfloor + i, , & \text{if } \frac{n+2}{2} \leq i \leq n \end{cases}$$

It is straightforward to ensure that all edge sums form the set of q consecutive integers

$\{n+4, n+5, \dots, n(m+2)+6+2\lfloor \frac{n-2}{2} \rfloor + \lceil \frac{n}{2} \rceil\}$, for $n \equiv 1(mod2)$

$\{n+4, n+5, \dots, n(m+3)+5+\lfloor \frac{n-1}{3} \rfloor\}$, for $n \equiv 0(mod2)$

Therefore by Lemma 1.1, ψ can be extended to a SEM total labeling. Hence, the graph G and H reveals a SEM total labeling. This substantiates that

$$\mu_s(\check{S}h_n \cup St_n \cup St_{\lceil \frac{n}{2} \rceil}) \leq \left\lfloor \frac{n-2}{2} \right\rfloor \ for \ n \equiv 1(mod2)$$

$$\mu_s(\check{S}h_n \cup St_n \cup St_{\frac{n}{2}}) \leq \frac{n}{2} + \left\lfloor \frac{n-1}{3} \right\rfloor \ for \ n \equiv 0(mod2)$$

In the theorem 3 we bring into being an upper bound for SEM deficiency of disjoint union of shrub graph with path.

THEOREM 1.3. *Let $m \geq 3, q \geq 4$, then*

$$\mu_s(\check{S}h_q \cup P_q) \leq \frac{3q-1}{2}, \ for \ q \equiv 1(mod2)$$
$$\mu_s(\check{S}h_q \cup P_q) \leq \frac{3q-2}{2}, \ for \ q \equiv 0(mod2)$$

Proof. First we describe the vertex and edge sets of shrub graph and path in the following approach.

$V(\check{S}h_q) = \{c, c_i, c_{i,j}; 1 \leq i \leq q,\ \ 1 \leq j \leq m\}$, $|V(\check{S}h_q)| = 1 + q + mq$

$E(\check{S}h_q) = \{cc_i, c_ic_{i,j}; 1 \leq i \leq q,\ \ 1 \leq j \leq m\}$, $|E(\check{S}h_q)| = q + mq$

$V(P_q) = \{x_i; 1 \leq i \leq q\}$, $|V(P_q))| = q$

$E(P_q) = \{x_ix_{i+1}; 1 \leq i \leq q-1\}$, $|E(P_q)| = q - 1$

- when $q \equiv 1(mod2)$

  Let $G \cong \check{S}h_q \cup P_q \cup (\frac{3q-1}{2})K_1$, then

  $V(G) = V(\check{S}h_q) \cup V(P_q) \cup \{z_r^1 : 1 \leq r \leq \frac{q-1}{2}\} \cup \{z_r^2 : 1 \leq r \leq q\}$ with
  $$|V(G)| = \frac{7q + 2mq + 1}{2}$$
  $E(G) = E(\check{S}h_q) \cup E(P_q)$ with $|E(G)| = 2q + mq - 1$.

  Now to prove the above statement we define a labeling $\psi : V(G) \rightarrow \{1, 2, \ldots, \frac{7q+2mq+1}{2}\}$ for $1 \leq j \leq m$ in the following way.

  $\psi(c) = \frac{3q+3}{2}, \psi(c_i) = i + \frac{q+1}{2}$, for $1 \leq i \leq q$

  $\psi(z_r^1) = \frac{3q+3}{2} + r$, for $1 \leq r \leq \frac{q-1}{2}$

  $\psi(z_r^2) = q(m + 2) + 1 + r$, for $1 \leq r \leq q$

  $$\psi(x_i) = \begin{cases} \frac{i+1}{2}, & \text{if } 1 \leq i \leq q, odd \\ n(m+3) + 1 + \frac{i}{2}, & \text{if } 1 \leq i \leq q, even \end{cases}$$

  $$\psi(c_{i,j}) = \begin{cases} 3q + 2(1-i) + q(j-1), & \text{if } 1 \leq i \leq \frac{q-1}{2} \\ 2(2q + 1 - i) + q(j-1), & \text{if } \frac{q+1}{2} \leq i \leq q \end{cases}$$

- when $q \equiv 0(mod2)$

  Let $H \cong \check{S}h_q \cup P_q \cup (\frac{3q-2}{2})K_1$, then

  $V(H) = V(\check{S}h_q) \cup V(P_q) \cup \{z_r^1 : 1 \leq r \leq \frac{q-2}{2}\} \cup \{z_r^2, z_r^3 : 1 \leq r \leq \frac{q}{2}\}$ with
  $$|V(H)| = \frac{7q + 2mq - 2}{2}$$
  $E(H) = E(\check{S}h_q) \cup E(P_q)$ with $|E(H)| = 2q + mq - 1$.

  Now to validate the above statement we term a labeling $\psi : V(H) \rightarrow \{1, 2, \ldots, \frac{7q+2mq-2}{2}\}$ for $1 \leq j \leq m$ in the following way.

  $\psi(c) = \frac{3q+2}{2}, \psi(c_i) = i + \frac{q}{2}$, for $1 \leq i \leq q$

  $\psi(z_r^1) = \frac{3q+2}{2} + r$, for $1 \leq r \leq \frac{q-2}{2}$

  $\psi(z_r^2) = 2q + m(q-1) + r$, for $1 \leq r \leq \frac{q}{2}$

$\psi(z_r^3) = \frac{q(2m+5)}{2} + r$, for $1 \leq r \leq \frac{q}{2}$

$$\psi(x_i) = \begin{cases} \frac{i+1}{2}, & \text{if } 1 \leq i \leq q, odd \\ n(m+3) + \frac{i}{2}, & \text{if } 1 \leq i \leq q, even \end{cases}$$

$$\psi(c_{i,j}) = \begin{cases} \frac{5q}{2} + m(q-1) + j, & \text{if } i = 1 \\ 2q + j(q-1) - 2i + 3, & \text{if } 2 \leq i \leq \frac{q}{2} \\ 3q + j(q-1) - 2i + 2, & \text{if } \frac{q+2}{2} \leq i \leq q \end{cases}$$

It is effortless to test out that all edge sums form the set of w consecutive integers

$\{2q+3, 2q+4, \ldots, q(m+4)+1\}$, for $q \equiv 1(mod 2)$

$\{2q+2, 2q+3, \ldots, q(m+4)\}$, for $q \equiv 0(mod 2)$

Therefore by Lemma 1.1, ψ can be extended to a SEM total labeling. Hence, the graph G and H admits a SEM total labeling. This shows that

$$\mu_s(\check{S}h_q \cup P_q) \leq \frac{3q-1}{2}, \;\; for \;\; q \equiv 1(mod 2)$$
$$\mu_s(\check{S}h_q \cup P_q) \leq \frac{3q-2}{2}, \;\; for \;\; q \equiv 0(mod 2)$$

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