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Heat Transfer in Magnetohydrodynamic Second Grade Fluid with Porous Impacts using Caputo-Fabrizoi Fractional Derivatives

Kashif Ali Abro
Department of Basic Sciences and Related Studies,
Mehran University of Engineering and Technology, Jamshoro, Pakistan,
Email: kashif.abro@faculty.muet.edu.pk

Muhammad Anwar Solangi
Department of Basic Sciences and Related Studies,
Mehran University of Engineering and Technology, Jamshoro, Pakistan,

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Abstract. This article leads to free convection problem of magnetohydrodynamic second grade fluid with porous medium using recently defined Caputo-Fabrizoi fractional derivatives. Analytical expressions for temperature distribution and velocity field have been investigated using Laplace transforms having inverses. The general expressions for Temperature distribution and velocity field are written in terms of Generalized Mittage-Leffler function $M^{P}_{Q,R}(Z)$ and Fox-H function $H^{1,P}_{P,Q+1}(Z)$ respectively. Both the solutions of Temperature distribution and velocity field satisfy implemented conditions as $V(y,0) = T(y,0) = 0$ and $V(0,t) = UH(t)cos(\omega t), V(0,t) = UH(t)sin(\omega t)$ and $T(0,t) = 1$. The general expressions have been reduced for limiting cases. Finally influences of material parameter, non-dimensional parameters, rheological parameters and Caputo-Fabrizoi fractional parameter are analyzed graphically by choosing distinct values on fluid flow.

AMS (MOS) Subject Classification Codes: 35S29; 40S70; 25U09
Key Words: Caputo-Fabrizoi fractional derivatives, Porous and magnetic materials, Heat transfer, Laplace transforms, Generalized functions.

1. Introduction

Flow of viscoelastic fluid with communal influence of heat and mass transfer has diverted the attention of several mathematicians and researchers due to its active industrial, engineering and technological applications. Such applications have vital role in different field, for instance thermal insulation, damage of crops due to freezing, chemical catalytic reactors, pollution of the environment, distributions of temperature, extraction of crude oil, roves of fruit trees, moisture over agricultural fields and many others. The applications of non-Newtonian fluid with heat and mass transfer have significant importance; such as ventilation and air conditioning, industrial heating or refrigeration, environmental chambers, chemical manufacturing, ice rinks and engine cooling, pharmaceutical etc. [1, 8, 11]. It is also well recognized fact in comparison to Newtonian fluid, non-Newtonian fluid has complicated and higher order differential equations. In order to understand and predict the behavior of non-Newtonian fluid's mathematical systems, various suggestions regarding constitutive models have been presented in literatures [9, 15, 23, 24, 14]. Fetecau et al. [7] provided unidirectional flows of second grade fluid for starting solutions. Asghar et al. [2] obtained Stoke's second problem for second grade fluid is analytically. Ali et al. [3] investigated viscoelastic fluids with heat and mass transfer in a porous channel for magnetohydrodynamics oscillatory flows. Tan et al. [22] have analyzed generalized second grade fluid. They computed analytical solutions between two parallel plates by using fractional derivatives. Qi et al. [20] obtained solutions in cylindrical geometries, in which they considered fractionalized viscoelastic fluid for rotating flows of coaxial cylinders. In brief, efforts for non-Newtonian fluids with or without fractional derivatives have been taken by several researchers, include references as Ali et al. [4], Zheng et al. [25], Kashif et al. [16], Fetecau et al. [10], Zhang et al. [26], Liu et al. [17], Kashif et al. [13], Shakir et al. [21], Hussain et al. [12]. Of course, the study of fractional derivatives and magnetohydrodynamics can be continued but we close it by adding few related references as well [27-30]. Observing the significance of above research work, the solutions existing in the literature, our solutions are new and more general expression in terms of special functions. Hence, we investigated free convection problem of magnetohydrodynamics second grade fluid with porous medium using recently defined Caputo-Fabrizoi fractional derivatives. Analytical expressions for Temperature distribution and velocity field have been investigated using Laplace transforms having inverses. The general expressions for Temperature distribution and velocity field are written in terms of Generalized Mittage-Leffler function $M^{P}_{Q,R}(Z)$ and Fox-H function $H^{1,P}_{P,Q+1}(Z)$ respectively. Both the solutions namely Temperature distribution and velocity field satisfy implemented initial and boundary conditions. The general expressions have been reduced for limiting cases having contrast of present solutions in literature. Finally influences of material parameter, non-dimensional parameters, rheological parameters and Caputo-Fabrizoi fractional parameter are analyzed graphically by choosing distinct values.

2. STATEMENT AND GOVERNING EQUATIONS OF PROBLEM

Let us assume the unsteady free convection flow of magnetohydrodynamics (MHD) fractionalized second grade fluid embedded in porous medium lying in an infinite oscillating plate situated in the x,y plane of a Cartesian coordinate system. Initially, with constant temperature plate and fluid are at rest. After $t = 0^{+}$ the plate oscillates in its own plane with the velocity $V(0,t) = UH(t)cos(\omega t)$ or $sin(\omega t)$, while the temperature T_w of plate is raised at the same time. Temperature distribution and velocity field are of the functions of t and y and constrain of incompressibility is identically fulfilled for such type flow. Using

Boussinesq's approximation and assumptions of flow problem, we formulated the governing equations for second grade fluid as [5]

$$\frac{\alpha_1}{\rho}\frac{\partial^3 V(y,t)}{\partial y^2 \partial t} + \nu\frac{\partial^2 V(y,t)}{\partial y^2} - \frac{\partial V(y,t)}{\partial t} + g\beta_T\{T(y,t) - T_\infty\}$$

$$-MV(y,t) - \frac{\phi}{K}V(y,t) = 0, t, y > 0, \tag{2. 1}$$

$$\frac{\partial^2 V(y,t)}{\partial y^2}\frac{k}{C_p\,\rho} - \frac{\partial T(y,t)}{\partial t} = 0, t, y > 0, \tag{2. 2}$$

where, α_1 is second grade parameter, ρ is constant density, ν is kinematic viscosity, g is gravitational acceleration, β_T is volumetric coefficient of thermal expansion, M is magnetic field, ϕ is porosity, K is permeability, C_p is heat capacity at constant pressure and k is thermal conductivity. The appropriate initial and boundary conditions are:

$$V(y,0) = 0, T(y,0) = T_\infty,\ y > 0, \tag{2. 3}$$

$$V(0,t) = UH(t)cos(\omega t),\ \ V(0,t) = UH(t)sin(\omega t),\ \ T(0,t) = 1,\ t \geq 0, \tag{2. 4}$$

$$V(y,t) \to 0,\ T(y,t) = T_\infty,\ as\ \ y \to \infty,\ t > 0, \tag{2. 5}$$

using dimensionless quantities and Caputo- Fabrizio time-fractional derivatives, we replace the time derivative of order one with Caputo- Fabrizio time-fractional derivative of order $0 < \xi < 1$ defined as [6]

$$D_t^\xi V(y,t) = \frac{1}{1-\xi}\int_0^t \exp\left(\frac{-\xi(t-\theta)}{1-\xi}\right)V'(\theta)d\theta, 0 < \xi < 1. \tag{2. 6}$$

The governing differential equations are:

$$\alpha_2 D_t^\xi \frac{\partial^2 V(y,t)}{\partial y^2} + \frac{\partial^2 V(y,t)}{\partial y^2} - D_t^\xi V(y,t) + G_r T(y,t)$$

$$-MV(y,t) - \frac{\phi}{K}V(y,t) = 0, t, y > 0, \tag{2. 7}$$

$$\frac{\partial^2 T(y,t)}{\partial y^2} - p_r D_t^\xi T(y,t) = 0,\ t, y > 0, \tag{2. 8}$$

3. SOLUTION OF TEMPERATURE FIELD

Applying the Laplace transform formula to equations (2.8), (2.3), (2.4) and (2.5) we get

$$\frac{\partial^2 T(y,t)}{\partial y^2} - \frac{\eta p_r q}{q+\eta\xi} = 0, \; q, y > 0, \tag{3. 9}$$

using the boundary conditions $\bar{T}(0,q) = \frac{1}{q}$, $\bar{T}(y,q) \to 0$ as $y \to \infty$ and letting $\eta = \frac{1}{1-\xi}$, we have

$$\bar{T}(y,q) = \frac{1}{q}\, e^{-\sqrt{\frac{\eta p_r q}{q+\eta\xi}}\, y}, \tag{3. 10}$$

equation (3.10) is written in equivalent form as

$$\bar{T}(y,q) = \frac{1}{q} + \sum_{l=1}^{\infty} \frac{(-y\sqrt{\eta p_r})^l}{l!} \sum_{p=0}^{\infty} \frac{(-\eta\xi)^p}{p\,!} \frac{\Gamma(p+\frac{l}{2})}{\Gamma(\frac{l}{2})} \frac{1}{q^{p+1}}, \tag{3. 11}$$

Applying inverse Laplace transform on equation (3.11),

$$T(y,t) = 1 + \sum_{l=1}^{\infty} \frac{(-y\sqrt{\eta p_r})^l}{l!} \sum_{p=0}^{\infty} \frac{(-\eta\xi)^p}{p\,!} \frac{\Gamma(p+\frac{l}{2})}{\Gamma(\frac{l}{2})\Gamma(p+1)} t^p, \tag{3. 12}$$

writing in terms of Generalized Mittage Leffler function [18], we find temperature distribution as

$$T(y,t) = 1 + \sum_{l=1}^{\infty} \frac{(-y\sqrt{\eta p_r})^l}{l!} M^{\frac{l}{2}}_{0,p+1}(-\eta\xi), \tag{3. 13}$$

Where, the Generalized Mittage Leffler function is described as

$$t^{\gamma-1} \sum_{k}^{\infty} \frac{(W)^k \Gamma(y+k)}{k!\Gamma(y)\Gamma(\beta k+\gamma)} = t^{\gamma-1} E^{y}_{\beta,\gamma}(W) = M^{y}_{\beta,\gamma}(W). \tag{3. 14}$$

4. SOLUTION OF VELOCITY FIELD

Applying discrete Laplace transform to equations (2.7) and (2.6), we get suitable expression

$$\frac{\eta\alpha_2 q}{q+\eta\xi} \frac{\partial^2 \bar{V}(y,q)}{\partial y^2} + \frac{\partial^2 \bar{V}(y,q)}{\partial y^2} - \frac{\eta q}{q+\eta\xi} \bar{V}(y,q) + G_r \bar{T}(y,q)$$

$$-M\bar{V}(y,q) - \frac{\phi}{K}\bar{V}(y,q) = 0 \tag{4. 15}$$

subject to conditions (2.3), $(2.4)_1$ and $(2.4)_2$ into equation (4.15), we obtain

$$\bar{V}(y,q) = \frac{Uq}{q^2+\omega^2}\, e^{-\sqrt{\frac{(q+\eta\xi)(M+\frac{\phi}{K})}{(q+\eta\xi+\alpha_2\eta\xi)}}\, y} + \frac{\eta q - (q+\eta\xi) G_r e^{-\sqrt{\frac{\eta p_r q}{q+\eta\xi}}\, y}}{q(M+\frac{\phi}{K})(q+\eta\xi)} \tag{4. 16}$$

reworking on equation (4.16) in terms of series form,

$$\bar{V}(y,q)=\frac{Uq}{q^2+\omega^2}+\frac{1}{(M+\frac{\phi}{K})}\left(\frac{\eta q}{q+\eta\xi}-\frac{e^{-\sqrt{\frac{\eta p_r q}{q+\eta\xi}}y}}{q}\right)$$

$$+U\sum_{l=1}^{\infty}\frac{\left(-\eta\alpha_2\sqrt{M+\frac{\phi}{K}}\right)^l}{l!}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\sum_{m=0}^{\infty}\frac{(-\eta\xi)^m}{m!}$$

$$\times\frac{\Gamma(m+l)\Gamma(k+l)}{\Gamma(l)\Gamma(l)}\frac{q}{q^m(q^2+\omega^2)}, \tag{4. 17}$$

Inverting equation (4.17) by Laplace transform with convolution product and expressing as Fox H-function and using the fact of transform as $\Phi(y,q,A,B)=\frac{1}{q}\ e^{-\sqrt{\frac{Aq}{q+b}}y}$ and $L^{-1}\Phi(y,q,A,B)=\varphi(y,t,A,B)$, we find expression for velocity field:

$$V_c(y,t)=UH(t)cos\omega t+\frac{\eta(1-\eta\xi e^{-\eta\xi t})-\varphi(y,t,\eta p_r,\eta\xi)}{(M+\frac{\phi}{K})}$$

$$+UH(t)\int_0^t\frac{cos\ \omega(t-z)}{t}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\sum_{l=1}^{\infty}\frac{\left(-\eta\alpha_2\sqrt{M+\frac{\phi}{K}}\right)^l t}{l!}$$

$$\times H^{1,2}_{2,3}\left[\eta\xi t\Bigg|^{(1-l,1)(1-k-l,0)}_{(0,1),(1-l,0),(1-l,0)(1,1)}\right]dw \tag{4. 18}$$

where, the Fox-H function is described as

$$\sum_{i}^{\infty}\frac{(-R)^i\Pi_{j=1}^{p}\Gamma(x_j+X_ji)}{i!\ \Pi_{j=1}^{q}\Gamma(z_j+Z_ji)}=H^{1,p}_{p,q+1}\left[Q\Bigg|^{(1-x_1,X_1),...,(1-x_p,X_p)}_{(0,1),(1-z_1,Z_1),...,(1-z_q,Z_q)}\right] \tag{4. 19}$$

Employing similar pattern, one can find corresponding solution for sine oscillation as

$$V_s(y,t)=UH(t)sin\omega t+\frac{\eta(1-\eta\xi e^{-\eta\xi t})-\varphi(y,t,\eta p_r,\eta\xi)}{(M+\frac{\phi}{K})}$$

$$+UH(t)\int_0^t \frac{sin\ \omega(t-z)}{t}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\sum_{l=1}^{\infty}\frac{\left(-\eta\alpha_2\sqrt{M+\frac{\phi}{K}}\right)^l t}{l!}$$

$$\times H_{2,3}^{1,2}\left[\eta\xi t\Bigg|_{(0,1),(1-l,0),(1-l,0),(1,1)}^{(1-l,1),(1-k-l,0)}\right]dw \tag{4. 20}$$

5. LIMITING CASES

5.1. SINE AND COSINE OSCILLATIONS OF SECOND GRADE FLUID IN THE ABSENCE OF MAGNETIC FIELD WHEN $M = 0$.

Letting $M = 0$ in equation (4.18), we have solution for velocity field with porous medium and without magnetic field as

$$V_c(y,t) = UH(t)cos\omega t + \frac{\eta(1-\eta\xi e^{-\eta\xi t}) - \varphi(y,t,\eta p_r,\eta\xi)}{\frac{\phi}{K}}$$

$$+UH(t)\int_0^t \frac{cos\ \omega(t-z)}{t}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\sum_{l=1}^{\infty}\frac{\left(-\eta\alpha_2\sqrt{\frac{\phi}{K}}\right)^l t}{l!}$$

$$\times H_{2,3}^{1,2}\left[\eta\xi t\Bigg|_{(0,1),(1-l,0),(1-l,0)(1,1)}^{(1-l,1)(1-k-l,0)}\right]dw \tag{5. 21}$$

$$V_s(y,t) = UH(t)sin\omega t + \frac{\eta(1-\eta\xi e^{-\eta\xi t}) - \varphi(y,t,\eta p_r,\eta\xi)}{\frac{\phi}{K}}$$

$$+UH(t)\int_0^t \frac{sin\ \omega(t-z)}{t}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\sum_{l=1}^{\infty}\frac{\left(-\eta\alpha_2\sqrt{\frac{\phi}{K}}\right)^l t}{l!}$$

$$\times H_{2,3}^{1,2}\left[\eta\xi t\Bigg|_{(0,1),(1-l,0),(1-l,0),(1,1)}^{(1-l,1),(1-k-l,0)}\right]dw \tag{5. 22}$$

5.2. **SINE AND COSINE OSCILLATIONS OF SECOND GRADE FLUID IN THE ABSENCE OF POROUS MEDIUM WHEN $\phi = 0$.**
Letting $\phi = 0$ in equation (4.18), we have solution for velocity field with magnetic field and without porous medium as

$$V_c(y,t) = UH(t)cos\omega t + \frac{\eta(1-\eta\xi e^{-\eta\xi t}) - \varphi(y,t,\eta p_r,\eta\xi)}{M}$$

$$+UH(t)\int_0^t \frac{cos\ \omega(t-z)}{t}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\sum_{l=1}^{\infty}\frac{\Big(-\eta\alpha_2\sqrt{M}\Big)^l t}{l!}$$

$$\times H_{2,3}^{1,2}\left[\eta\xi t\Bigg|_{(0,1),(1-l,0),(1-l,0)(1,1)}^{(1-l,1)(1-k-l,0)}\right]dw \qquad (5.\ 23)$$

$$V_s(y,t) = UH(t)sin\omega t + \frac{\eta(1-\eta\xi e^{-\eta\xi t}) - \varphi(y,t,\eta p_r,\eta\xi)}{M}$$

$$+UH(t)\int_0^t \frac{sin\ \omega(t-z)}{t}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\sum_{l=1}^{\infty}\frac{\Big(-\eta\alpha_2\sqrt{M}\Big)^l t}{l!}$$

$$\times H_{2,3}^{1,2}\left[\eta\xi t\Bigg|_{(0,1),(1-l,0),(1-l,0),(1,1)}^{(1-l,1),(1-k-l,0)}\right]dw \qquad (5.\ 24)$$

5.3. **SECOND GRADE FLUID IN THE ABSENCE OF CAPUTO-FABRIZOI DERIVATIVE WHEN $\xi = 1$.**
Substituting $\xi = 1$ in equation (4.13) and (4.18), we have solution for Temperature profile and velocity field with magnetic field and porous medium for ordinary differential equations as

$$V_c(y,t) = UH(t)cos\omega t + \frac{\eta(1-\eta e^{-\eta t}) - \varphi(y,t,\eta p_r,\eta)}{(M+\frac{\phi}{K})}$$

$$+UH(t)\int_0^t \frac{cos\ \omega(t-z)}{t}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\sum_{l=1}^{\infty}\frac{\Big(-\eta\alpha_2\sqrt{M+\frac{\phi}{K}}\Big)^l t}{l!}$$

$$\times H_{2,3}^{1,2}\left[\eta t\Bigg|_{(0,1),(1-l,0),(1-l,0)(1,1)}^{(1-l,1)(1-k-l,0)}\right]dw \qquad (5.\ 25)$$

$$V_s(y,t) = UH(t)sin\omega t + \frac{\eta(1-\eta e^{-\eta t}) - \varphi(y,t,\eta p_r,\eta)}{(M+\frac{\phi}{K})}$$

$$+UH(t)\int_0^t \frac{sin\;\omega(t-z)}{t}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\sum_{l=1}^{\infty}\frac{\left(-\eta\alpha_2\sqrt{M+\frac{\phi}{K}}\right)^l t}{l!}$$

$$\times H^{1,2}_{2,3}\left[\eta t\Big|^{(1-l,1),(1-k-l,0)}_{(0,1),(1-l,0),(1-l,0),(1,1)}\right]dw \tag{5. 26}$$

$$T(y,t) = 1 + \sum_{l=1}^{\infty}\frac{(-y\sqrt{\eta p_r})^l}{l!}M^{\frac{l}{2}}_{0,p+1}(-\eta). \tag{5. 27}$$

It is also pointed out that when $\alpha_2 = 0$, the fluid is transferred from second grade to Newtonian fluid and when $\omega = 0$, we obtain solutions for first problem of stokes.

6. DISCUSSIONS AND CONCLUSIONS

The main purpose of this paper is to investigate the exact solutions for temperature distribution and velocity field by employing recently defined Caputo-Fabrizoi fractional derivatives. The general solutions are satisfied by the imposed conditions i-e initial, boundary and natural conditions. These solutions have been specifically investigated for limiting cases in which effects of magnetic field and porous medium are shown. It is also pointed out that set of governing partial differential equation have been converted from newly defined Caputo-Fabrizoi fractional derivatives to ordinary derivatives. Both solutions have interesting results which are analyzed by graphical illustrations. The effects of rheological parameters such as material parameter, magnetic parameter, porosity, permeability, viscosity, etc. have been presented by graphical illustrations. Finally it is worth pointing out that when $M = \phi = 0$, we retrieved the results obtained by Nehad and et al. [26, see Eqs. 22 and 42]. It is also noted that four models have been analyzed with comparison, they are (i) fractionalized second grade fluid in presence of magnetic field and porosity (ii) fractionalized second grade fluid in the absence of magnetic field and porosity (iii) Ordinary second grade fluid in the presence of magnetic field and porosity and (iv) Ordinary second grade fluid in absence of magnetic field and porosity. However, the major outcomes are listed below:

(i) Figure 1 shows that, when time t increases the velocity as well as temperature increase. The physical aspect of this figure is as time increases the thickness of fluid varies.

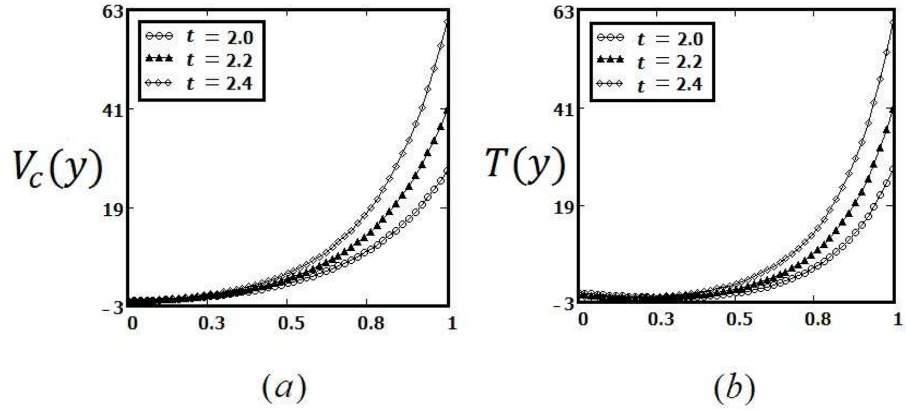

FIGURE 1. Velocity and temperature profiles for different values of t at $\xi = 0.3,\ p_r = 1.9,\ M = 1.11,\ k = 2,\ \phi = 0.5$ and $a_2 = 3.5$.

(ii) Figure 2 shows the influence on material parameter in which behavior of fluid is decreasing function with increasing values. It is pointed out that profiles of velocity and temperature are enhanced for fluid flows.

(iii) The behavior of fluid flows is plotted for prandtl number in fig. 3 in which it is seen that tendency of reduction in velocity field and temperature has decreasing profiles.

(iv) Figure 4 represents the variation in caputo-Fabrizo fractional derivative has strong behavior having much more consistent in both velocity field and temperature distribution. Here our results relates for caputo-Fabrizo fractional derivative when we increases caputo-Fabrizo fractional derivative, velocity field and temperature distribution have similar effects as investigated by [19 (see fig. (a) from temperature and fluid velocity)].

(v) In comparison with figure 4, figure 5 indicates that the magnetic parameter have reciprocal response of fluid flow. This is due to fact that increasing magnetic field produces slowness in the behavior of flow.

(vi) Figure 6 is drawn for four fractionalized models of fluid, (i) fractionalized Second grade fluid without magnetic field, (ii) fractionalized Second grade fluid with magnetic field, (iii) fractionalized Second grade fluid without porosity and (iv) fractionalized Second grade fluid with porosity. It is noted in velocity profile fractionalized Second grade fluid with porosity vicissitudes faster than remaining all models. In contrast with temperature distribution, fractionalized Second grade fluid without magnetic field has reciprocal influences throughout the vicinity of plate. Same fact can be seen in figure 7 for ordinary fluid i-e in the absence of caputo-Fabrizo fractional derivative.

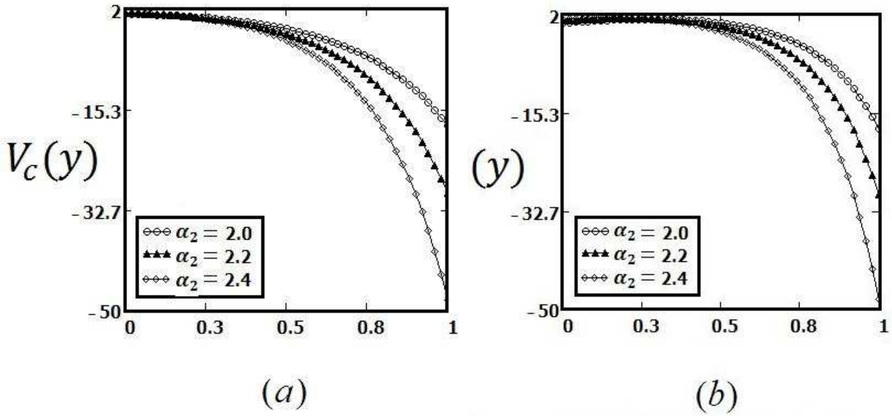

FIGURE 2. Velocity and Temperature profiles for different values of a_2 at $\xi = 0.1$ $p_r = S$. $M = 6.1$. $k = 0.5$, $\phi = 0.1$ and t = 2S.

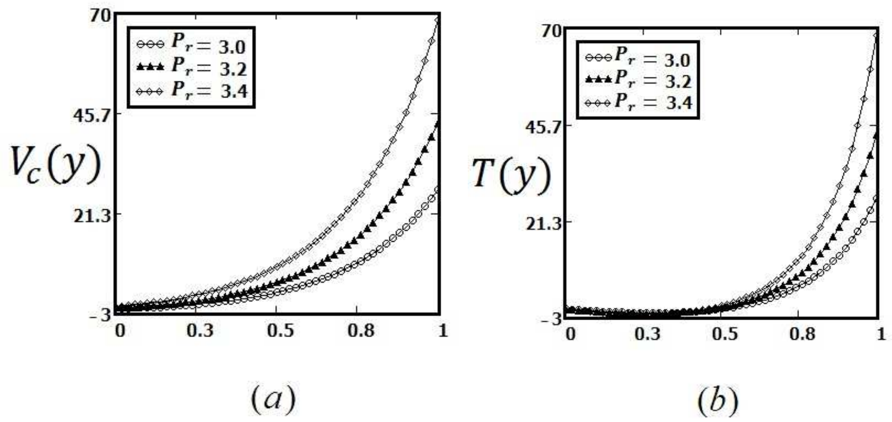

FIGURE 3. Velocity and Temperature profiles for different values of p_r at $\xi = 0.7$,a$_2 = 4.22$ M=2.4. k=11, $\phi = 39$ and t = 2S.

7. ACKNOWLEDGMENTS

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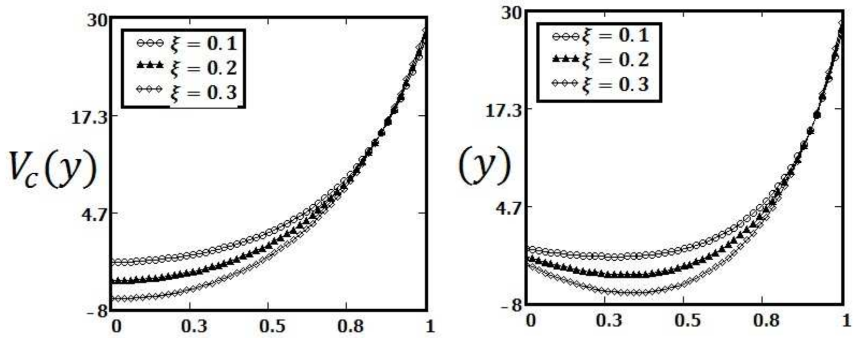
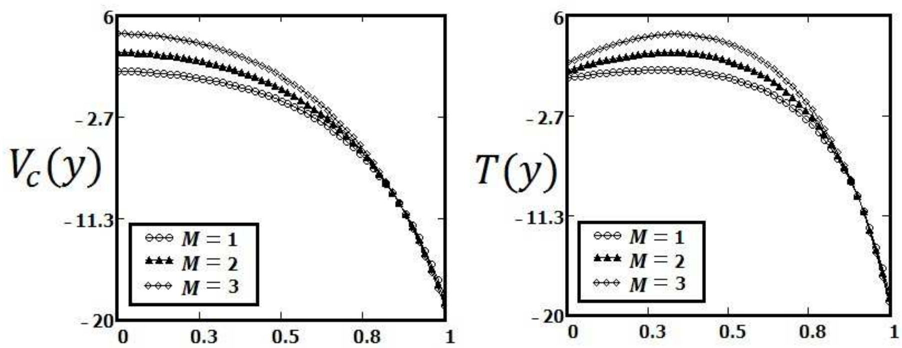


(a) (b)

FIGURE 4. Velocity and Temperature profiles for different values of ξ at $\mathrm{p}_r = 2.79$,$\mathrm{a}_2 = 7.3$, M=1.2, k=11, $\phi = 2.8$ and t = 2S.



(a) (b)

FIGURE 5. Velocity and Temperature profiles for different values of M at $\mathrm{p}_r = 11$,$\mathrm{a}_2 = 4.1$, $\xi = 0.2$ k=2.5, $\phi = 0.2$ and t = 2S.

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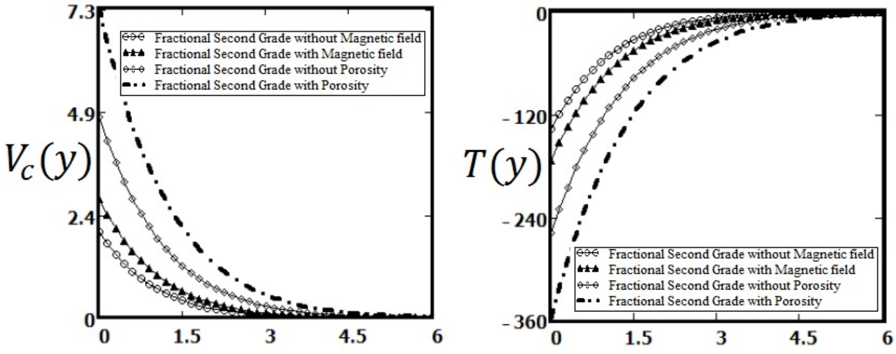
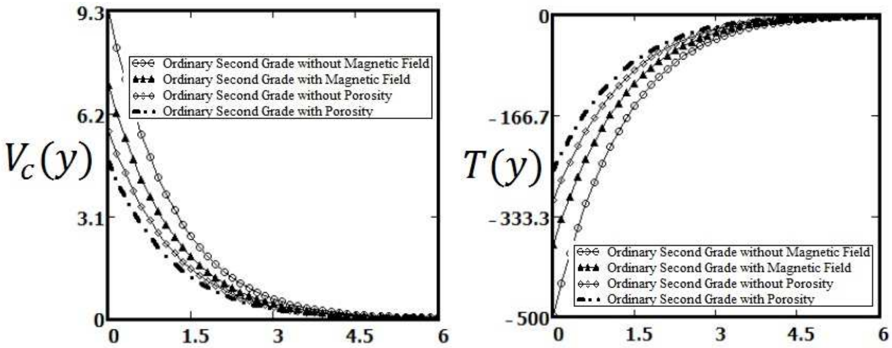


(a) (b)

FIGURE 6. Comparison of Velocity and temperature profiles for M=3.7, $p_r = 4.1$,$a_2 = 13.6$ $\xi = 0.7$, $k = 21.23$, $\phi = 7.11$ and t = 2S.



(a) (b)

FIGURE 7. Comparison of Velocity and temperature profiles for M=3.7, $p_r = 4.1$,$a_2 = 13.6$ $\xi = 0.7$, $k = 21.23$, $\phi = 7.11$ and t = 2S.

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