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On a New Type of Soft Topological Spaces via Soft Ideals

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Abstract. Firstly, we give a definition called $soft\ \tilde{J}$-$extremally\ disconnected\ space$ (briefly, $S_{\tilde{J}}.E.D.S$). Secondly, to obtain some characterizations of $S_{\tilde{J}}.E.D.S$ we introduce the notion of $soft\ weak\ regular$-$\tilde{J}$-$closed\ set$. In addition, we give some properties of $S_{\tilde{J}}.E.D.S$. Finally, we give to coincidence some soft sets types in which is $S_{\tilde{J}}.E.D.S$.

AMS (MOS) Subject Classification Codes: 54A05; 54B05
Key Words: Soft $\tilde{J}$-extremally disconnected space, Soft weak regular-$\tilde{J}$-closed set, Soft strong β-$\tilde{J}$-open set, Soft almost strong $\tilde{J}$-open set.

1. Introduction

All the theories introduced in the field of mathematics has been urged by necessity. The concept of Fuzzy Logic[25], whose historical development goes back to ancient times, was first introduced by Zadeh in modern sense. And this concept soon turned into a fundamental issue in the solution of problems in a lot of fields such as medicine, engineering, mathematics, economics, artificial intelligence, intelligent systems, robotics, signal processing and transportation problems. Similarly, the Rough Set Theory[18] which Pawlak prosposed (1982), has become a theory used in areas such as artificial intelligence, learning machines, knowledge acquisition, decision analysis, research on information in databases, specialized systems and reasoning. The concept of Soft Set Theory[17] we use in our study was introduced by Molodtsov in 1999. Molodtsov pointed out that teories such as fuzzy sets, probability and interval mathematics, which are used to solve uncertainties in some fields such as engineering, medicine, economics and environmental science, are insufficient to describe the objects used. And he introduced this new theory which will also take into account the properties of elements of universe set. He also successfully applied this theory to many areas such as Riemann integral, game theory and measurement theory. This works of application has been continued by many scientists([14], [16], [20], [21]). In [13], researchers gave concept of measurable soft mappings and studied the concept in detail. Following the first results of the soft set, Maji et al.[15] gave the basic soft set concepts and some propositions about them.

Topological structures on soft sets were first studied by Shabir and Naz[19]. Notions of soft neighborhood and soft closure are defined by using soft topology notion and many features and propositions related to soft closure concept are given. In addition, the separation axioms known in general topology are applied to soft topological spaces and soft T_0, soft T_1, soft T_2, soft T_3, soft T_4 and soft normal space concepts are given and their relation to each other is examined in detail. Hussain and Ahmad[10] introduced an inclusion of an element in a soft set, soft interior point, soft exterior, soft interior and soft boundary. The first study on soft functions was done by Ahmad and Kharal[1]. Zorlutuna et al.[26] have given definitions such as the union and intersection of any number of soft sets. Although different definitions have been made about the notion of soft point, the definition given by Bayramov and Aras[7] is used widely. We used this definition in our work.

The first studies on soft weak open(closed) sets have started with Chen[8]. Chen gave definitions of soft semi-open set, soft semi-closed set, and also defined soft semi-interior and soft semi-closure concepts by using these definitions. In addition, the relationships between these types of weak soft set and soft open and soft closed sets are examined. Arockiarani and Lancy[5] introduced soft $g\beta$-closed and soft $gs\beta$-closed sets and examined their some properties. Yuksel et al.[22] have studied soft regular generalized closed(open) sets. Yumak and Kaymakci[23] studied on soft β-open sets and made some research on the relations between this new soft sets and other soft sets in the literature. In addition, new soft weak continuity types have been introduced with the help of soft β-open sets and their properties have been examined.

Ideal concept on soft sets is given by Kandil et al.[11] in 2014. The concept of soft local function is also given by these authors first, and properties of soft local function are shown. With the help of soft local function concept, soft star closure operator, a new concept, has been introduced and its properties have been given. Kandil et al.[12] obtained some new types of soft sets that are weaker than the soft open sets in soft ideal topological spaces by using soft interior, soft closure, soft local function and soft star closure operations. In addition they shown relations between each other and under what conditions they are equivalent. Also in 2017, Aras and et al.[4] studied the notions of $\tilde{I}_{\tilde{c}}$ soft free ideal and soft $\tilde{c}$-ideals. Here they investigated the soft ideal extension of a given soft topological space via the concept of soft ideals.

In this work, we first defined two new soft sets called soft strong β-$\tilde{J}$-open and soft almost strong $\tilde{J}$-open. Then, we studied reletionships between these definitions and the existing soft set types. And we showed all these reletionships by Diagram 2. Second, we gave soft weak regular-$\tilde{J}$-closed and soft I-exremally disconnected space. With the aid of this soft space type we have defined, we have re-examined the relations between the existing soft sets and obtained some equivalents between them. Finaly, we used all these acquired properties on soft continuity types.

2. Notations and Preliminaries

Given a universe set U and a parameter set E that contain all the possible properties of elements in U. Besides, let $P(U)$ be collection of all subsets of U and $A \subseteq E$.

Definition 2.1. [17] *Given a F mapping defined as $F : A \longrightarrow P(U)$. In this case, the (F, A) (or F_A) pair is called a soft set on U. The family of all soft sets on U is denoted by $SS(U)_A$.*

Definition 2.2. [15] *Let $A, B \subseteq E$ and $F_A, G_B \in SS(U)_E$. If (i) $A \subseteq B$, and (ii) $\forall e \in A, F(e) \subseteq G(e)$, then F_A is a soft subset of G_B and we can write $F_A \tilde{\subseteq} G_B$.*

Definition 2.3. [15] *$F_E \in SS(U)_E$ is called to be (i) null soft set indicated by Φ, if $\forall e \in E, F(e) = \phi$, (ii) absolute soft set indicated by $\tilde{U}$, if $\forall e \in E,\ F(e) = U$.*

Definition 2.4. *Let $A_1, A_2 \subseteq E$ and $F_{A_1}, G_{A_2} \in SS(U)_E$*

$a)$ [15] *soft union of F_{A_1} and G_{A_2} is equal to K_{A_3}, where $A_3 = A_1 \cup A_2$ and $\forall\ e \in A_3$,*

$$K(e) = \begin{cases} F(e) & , \quad \text{if } e \in A_1 - A_2 \\ G(e) & , \quad \text{if } e \in A_2 - A_1 \\ F(e) \cup G(e) & , \quad \text{if } e \in A_1 \cap A_2 \end{cases}$$

We write $F_{A_1} \tilde{\cup}\, G_{A_2} = K_{A_3}$.

$b)$ [9] *soft intersection of F_{A_1} and G_{A_2} is equal to K_{A_3}, where $A_3 = A_1 \cap A_2$, and $\forall\ e \in A_3$, $K(e) = F(e) \cap G(e)$. We write $F_{A_1} \tilde{\cap}\, G_{A_2} = K_{A_3}$.*

Definition 2.5. [19] *Let $u \in U$ and $F_E \in SS(U)_E$. Then,*
$i)$ $u \in F_E \Longleftrightarrow u \in F(e), \forall\ e \in E$,
$ii)$ $u \notin F_E \Longleftrightarrow u \notin F(e), \exists\ e \in E$.

Definition 2.6. [3] *The relative complement of H_E soft set is denoted by $(H_E)'$ (or (H', E)) where $H' : E \longrightarrow P(U)$ is a map with by $H'(e) = U - H(e)$ for all $e \in E$.*

Definition 2.7. [19] *Let $\lambda \tilde{\subseteq} SS(U)_E$. In this situation, λ and (U, λ, E) are said to be soft topology and soft topological space (briefly, STS) over U respectively if*

1) Φ, $\tilde{U} \in \lambda$,

2)*For any number of soft sets in λ, the union of them belongs to λ,*

3) *For any two soft sets in λ, the intersection of them belongs to λ.*

If $H_E \in \lambda$, H_E is called a soft open sets in U.

Definition 2.8. [19] *Let (U, λ, E) be a STS over U. A soft set F_E over U is said to be a soft closed set in U, if its relative complement $(F_E)'$ belongs to λ.*

Throughout this article $SO(U)$ ($SC(U)$) will identify all soft open(closed) sets.

Definition 2.9. *Let (U, λ, E) be a STS and $F_E \in SS(U)_E$. In this case,*

a) [10] $int(F_E) = \tilde{\bigcup} \{F_E \tilde{\supseteq} G_E : G_E \in \lambda\}$,

b) [19] $cl(F_E) = \tilde{\bigcap} \{G_E \tilde{\supseteq} F_E : G_E \in \lambda'\}$.

Proposition 2.10. [10] *Let (U, λ, E) be a STS and H_E, $L_E \in SS(U)_E$. Then,*

i) $int(int(H_E)) = int(H_E)$,
ii) $H_E \tilde{\subseteq} L_E \Longrightarrow int(H_E) \tilde{\subseteq} int(L_E)$,
iii) $cl(cl(H_E)) = cl(H_E)$,
iv) $H_E \tilde{\subseteq} L_E \Longrightarrow cl(H_E) \tilde{\subseteq} cl(L_E)$.

Definition 2.11. [7] *If for any $e \in E$, $L(e) = \{u\}$ and for all $e' \in (E - \{e\})$ $L(e') = \phi$, the L_E soft set which is in $SS(U)_E$ is named a soft point and it is presented by (u_e, E) or u_e.*

Definition 2.12. [26] *Let $u_e = L_E$ be a $soft\ point$ over U. $u_e \in H_E \Longleftrightarrow \forall\ e \in E$, $L(e)$ $\tilde{\subseteq}$ $H(e)$.*

Definition 2.13. [26] *Let (U, λ, E) be a STS and $G_E \in SS(U)_E$. G_E is named a soft neighborhood of the soft point u_e if there exists an open soft set H_E such that $u_e \in H_E$ $\tilde{\subseteq}$ G_E. A soft set G_E in a STS (U, λ, E) is called a soft neighborhood of the soft set F_E if there is an open soft set H_E satisfying $F_E \tilde{\subseteq} H_E \tilde{\subseteq} G_E$. All soft neighborhood families of soft point u_e are indicated by $N_\lambda(u_e)$.*

Definition 2.14. [1] *Assume $SS(U)_A$ and $SS(Y)_B$ be soft set families with mappings $u : U \to Y$ and $p : A \to B$. Also assume $f_{pu} : SS(U)_A \to SS(Y)_B$ be mapping. Then,*

1) If H_A is in $SS(U)_A$, then under mapping f_{pu}, H_A image which is written as $f_{pu}(H_A)$ $= (f_{pu}(H), p(A))$, is a soft set in $SS(Y)_B$ such that

$$f_{pu}(H)(b) = \begin{cases} \cup_{a \in p^{-1}(b) \cap A} u(H(a)) & , \quad p^{-1}(b) \cap A \neq \phi \\ \phi & , \quad \text{otherwise.} \end{cases}$$

for all $b \in B$.

2) Let $L_B \in SS(Y)_B$. Under f_{pu}, L_B inverse image , written as $f_{pu}^{-1}(L_B) = (f_{pu}^{-1}(L), p^{-1}(B))$, is a soft set in $SS(U)_A$ such that

$$f_{pu}^{-1}(L)(a) = \begin{cases} u^{-1}(L(p(a))) & , \quad p(a) \in B \\ \phi & , \quad \text{otherwise.} \end{cases}$$

for all $a \in A$.

Definition 2.15. [11] *Let $\tilde{J} \subseteq SS(U)_E$ and $\tilde{J} \neq \Phi$, then $\tilde{J}$ is named a soft ideal on U and with a fixed set E if*

i) If H_E and L_E are in $\tilde{J}$, then the union of H_E and L_E are in $\tilde{J}$,

ii) If H_E is in $\tilde{J}$ and $L_E \tilde{\subseteq} H_E$, then L_E is in $\tilde{J}$.

Definition 2.16. [11] *Let $(U, \lambda, \tilde{J}, E)$ be a soft ideal topological space (briefly, SITS). Then,*

$$(H_E)^*_{(\tilde{J},\lambda)} = (H_E)^* = \widetilde{\bigcup}\{u_e : O_{u_e} \tilde{\cap} H_E \notin \tilde{J}, \forall O_{u_e} \in \lambda\}$$

is named the soft local function of H_E related to $\tilde{J}$, λ and also $u_e \in O_{u_e} \tilde{\subseteq} \lambda$.

Theorem 2.17. [11] *For a SITS $(U, \lambda, \tilde{J}, E)$, the $cl^* : SS(U)_E \to SS(U)_E$ soft closure operator which is describe by: $cl^*(H_E) = H_E \tilde{\cup} (H_E)^*$ satisfies the axioms of Kuratowski.*

Lemma 2.18. [11] *Let* $(U, \lambda, \tilde{J}, E)$ *be a SITS and* H_E, $L_E \in SS(U)_E$. *In this case, we can say the following features:*

$a)$ $H_E \tilde{\subseteq} L_E \Rightarrow (H_E)^* \tilde{\subseteq} (L_E)^*$,
$b)$ $(H_E)^* = cl((H_E)^*) \tilde{\subseteq} cl(H_E)$,
$c)$ $((H_E)^*)^* \tilde{\subseteq} (H_E)^*$,
$d)$ $(H_E \tilde{\cup} L_E)^* = (H_E)^* \tilde{\cup} (L_E)^*$,
$e)$ $K_E \in \lambda \Rightarrow K_E \tilde{\cap} (H_E)^* \tilde{\subseteq} (H_E \tilde{\cap} K_E)^*$.

Definition 2.19. *Let* $(U, \lambda, \tilde{J}, E)$ *be a SITS and* $F_E \in SS(U)_E$. *Then* F_E *is called*

$a)$ [2] *soft* $\tilde{J}$*-open if* $F_E \tilde{\subseteq} int((F_E)^*)$,
$b)$ [12] *soft pre-*$\tilde{J}$*-open if* $F_E \tilde{\subseteq} int(cl^*(F_E))$,
$c)$ [12] *soft* α*-*$\tilde{J}$*-open if* $F_E \tilde{\subseteq} int(cl^*(int(F_E)))$,
$d)$ [12] *soft semi-*$\tilde{J}$*-open if* $F_E \tilde{\subseteq} cl^*(int(F_E))$,
$e)$ [12] *soft* β*-*$\tilde{J}$*-open if* $F_E \tilde{\subseteq} cl(int(cl^*(F_E)))$,
$f)$ [24] $almost\ soft$-$\tilde{J}$-$open$ *if* $F_E \tilde{\subseteq} cl(int((F_E)^*))$.

The relationship obtained in [12] for some of the soft sets described above are given in the following diagram.

$$\begin{array}{ccccc} soft\ open & \rightarrow & soft\ \alpha\text{-}\tilde{J}\text{-}open & \rightarrow & soft\ semi\text{-}\tilde{J}\text{-}open \\ & & \downarrow & & \downarrow \\ soft\ \tilde{J}\text{-}open & \rightarrow & soft\ pre\text{-}\tilde{J}\text{-}open & \rightarrow & soft\ \beta\text{-}\tilde{J}\text{-}open \end{array}$$

$Diagram\ I$

Definition 2.20. *Let* $(X, \lambda, \tilde{J}, E)$ *be a SITS and* $F_E \in SS(X)_E$. *In this case* F_E *is called*

$a)$ *soft strong* β*-*$\tilde{J}$*-open if* $F_E \tilde{\subseteq} cl^*(int(cl^*(F_E)))$,
$b)$ *soft almost strong* $\tilde{J}$*-open if* $F_E \tilde{\subseteq} cl^*(int((F_E)^*))$.

We denoted by $S_{\tilde{J}}O(X)$ (resp. $SP_{\tilde{J}}O(X)$, $S\alpha_{\tilde{J}}O(X)$, $SS_{\tilde{J}}O(X)$, $S\beta_{\tilde{J}}O(X)$, $aS_{\tilde{J}}O(X)$, $Ss\beta_{\tilde{J}}O(X)$, $Sas_{\tilde{J}}O(X)$) the family of all $soft\ \tilde{J}$-$open$ (resp. $soft\ pre$-$\tilde{J}$-$open$, $soft\ \alpha$-$\tilde{J}$-$open$, $soft\ semi$-$\tilde{J}$-$open$, $soft\ \beta$-$\tilde{J}$-$open$, $almost\ soft\ \tilde{J}$- $open$, $soft\ strong\ \beta$ -$\tilde{J}$-$open$, $soft\ almost\ strong\ \tilde{J}$-$open$) soft subsets of $(X, \lambda, \tilde{J}, E)$.

Proposition 2.21. *Let* $(X, \lambda, \tilde{J}, E)$ *be a SITS and* $F_E \in SS(X)_E$. *In this case,*

a) Every $soft\ \tilde{J}$-$open$ *set is a* $soft\ almost\ strong\ \tilde{J}$-$open$ *set.*
b) Every $soft\ pre$-$\tilde{J}$-$open$ *set is a* $soft\ strong\ \beta$-$\tilde{J}$-$open$ *set.*
c) Every $soft\ almost\ strong\ \tilde{J}$-$open$ *set a is* $soft\ strong\ \beta$-$\tilde{J}$-$open$ *set.*
d) Every $soft\ semi$-$\tilde{J}$-$open$ *set a is* $soft\ strong\ \beta$-$\tilde{J}$-$open$ *set.*
e) Every $almost\ soft\ \tilde{J}$-$open$ *set is a* $soft\ \beta$-$\tilde{J}$-$open$ *set.*
f) Every $soft\ strong\ \beta$-$\tilde{J}$-$open$ *set a is* $soft\ \beta$-$\tilde{J}$-$open$ *set.*
g) Every $soft\ almost\ strong\ \tilde{J}$-$open$ *set is an* $almost\ soft\ \tilde{J}$-$open$ *set.*

Proof. a) Let F_E be a $soft\ \tilde{J}$-$open$ set. Then, $F_E \tilde{\subseteq} int((F_E)^*) \Rightarrow F_E \tilde{\subseteq} cl^*(F_E) \tilde{\subseteq} cl^*(int(F_E)^*)$. Therefore, F_E is $soft\ almost\ strong\ \tilde{J}$-$open$ set.

b) Let F_E be a $soft$ pre-$\tilde{J}$-$open$ set. Then, $F_E \tilde{\subseteq} int(cl^*(F_E)) \Rightarrow F_E \tilde{\subseteq} cl^*(F_E) \tilde{\subseteq} cl^*(int(cl^*(F_E)))$. Therefore, F_E is $soft$ $strong$ β-$\tilde{J}$-$open$ set.

c) Let F_E be a $soft$ $almost$ $strong$ $\tilde{J}$-$open$ set. Then, $F_E \tilde{\subseteq} cl^*(int((F_E)^*))$. We know well that $(F_E)^* \tilde{\subseteq} cl^*(F_E)$. Hence, $F_E \tilde{\subseteq} cl^*(int((F_E)^*)) \tilde{\subseteq} cl^*(int(cl^*\ (F_E)))$. Therefore, F_E is $soft$ $strong$ β-$\tilde{J}$-$open$ set.

d) Let F_E be a $soft$ $semi$-$\tilde{J}$-$open$ set. Then, $F_E \tilde{\subseteq} cl^*(int(F_E))$. We know well that $F_E \tilde{\subseteq} cl^*(F_E)$. Hence, $F_E \tilde{\subseteq} cl^*(int(F_E)) \tilde{\subseteq} cl^*(int(cl^*(F_E)))$. Therefore, F_E is $soft$ $strong$ β-$\tilde{J}$-$open$ set.

e) Let F_E be an $almost$ $soft$ $\tilde{J}$-$open$ set. Then, $F_E \tilde{\subseteq} cl(int((F_E)^*))$. We know well that $(F_E)^* \tilde{\subseteq} cl^*(F_E)$. Hence, $F_E \tilde{\subseteq} cl(int((F_E)^*)) \tilde{\subseteq} cl(int(cl^*(F_E)))$. Therefore, F_E is $soft$ β-$\tilde{J}$-$open$ set.

f) Let F_E be a $soft$ $strong$ β-$\tilde{J}$-$open$ set. Then, $F_E \tilde{\subseteq} cl^*(int(cl^*(F_E)))$. We know well that $cl^*(F_E) \tilde{\subseteq} cl(F_E)$. Hence, $F_E \tilde{\subseteq} cl^*(int(cl^*(F_E))) \tilde{\subseteq} cl(int(cl^*(F_E)))$. Therefore, F_E is $soft$ β-$\tilde{J}$-$open$ set.

g) Let F_E be a $soft$ $almost$ $strong$ $\tilde{J}$-$open$ set. Then, $F_E \tilde{\subseteq} cl^*(int((F_E)^*))$. We know well that $cl^*(F_E) \tilde{\subseteq} cl(F_E)$. Hence, $F_E \tilde{\subseteq} cl^*(int((F_E)^*)) \tilde{\subseteq} cl(int\ ((F_E)^*))$. Therefore, F_E is $almost$ $soft$ $\tilde{J}$-$open$ set. □

Remark 2.22. *The following examples show that the inverse of the statements in Proposition* 2.21 *is not generally correct.*

Example 2.23. *Let* $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ *and* $\lambda = \{\Phi, \tilde{X}, F_E^1, F_E^2, F_E^3\}$, $\tilde{J} = \{\Phi, G_E^1, G_E^2, G_E^3\}$, *where* F_E^1, F_E^2, F_E^3, G_E^1, G_E^2, G_E^3 *are soft sets such that* $F^1(e_1) = \{x_1, x_3\}$, $F^1(e_2) = \phi$, $F^2(e_1) = \{x_4\}$, $F^2(e_2) = \{x_4\}$, $F^3(e_1) = \{x_1, x_3, x_4\}$, $F^3(e_2) = \{x_4\}$, $G^1(e_1) = \{x_4\}$, $G^1(e_2) = \phi$, $G^2(e_1) = \phi$, $G^2(e_2) = \{x_4\}$, $G^3(e_1) = \{x_4\}$, $G^3(e_2) = \{x_4\}$.

a) Let $L_E = \{\{x_1, x_2\}, \phi\} \in SS(X)_E$. *Since* $L_E \tilde{\nsubseteq} int((L_E)^*) = \{\{x_1, x_3\}, \phi\}$ *and* $L_E \tilde{\subseteq} cl^*(int((L_E)^*)) = \{\{x_1, x_2, x_3\}, \{x_1, x_2, x_3\}\}$, L_E *is* $soft$ $almost$ $strong$ $\tilde{J}$-$open$ *but not* $soft$ $\tilde{J}$-$open$.

b) Let $L_E = \{\{x_1, x_2\}, \phi\} \in SS(X)_E$. *Since* $L_E \tilde{\nsubseteq} int(cl^*(L_E)) = \{\{x_1, x_3\}, \phi\}$ *and* $L_E \tilde{\subseteq} cl^*(int(cl^*(L_E))) = \{\{x_1, x_2,\ x_3\}, \{x_1, x_2, x_3\}\}$, L_E *is* $soft$ $strong$ β-$\tilde{J}$-$open$ *but not* $soft$ pre-$\tilde{J}$-$open$.

c) Let $L_E = \{\{x_1, x_4\}, \{x_4\}\} \in SS(X)_E$. *Since* $L_E \tilde{\nsubseteq} cl^*(int((L_E)^*)) = \{\{x_1, x_2, x_3\}, \{x_1, x_2, x_3\}\}$ *and* $L_E \tilde{\subseteq} cl^*(int(cl^*(L_E))) = \tilde{X}$, L_E *is* $soft$ $strong$ β-$\tilde{J}$-$open$ *but not* $soft$ $almost$ $strong$ $\tilde{J}$-$open$.

d) Let $L_E = \{\{x_1, x_4\}, \{x_4\}\} \in SS(X)_E$. *Since* $L_E \tilde{\nsubseteq} cl^*(int(L_E)) = \Phi$ *and* $L_E \tilde{\subseteq} cl^*(int(cl^*(L_E))) = \tilde{X}$, L_E *is* $soft$ $strong$ β-$\tilde{J}$-$open$ *but not* $soft$ $semi$-$\tilde{J}$-$open$.

e) Let $L_E = \{\{x_1, x_4\}, \{x_4\}\} \in SS(X)_E$. *Since* $L_E \tilde{\nsubseteq} cl(int((L_E)^*)) = \{\{x_1, x_2, x_3\}, \{x_1, x_2, x_3\}\}$ *and* $L_E \tilde{\subseteq} cl(int(cl^*(L_E))) = \tilde{X}$, L_E *is* $soft$ β-$\tilde{J}$-$open$ *but not* $almost$ $soft$ $\tilde{J}$-$open$.

Example 2.24. *Let* $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ *and* $\lambda = \{\Phi, \tilde{X}, F_E^1, F_E^2, F_E^3\}$, $\tilde{J} = \{\Phi, G_E^1, G_E^2, G_E^3\}$, *where* F_E^1, F_E^2, F_E^3, G_E^1, G_E^2, G_E^3 *are soft sets such that* $F^1(e_1) = \{x_1, x_3\}$, $F^1(e_2) = \phi$, $F^2(e_1) = \{x_4\}$, $F^2(e_2) = \phi$, $F^3(e_1) = \{x_1, x_3, x_4\}$, $F^3(e_2) = \phi$, $G^1(e_1) = \{x_3\}$, $G^1(e_2) = \phi$, $G^2(e_1) = \{x_4\}$, $G^2(e_2) = \phi$, $G^3(e_1) = \{x_3, x_4\}$, $G^3(e_2) = \phi$.

Let $L_E = \{\{x_2, x_4\}, \phi\} \in SS(X)_E$. *Since* $L_E \tilde{\nsubseteq} cl^*(int(cl^*(L_E))) = \{\{x_4\}, \phi\}$ *and* $L_E \tilde{\subseteq} cl(int(cl^*(L_E)))$ $=\{\{x_2, x_4\}, X\}$, L_E *is* $soft\ \beta\text{-}\tilde{J}\text{-}open$ *but not* $soft\ strong\ \beta\text{-}\tilde{J}\text{-}open$.

We have obtained the following diagram by using Diagram 1, Proposition 2.21 and counterexamples.

$$\begin{array}{ccccc}
soft\ open & & & & \\
\downarrow & & & & \\
soft\ \alpha\text{-}\tilde{J}\text{-}open & \rightarrow & soft\ semi\text{-}\tilde{J}\text{-}open & & \\
\downarrow & & \downarrow & & \\
soft\ pre\text{-}\tilde{J}\text{-}open & \rightarrow & soft\ strong\ \beta\text{-}\tilde{J}\text{-}open & \rightarrow & soft\ \beta\text{-}\tilde{J}\text{-}open \\
\uparrow & & \uparrow & & \uparrow \\
soft\ \tilde{J}\text{-}open & \rightarrow & soft\ almost\ strong\ \tilde{J}\text{-}open & \rightarrow & almost\ soft\ \tilde{J}\text{-}open
\end{array}$$

$Diagram$ 2

3. Soft $\tilde{J}$-Exremally Disconnected Spaces

Definition 3.1. *Let* $(X, \lambda, \tilde{J}, E)$ *be a SITS and* $F_E \in SS(X)_E$. *Then* F_E *is called soft weak regular-*$\tilde{J}$*-closed if* $F_E = cl^*(int(F_E))$.

We denoted by $SwR_{\tilde{J}}C(X)$ the family of all $soft\ weak\ regular\text{-}\tilde{J}\text{-}closed$ soft subsets of $(X, \lambda, \tilde{J}, E)$.

Definition 3.2. [6] *Let* (X, λ, E) *be a STS. Then* (X, λ, E) *is called as soft extremally disconnected (briefly* $S.E.D.S$*) if* $cl(F_E) \in \lambda$ *for each* $F_E \in \lambda$.

Definition 3.3. $(X, \lambda, \tilde{J}, E)$ *is called as soft* $\tilde{J}$*-extremally disconnected (briefly* $S_{\tilde{J}}.E.D.S$*) if* $cl^*(F_E) \in \lambda$ *for each* $F_E \in \lambda$.

Proposition 3.4. *For a soft ideal topological space* $(X, \lambda, \tilde{J}, E)$, *the following properties are equivalent:*

$a)$ $(X, \lambda, \tilde{J}, E)$ *is* $S_{\tilde{J}}.E.D.S$,
$b)$ $SS_{\tilde{J}}O(X) \tilde{\subseteq} SP_{\tilde{J}}O(X)$,
$c)$ $SwR_{\tilde{J}}C(X) \tilde{\subseteq} \lambda$.

Proof. $a) \Rightarrow b)$ Let $F_E \in SS_{\tilde{J}}O(X)$. Then $F_E \tilde{\subseteq} cl^*(int(F_E))$ and by $a)$ $cl^*(int(F_E)) \in \lambda$. Therefore, we have $F_E \tilde{\subseteq} cl^*(int(F_E)) = int(cl^*(int(F_E))) \tilde{\subseteq} int(cl^*(F_E))$. This shows that $F_E \in SP_{\tilde{J}}O(X)$.

$b) \Rightarrow c)$ Let $F_E \in SwR_{\tilde{J}}C(X)$. Then $F_E = cl^*(int(F_E))$ and hence $F_E \in SS_{\tilde{J}}O(X)$. By $b)$, $F_E \in SP_{\tilde{J}}O(X)$ and $F_E \tilde{\subseteq} int(cl^*(F_E))$. Morever, F_E is soft $\lambda^*\text{-}closed$ and F_E

$\tilde{\subseteq}$ $int(cl^*(F_E)) = int(F_E)$. Therefore, we obtain $F_E \in \lambda$.

$c) \Rightarrow a)$ For $F_E \in \lambda$, we need to show that $cl^*(F_E) \in SwR_{\tilde{J}}C(X)$. Since $int(cl^*(F_E))$ $\tilde{\subseteq}$ $cl^*(F_E)$, we have $(int(cl^*(F_E)))^* \tilde{\subseteq} (cl^*(F_E))^* = ((F_E) \tilde{\cup} (F_E)^*)^* = (F_E)^* \tilde{\cup} ((F_E)^*)^* \tilde{\subseteq} (F_E)^* \tilde{\cup} (F_E)^* = (F_E)^* \tilde{\subseteq} cl^*(F_E)$ by using Lemma 2.18 $d), c)$ respectively and hence $(int(cl^*(F_E)))^* \tilde{\subseteq} cl^*(F_E)$. So, we have $cl^*(int(cl^*(F_E))) = int(cl^*(F_E)) \tilde{\cup} (int(cl^*(F_E)))^* \tilde{\subseteq} cl^*(F_E)$ and hence

$$cl^*(int(cl^*(F_E))) \tilde{\subseteq} cl^*(F_E). \quad (3.1)$$

On the other hand, since F_E is soft open, according to $Diagram\ I$, it is a $soft\ pre\text{-}\tilde{J}\text{-}open\ set$ and hence we have $F_E \tilde{\subseteq} int(cl^*(F_E))$. Then, we have

$$cl^*(F_E) \tilde{\subseteq} cl^*(int(cl^*(F_E))). \quad (3.2)$$

By using (3.1) and (3.2), we have $cl^*(F_E) = cl^*(int(cl^*(F_E)))$. This shows that $cl^*(F_E)$ is $soft\ weak\ regular\text{-}\tilde{J}\text{-}closed$ by using Definition 2.20 c). Furthermore, since $SwR_{\tilde{J}}C(X) \tilde{\subseteq} \lambda$, we have $cl^*(F_E) \in \lambda$. This shows that $(X, \lambda, \tilde{J}, E)$ is $S_{\tilde{J}}.E.D.S$ by Definition 3.3.

□

Example 3.5. *Let $(X, \lambda, \tilde{J}, E)$ is a SITS. If $\tilde{J} = SS(X)_E$, then $(X, \lambda, \tilde{J}, E)$ is $S_{\tilde{J}}.E.D.S.$*

Remark 3.6. *In the following examples, we showed that soft $\tilde{J}$-extremally disconnectedness and soft extremally disconnectedness are independent of each other.*

Example 3.7. *Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\lambda = \{\Phi, \tilde{X}, F_E^1, F_E^2, F_E^3\}$, $\tilde{J} = \{\Phi, \tilde{X}, F_E^1, F_E^2, F_E^3\}$ where F_E^1, F_E^2, F_E^3 are soft sets such that $F^1(e_1) = \{x_1\}$, $F^1(e_2) = \phi$, $F^2(e_1) = \{x_2\}$, $F^2(e_2) = \phi$, $F^3(e_1) = \{x_1, x_2\}$, $F^3(e_2) = \phi$. Then $(X, \lambda, \tilde{J}, E)$ is a $S_{\tilde{J}}.E.D.S$ which is not $S.E.D.S$. For $F_E \in \lambda$, since $(F_E)^* = \Phi$, we have $cl^*(F_E) = F_E$ $\tilde{\cup}$ $(F_E)^* = F_E$. This shows that $(X, \lambda, \tilde{J}, E)$ is a $S_{\tilde{J}}.E.D.S.$ On the other hand, for F_E^1 $= \{\{x_1\}, \phi\} \in \lambda$, $cl(F_E^1) = \{\{x_1, x_3\}, X\} \notin \lambda$. Hence, $(X, \lambda, \tilde{J}, E)$ is not $S.E.D.S.$*

Example 3.8. *Let $X = \{x_1, x_2, x_3, x_4, x_5\}$, $E = \{e_1, e_2\}$ and $\lambda = \{\Phi, \tilde{X}, F_E^1, F_E^2, F_E^3, F_E^4\}$, $\tilde{J} = \{\Phi, G_E^1, G_E^2, G_E^3\}$, where $F_E^1, F_E^2, F_E^3, F_E^4$ G_E^1, G_E^2, G_E^3 are soft sets such that $F^1(e_1) = \{x_1\}$, $F^1(e_2) = \phi$, $F^2(e_1) = \{x_1, x_3\}$, $F^2(e_2) = \phi$, $F^3(e_1) = \{x_1, x_2, x_4\}$, $F^3(e_2) = \phi$, $F^4(e_1) = \{x_1, x_2, x_3, x_4\}$, $F^4(e_2) = \phi$, $G^1(e_1) = \{x_1\}$, $G^1(e_2) = \phi$, $G^2(e_1)$ $= \{x_4\}$, $G^2(e_2) = \phi$, $G^3(e_1) = \{x_1, x_4\}$, $G^3(e_2) = \phi$. Then $(X, \lambda, \tilde{J}, E)$ is a $S.E.D.S$ which is not $S_{\tilde{J}}.E.D.S.$ For $F_E \in \lambda$, since $cl(F_E) = X$, $(X, \lambda, \tilde{J}, E)$ is a $S.E.D.S.$ On the other hand, for $F_E^3 \in \lambda$, since $(F_E^3)^* = \{\{x_2, x_4, x_5\}, X\}$, we have $cl^*(F_E^3) = F_E^3$ $\tilde{\cup}$ $(F_E^3)^* = \{\{x_1, x_2, x_4, x_5\}, X\} \notin \lambda$. This shows that $(X, \lambda, \tilde{J}, E)$ is not $S_{\tilde{J}}.E.D.S.$*

Proposition 3.9. *Let $(X, \lambda, \tilde{J}, E)$ be a SITS and $\tilde{J} = \{\Phi\}$. Then $(X, \lambda, \tilde{J}, E)$ is a $S_{\tilde{J}}.E.D.S$ iff $(X, \lambda, \tilde{J}, E)$ is a $S.E.D.S.$*

Proof. If $\tilde{J} = \{\Phi\}$, then is is well-known that $(F_E)^* = cl(F_E)$ and $cl^*(F_E) = F_E \tilde{\cup} (F_E)^*$ $= F_E \tilde{\cup} cl(F_E) = cl(F_E)$. Consequently, we obtain $cl(F_E) = cl^*(F_E) \in \lambda$ for every F_E $\in \lambda$. This shows that $(X, \lambda, \tilde{J}, E)$ is a $S_{\tilde{J}}.E.D.S$ iff it is $S.E.D.S$.

□

Lemma 3.10. *Let $(X, \lambda, \tilde{J}, E)$ be a SITS. If $F_E \tilde{\cap} G_E = \Phi$ for every F_E, $G_E \in \lambda$, then $F_E \tilde{\cap} cl^*(G_E) = \Phi$.*

Proof. Since $F_E \tilde{\cap} G_E = \Phi$, we have $F_E \tilde{\cap} cl^*(G_E) = F_E \tilde{\cap} [G_E \tilde{\cup} (G_E)^*] = [F_E \tilde{\cap} G_E] \tilde{\cup} [F_E \tilde{\cap} (G_E)^*] \tilde{\subseteq} [F_E \tilde{\cap} G_E] \tilde{\cup} [F_E \tilde{\cap} G_E]^* = cl^*[F_E \tilde{\cap} G_E]$ by using Lemma 2.18.e). On the other hand, since $\Phi^* = \Phi$ and $cl^*(\Phi) = \Phi$, we have $F_E \tilde{\cap} cl^*(G_E) \tilde{\subseteq} cl^*[F_E \tilde{\cap} G_E] = \Phi$. Thus, we obtain that $F_E \tilde{\cap} cl^*(G_E) = \Phi$.

□

Lemma 3.11. *Let $(X, \lambda, \tilde{J}, E)$ be a $S_{\tilde{J}}.E.D.S$. If $F_E \tilde{\cap} G_E = \Phi$ for every F_E, $G_E \in \lambda$, then $cl^*(F_E) \tilde{\cap} cl^*(G_E) = \Phi$.*

Proof. By the aid of Definition 3.3 and Lemma 3.10, the proof is clear. □

Lemma 3.11 is important because it is given that in any $S_{\tilde{J}}.E.D.S$ every two disjoint $soft$ λ-$open\ sets$ have disjoint $soft$ λ^*-$closures$.

Lemma 3.12. *Let $(X, \lambda, \tilde{J}, E)$ be a SITS. If $cl^*(F_E) \tilde{\cap} cl^*(G_E) = \Phi$ for any soft subsets F_E and G_E, then $F_E \tilde{\cap} G_E = \Phi$.*

Proof. Since $F_E \tilde{\subseteq} cl^*(F_E)$ and $G_E \tilde{\subseteq} cl^*(G_E)$, we have $F_E \tilde{\cap} G_E \tilde{\subseteq} cl^*(F_E) \tilde{\cap} cl^*(G_E) = \Phi$. Then, we have $F_E \tilde{\cap} G_E = \Phi$.

□

Theorem 3.13. *Let $(X, \lambda, \tilde{J}, E)$ be a $S_{\tilde{J}}.E.D.S$. For every F_E, $G_E \in \lambda$, the following property are satisfied: $F_E \tilde{\cap} G_E = \Phi$ iff $cl^*(F_E) \tilde{\cap} cl^*(G_E) = \Phi$.*

Proof. By the aid of lemma 3.11 and 3.12,the proof is clear. □

Proposition 3.14. *Let $(X, \lambda, \tilde{J}, E)$ be a $S_{\tilde{J}}.E.D.S$ and $F_E \in SS(X)_E$. In this case,*

a) $F_E \in SS_{\tilde{J}}O(X)$ iff $F_E \in S\alpha_{\tilde{J}}O(X)$,
b) $F_E \in SP_{\tilde{J}}O(X)$ iff $F_E \in Ss\beta_{\tilde{J}}O(X)$,
c) $F_E \in S\tilde{J}O(X)$ iff $F_E \in Sas_{\tilde{J}}O(X)$.

Proof. a) Sufficient condition is given in [12]. On the other hand, let $F_E \in SS_{\tilde{J}}O(X)$. Then, we have $F_E \tilde{\subseteq} cl^*(int(F_E))$. Since $(X, \lambda, \tilde{J}, E)$ be a $S_{\tilde{J}}.E.D.S$, for $int(F_E) \in \lambda$, we have $cl^*(int(F_E)) \in \lambda$. Therefore , we have $F_E \tilde{\subseteq} cl^*(int(F_E)) = int(cl^*(int\ (F_E)))$, and hence F_E is $soft$ α-$\tilde{J}$-$open$.

b) Necessary condition is given Proposition 2.21 b). On the other hand, let $F_E \in Ss\beta_{\tilde{J}}O(X)$ and hence $F_E \tilde{\subseteq} cl^*(int(cl^*(F_E)))$. Since $(X, \lambda, \tilde{J}, E)$ is a $S_{\tilde{J}}.E.D.S$, for $int(cl^*(F_E)) \in \lambda$, we have $cl^*(int(cl^*(F_E))) \in \lambda$. So, we have $F_E \tilde{\subseteq} cl^*(int(cl^*(F_E))) = int(cl^*(int(cl^*(F_E))))$, that is

$$F_E \tilde{\subseteq} int(cl^*(int(cl^*(F_E)))). \tag{3.3}$$

Besides, since $int(cl^*(F_E)) \tilde{\subseteq} cl^*(F_E)$ and cl^* is Kuratowski closure operator, we have $cl^*(int(cl^*(F_E))) \tilde{\subseteq} cl^*(cl^*(F_E)) = cl^*(F_E)$ and hence

$$int(cl^*(int(cl^*(F_E)))) \tilde{\subseteq} int(cl^*(F_E)). \tag{3.4}$$

Consequently, by using (3.3) and (3.4) we have $F_E \tilde{\subseteq} int(cl^*(F_E))$ and hence F_E is $soft\ pre$-$\tilde{J}$-$open$.

c) Necessity condition is given Proposition 2.21 a). On the other hand, let $F_E \in Sas_{\tilde{J}}O(X)$, then we have $F_E \tilde{\subseteq} cl^*(int((F_E)^*))$. Since $(X, \lambda, \tilde{J}, E)$ is a $S_{\tilde{J}}.E.D.S$, for $int(F_E)^* \in \lambda$, we have $cl^*(int((F_E)^*)) \in \lambda$. Then, we have $F_E \tilde{\subseteq} cl^*(int((F_E)^*)) = int(cl^*(int((F_E)^*))) \tilde{\subseteq} int(cl^*((F_E)^*)) = int[(F_E)^* \tilde{\cup} ((F_E)^*)^*] \tilde{\subseteq} int[(F_E)^* \tilde{\cup} (F_E)^*] = int((F_E)^*)$ and hence $F_E \tilde{\subseteq} int((F_E)^*)$. So it can be said F_E is $soft\ \tilde{J}$-$open$. □

We recall that a soft subset F_E of a STS (X, λ, E) is said to be $soft\ pre$-$open$ if $F_E \tilde{\subseteq} int(cl(F_E))$ [5]. The family of all $soft\ pre$-$open\ sets$ of (X, λ, E) is shown by $SPO(X)$.

Proposition 3.15. *Let $(X, \lambda, \tilde{J}, E)$ be a $S.E.D.S$ and $F_E \in SS(X)_E$. In this case,*

a) $F_E \in S_{\tilde{J}}O(X)$ iff $F_E \in aS_{\tilde{J}}O(X)$,
b) If $F_E \in S\beta_{\tilde{J}}O(X)$, then $F_E \in SPO(X)$.

Proof. a) Necessary condition is obvious from Diagram 2. On the other hand, let $F_E \in aS_{\tilde{J}}O(X)$. Since $(X, \lambda, \tilde{J}, E)$ is a $S.E.D.S$, for $int((F_E)^*) \in \lambda$, we have $cl(int((F_E)^*)) \in \lambda$. Since $F_E \in aS_{\tilde{J}}O(X)$, we obtain $F_E \tilde{\subseteq} cl(int((F_E)^*)) = int(cl(int((F_E)^*))) \tilde{\subseteq} int(cl((F_E)^*)) = int((F_E)^*)$, from Lemma 2.18 b) it can be said F_E is $soft\ \tilde{J}$-$open$.

b) Let $F_E \in S\beta_{\tilde{J}}O(X)$, then we have $F_E \tilde{\subseteq} cl(int(cl^*(F_E)))$. Since $(X, \lambda, \tilde{J}, E)$ is a $S.E.D.S$, for $int(cl^*(F_E)) \in \lambda$, we have $cl(int(cl^*(F_E))) \in \lambda$. So we have $F_E \tilde{\subseteq} cl(int(cl^*(F_E))) = int(cl(int(cl^*(F_E)))) \tilde{\subseteq} int(cl(cl^*(F_E))) = int(cl[F_E \tilde{\cup} (F_E)^*]) = int[cl(F_E) \tilde{\cup} cl((F_E)^*)] = int(cl(F_E))$ by using Lemma 2.18 b). Therefore, $F_E \tilde{\subseteq} int(cl(F_E))$ and hence F_E is $soft\ pre$-$open$. □

Corollary 3.16. *Let $(X, \lambda, \tilde{J}, E)$ be a $S_{\tilde{J}}.E.D.S$ such that $\tilde{J} = \{\tilde{\Phi}\}$ and $F_E \in SS(X)_E$. In this case,*

a) $F_E \in S_{\tilde{J}}O(X)$ iff $F_E \in aS_{\tilde{J}}O(X)$,
b) If $F_E \in S\beta_{\tilde{J}}O(X)$, then $F_E \in SPO(X)$,

Proof. From propositions 3.9 and 3.15, the proof is clear. □

4. Soft Functions on S.I.E.D. Spaces

Definition 4.1. *A function $f_{pu} : (X, \lambda, \tilde{J}, A) \to (Y, \vartheta, B)$ is said to be $soft\ almost\ strongly\ \tilde{J}$-$continuous$ ($soft\ weakly\ regular\ \tilde{J}$-$continuous$) if for every $G_B \in \vartheta$, $f_{pu}^{-1}(G_B)$ is $soft\ almost\ strong\ \tilde{J}$-$open$ ($soft\ weak\ regular\ \tilde{J}$-$closed$) in $(X, \lambda, \tilde{J}, A)$.*

Definition 4.2. *A soft function $f_{pu} : (X, \lambda, \tilde{J}, A) \to (Y, \vartheta, B)$ is said to be $almost\ soft\ \tilde{J}$-$continuous$ (resp. $soft\ \tilde{J}$-$continuous$ [12], $soft\ pre$-$\tilde{J}$-$continuous$ [12], $soft\ semi\ \tilde{J}$-$continuous$ [12], $soft\ \alpha$-$\tilde{J}$-$continuous$ [12], $soft\ strongly\ \beta$-$\tilde{J}$- $continuous$) if for every $G_B \in \vartheta$, $f_{pu}^{-1}(G_B)$ is $almost\ soft\ \tilde{J}$-$open$ (resp. $soft\ \tilde{J}$- $open$, $soft\ pre$-$\tilde{J}$-$open$, $soft\ semi$-$\tilde{J}$-$open$, $soft\ \alpha$-$\tilde{J}$-$open$, $soft\ strongly\ \beta$-$\tilde{J}$- $open$) in $(X, \lambda, \tilde{J}, A)$.*

Theorem 4.3. *Let $(X, \lambda, \tilde{J}, A)$ be a $S_{\tilde{J}}.E.D.S$. For a soft function $f_{pu} : (X, \lambda, \tilde{J}, A) \to (Y, \vartheta, B)$, the properties below are satisfied.*

a) If f_{pu} is $soft\ semi$-$\tilde{J}$-$continuous$, then it is $soft\ pre$-$\tilde{J}$-$continuous$,
b) If f_{pu} is $soft\ weakly\ regular\ \tilde{J}$-$continuous$, then it is $continuous$.

Proof. From Propositions 3.4, the proofis clear. □

Theorem 4.4. *Let $(X, \lambda, \tilde{J}, A)$ be a $S_{\tilde{J}}.E.D.S$. For a soft function $f_{pu} : (X, \lambda, \tilde{J}, A) \to (Y, \vartheta, B)$, then the following properties are satisfied:*

a) If f_{pu} is $soft\ semi$-$\tilde{J}$-$continuous$ iff it is $soft\ \alpha$-$\tilde{J}$-$continuous$,
b) If f_{pu} is $soft\ pre$-$\tilde{J}$-$continuous$ iff it is $soft\ strongly\ \beta$-$\tilde{J}$-$continu\ ous$,
c) If f_{pu} is $soft\ \tilde{J}$-$continuous$ iff it is $soft\ almost\ strongly\ \tilde{J}$-$continu\ ous$.

Proof. The proof is obvious from Propositions 3.14 □

If the definition below is given again: A soft function $f_{pu} : (X, \lambda, A) \to (Y, \vartheta, B)$ is said to be $soft\ pre$-$continuous$ [12] if for every $G_B \in \vartheta$, $f_{pu}^{-1}(G_B)$ is $soft\ pre$- $open$ in (X, λ, A).

Theorem 4.5. *Let $(X, \lambda, \tilde{J}, A)$ be a $S.E.D.S$ and $S_{\tilde{J}}.E.D.S$ such that $\tilde{J} = \{\tilde{\Phi}\}$, respectively. For a soft function $f_{pu} : (X, \lambda, \tilde{J}, A) \to (Y, \vartheta, B)$, properties below are satisfied:*

a) f_{pu} is $soft\ \tilde{J}$-$continuous$ iff it is $almost\ soft\ \tilde{J}$-$continuous$,
b) If f_{pu} is $soft\ \beta$-$\tilde{J}$-$continuous$, then it is $soft\ pre$-$continuous$.

Proof. From Propositions 3.15 and Corallary 3.16, the proof is clear. □

5. Conclusion

In our study, we have not only defined many new types of soft sets, but also examined the characterizations of some of them. Researches on soft ideal topological spaces can do similar studies for others. Also, with the help of these soft sets, soft subspaces, soft compactness, soft connectedness and soft separation axioms can be discussed. Similarly, new characterizations of soft sets types in literature can be obtained by using soft space types.

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