Punjab University
Journal of Mathematics (ISSN 1016-2526)
Vol. 50(2)(2018) pp. 131-145

Fuzzy Parameterized Fuzzy Soft Compact Spaces with Decision-Making

Muhammad Riaz
Department of Mathematics,
University of the Punjab Lahore, Pakistan,
Email: mriaz.math@pu.edu.pk

Masooma Raza Hashmi
Department of Mathematics and Statistics,
The University of Lahore, Pakistan.
Email: masoomaraza25@gmail.com

Received: 04 April, 2017 / Accepted: 12 December, 2017 / Published online: 01 February, 2018

Abstract. Some novel concepts including fuzzy parameterized fuzzy soft neighborhood germ, $\mathfrak{fpfs}$-S-neighborhood, $\mathfrak{fpfs}$-Lindelöf property and $\mathfrak{fpfs}$-Ω accumulation point of $\mathfrak{fpfs}$-compact spaces are demonstrated with some important results. We delineate dual $\mathfrak{fpfs}$-point and the Bolzano Weierstrass property for $\mathfrak{fpfs}$-sets. We introduce modified form of an algorithm based on $\mathfrak{fpfs}$-compact topological space to the decision-making problem.

AMS (MOS) Subject Classification Codes: 54A05, 11B05, 54D30, 54A40, 06D72,03E72, 03E75, 62C86
Key Words: $\mathfrak{fpfs}$-neighborhood germ, $\mathfrak{fpfs}$-S-neighborhood, dual $\mathfrak{fpfs}$-point, countability of $\mathfrak{fpfs}$-space, $\mathfrak{fpfs}$-Ω-accumulation point, $\mathfrak{fpfs}$-Lindelöf space, countable $\mathfrak{fpfs}$-compact space, Bolzano Weierstrass property for $\mathfrak{fpfs}$-space, decision-making.

1. Introduction

The property of being a "bounded set in a metric space is not conserved by homeomorphisms. So to generalize theorems in Real analysis like a continuous function on a closed bounded interval is bounded we need a new concept. This is the idea of compactness. Geometrically speaking, in finite dimensions, compact sets are those sets that are closed and bounded. This fundamentally means that, in a certain sense, they can't have infinite structure, which is a desirable feature to have. Most of the problems in engineering, medical science, economics, environments etc. have various uncertainties". To deal with uncertainties there are different theories including, fuzzy set theory introduced by Zadeh [42], soft set theory introduced by Molodtsov [23], fuzzy soft set theory ($\mathfrak{fs}$-set) introduced by Maji *et al.*

[25] and fuzzy parameterized fuzzy soft set theory ($\mathfrak{fpfs}$-set) [15, 41] introduced by Cagman *et al.* Intuitionistic fuzzy set ($\mathfrak{if}$-set) introduced by Atanassov [10] as an abstraction of fuzzy set and intuitionistic fuzzy soft set ($\mathfrak{ifs}$-set) was introduced by Maji *et al.* [24] as an abstraction of fuzzy soft set. Chang [14] in (1968) introduced fuzzy topology by using fuzzy sets. Abbas *et al.* [1] presented upper and lower contra-continuous fuzzy Multi-functions. Akram *et al.* [2, 3, 4, 5] introduced certain types of soft graphs and novel applications of m-polar fuzzy hypergraphs. Aslam and Riaz [8, 9] studied G-subsets and G-orbits of under action of the Modular Group. Cagman *et al.* [16, 17, 18] proposed soft topology, FS-set theory, $\mathfrak{fpfs}$-set theory and presented some applications to decision-making problems. Maji *et al.* [24, 25, 26, 27] introduced intuitionistic fuzzy soft sets, fuzzy soft sets, operations on Soft sets and application of soft sets in decision making problem. Riaz *et al.* [31, 32, 33, 34, 35] introduced some concepts of soft sets together with soft algebra, soft σ-algebra, soft σ-ring and measurable soft mappings. They found certain properties of soft metric spaces and studied $\mathfrak{fpfs}$-set and $\mathfrak{fpfs}$-topology with some important propositions and inaugurated certain applications of $\mathfrak{fpfs}$-set to the decision-making problems. Zorlutuna and Atmaca [41] presented $\mathfrak{fpfs}$-topology with some important results and $\mathfrak{fpfs}$-mappings. Zimmermann [43] established some applications of FS-set theory. Fuzzy set theory, soft set theory, fuzzy soft set theory with applications to the decision-making have studied in the last decade (See [6, 7, 11, 12, 13, 19, 20, 21, 22, 25, 28, 29, 30, 36, 37, 38, 39, 40]). ”We have extended some ideas in the present work. We have continued to study the $\mathfrak{fpfs}$-compact spaces. We introduce some new concepts for $\mathfrak{fpfs}$-compact space such as $\mathfrak{fpfs}$-neighborhood germ, $\mathfrak{fpfs}$-S-neighborhood, dual $\mathfrak{fpfs}$-point and countability of $\mathfrak{fpfs}$-space. Moreover, we present $\mathfrak{fpfs}$-Ω-accumulation point, $\mathfrak{fpfs}$-Lindelöf space and Bolzano Weierstrass property for $\mathfrak{fpfs}$-space. We establish an application of $\mathfrak{fpfs}$-compact space to the decision-making problem. This paper can form the striking foundation for further applications of $\mathfrak{fpfs}$-topology on $\mathfrak{fpfs}$-sets”.

2. Preliminaries

In this section, we recall some basic ideas of $\mathfrak{fpfs}$-topology. Throughout this paper X represent initial universe and R represent the set of decision variables or attributes.

Definition 2.1. [15, 41] ”Let X be the universal set and $\widetilde{P}(X)$ be the set of all fuzzy subsets of X, i.e. $\widetilde{P}(X)$ is the set $[o,1]^X$ of all functions from X to [0,1], R be the set of attributes and $A \subseteq R$. A fuzzy parameterized fuzzy soft set ($\mathfrak{fpfs}$-set) is characterized by multivalued mapping $\gamma_A : A \to \widetilde{P}(X)$ such that $\gamma_A(\varpi) = \phi$ if $\mu_A(\varpi) = 0$ for $\varpi \in A \subseteq R$. The $\mathfrak{fpfs}$-set is denoted and defined by
$F_A = \{(\mu_A(\varpi)/\varpi, \gamma_A(\varpi)) : \varpi \in A \subseteq R, \gamma_A(\varpi) \in \widetilde{P}(X); \mu_A(\varpi), \gamma_A(\varpi)(\vartheta) \in [0,1], \vartheta \in X\}$.
The value $\gamma_A(\varpi)$ is a fuzzy set known as ϖ-element of $\mathfrak{fpfs}$-set F_A $\forall\, \varpi \in A \subseteq R$, where $\mu_A(\varpi)$ and $\gamma_A(\varpi)(\vartheta)$ are the degrees of memberships of elements of set $A \subseteq R$ and universal set X respectively”.

Definition 2.2. [15, 41] ”Let F_A be an $\mathfrak{fpfs}$-set over X. If $\gamma_A(\varpi) = \phi\, \forall\, \varpi \in R$ i.e. $\gamma_A(\varpi)$ is an empty fuzzy set for each parameter ϖ, then F_A is known as A-empty $\mathfrak{fpfs}$-set. It is represented as F_{ϕ_A}. If $A = \phi$, then A-empty $\mathfrak{fpfs}$-set is called empty $\mathfrak{fpfs}$-set denoted as F_ϕ”.

Definition 2.3. [15, 41] "Let F_A be an $\mathfrak{fpfs}$-set over X. If $\gamma_A(\varpi) = X$ and $\mu_A(\varpi) = 1$ $\forall\ \varpi \in R$ then F_A is known as A-universal $\mathfrak{fpfs}$-set and it is represented as $F_{\widetilde{A}}$.
If $A = R$, then A-universal $\mathfrak{fpfs}$-set is said to be universal or absolute $\mathfrak{fpfs}$-set and it is written as $F_{\widetilde{R}}$".

Now we present the definition of $\mathfrak{fpfs}$-topology by using elementary operations for $\mathfrak{fpfs}$-sets as given in [15, 41]. We use tilde $\sim$ for these operations in $\mathfrak{fpfs}$-set theory to differentiate from crisp set theory.

Definition 2.4. [34, 41] Let $F_{\widetilde{R}}$ be an absolute $\mathfrak{fpfs}$-set and $\mathfrak{fpfs}(F_{\widetilde{R}})$ is the family of all $\mathfrak{fpfs}$-subsets of $F_{\widetilde{R}}$. Let $\widetilde{\tau}$ be a subfamily of $\mathfrak{fpfs}(F_{\widetilde{R}})$ and $A, B, C \subseteq R$. Then $\widetilde{\tau}$ is known as $\mathfrak{fpfs}$-topology on $F_{\widetilde{R}}$ if the given conditions are satisfied:
(i) $F_\phi, F_{\widetilde{R}} \,\widetilde{\in}\, \widetilde{\tau}$,
(ii) if $F_A, F_B \widetilde{\in} \widetilde{\tau}$ then $F_A \widetilde{\cap} F_B \,\widetilde{\in}\, \widetilde{\tau}$,
(iii) if $(F_C)_\lambda \widetilde{\in} \widetilde{\tau}$, $\forall\ \lambda \in J$, J is arbitrary indexing set, then $\widetilde{\cup}_{\lambda\in J}(F_C)_\lambda \,\widetilde{\in}\, \widetilde{\tau}$.
Members of $\widetilde{\tau}$ are known as $\mathfrak{fpfs}$-open sets and $\mathfrak{fpfs}$-complement of $\mathfrak{fpfs}$-open set is called $\mathfrak{fpfs}$-closed set.

Definition 2.5. [34, 41] An $\mathfrak{fpfs}$-set F_A is said to be an $\mathfrak{fpfs}$-point, denoted by $\varpi(F_A)$, if $A \subseteq R$ is singleton fuzzy subset given as $A = \{\mu_{F_A}(\varpi)/\varpi : \varpi \in R\}$ and $F(\mu_{F_A}(\varpi)/\varpi) = \gamma^{\varpi}_{F_A}(\vartheta)$ is the image of A under multivalued mapping which is always a fuzzy set. Such that $\gamma^{\varpi}_{F_A}(\vartheta) \neq \phi$ and $F(\mu(\acute{\varpi})/\acute{\varpi}) = \phi$, $\forall\ \acute{\varpi} \in R \setminus \{\varpi\}$.

Example 2.6. Let $X = \{\vartheta_1, \vartheta_2, \vartheta_3\}$ be the universal set and let $R = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$ be the set of attributes. If $B = \{0.3/\varpi_1, 0.7/\varpi_2, 0.9/\varpi_4\} \subseteq R$, where B the function represented as $B : R \to [0, 1]$ i.e. B is a fuzzy subset of R. Similarly A is a fuzzy subset of R.
$A = \{0.1/\varpi_1\} \subseteq R$ with the $\mathfrak{fpfs}$-sets, $F_B = \{(0.3/\varpi_1, \{0.1/\vartheta_1, 0.5/\vartheta_2, 0.3/\vartheta_3\}),$
$(0.7/\varpi_2, \{0.1/\vartheta_1, 0.2/\vartheta_2, 0.5/\vartheta_3\}), (0.9/\varpi_4, \{0.4/\vartheta_1, 0.3/\vartheta_2, 0.6/\vartheta_3\})\}$
$F_A = \{(0.1/\varpi_1, \{0.1/\vartheta_1, 0.3/\vartheta_2, 0.2/\vartheta_3\}), (0/\varpi_2, \{0/\vartheta_1, 0/\vartheta_2, 0/\vartheta_3\}),$
$(0/\varpi_3, \{0/\vartheta_1, 0/\vartheta_2, 0/\vartheta_3\}), (0/\varpi_4, \{0/\vartheta_1, 0/\vartheta_2, 0/\vartheta_3\})\}$
then $\varpi(F_A)$ is called $\mathfrak{fpfs}$-point of F_B. Clearly $\mu_{F_A}(\varpi) \leq \mu_{F_B}(\varpi); \forall\varpi \in R$ and $\gamma^{\varpi}_{F_A}(\vartheta) \leq \gamma^{\varpi}_{F_B}(\vartheta); \forall\ \vartheta \in X$. So, $\varpi(F_A)\widetilde{\in}F_B$.

Definition 2.7. [34, 41] Let $\varpi(F_{A_1})$ and $F_{A_2}\widetilde{\in}\mathfrak{fpfs}(F_{\widetilde{R}})$. Then $\varpi(F_{A_1})$ is called $\mathfrak{fpfs}$ quasi-coincident with F_{A_2} written as $\varpi(F_{A_1})qF_{A_2}$, if
$\mu_{F_{A_1}}(\varpi) + \mu_{F_{A_2}}(\varpi) > 1; \varpi \in R$ and $\gamma^{\varpi}_{F_{A_1}}(\vartheta) + \gamma^{\varpi}_{F_{A_2}}(\vartheta) > 1$; for some $\vartheta \in X$.
where $\mu_{F_{A_1}}(\varpi)$ and $\mu_{F_{A_2}}(\varpi)$ are degrees of memberships for parameter ϖ in $\varpi(F_{A_1})$ and F_{A_2} respectively. Similarly, $\gamma^{\varpi}_{F_{A_1}}(\vartheta)$ and $\gamma^{\varpi}_{F_{A_2}}(\vartheta)$ represent the degrees of memberships for elements $\vartheta \in X$.
If $\varpi(F_{A_1})$ is not $\mathfrak{fpfs}$ quasi-coincident with F_{A_2}, then we write $\varpi(F_{A_1})\overline{q}F_{A_2}$.

Example 2.8. Let $X = \{\vartheta_1, \vartheta_2, \vartheta_3\}$ be the set of universe and let $R = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$ be the set of attributes. If $A_1 = \{0.6/\varpi_1\} \subseteq R$, $A_2 = \{0.7/\varpi_1, 0.8/\varpi_4\} \subseteq R$ with the $\mathfrak{fpfs}$-point and $\mathfrak{fpfs}$-set,
$\varpi(F_{A_1}) = \{(0.6/\varpi_1, \{0.6/\vartheta_1, 0.7/\vartheta_2, 0.8/\vartheta_3\})\}$,
$F_{A_2} = \{(0.7/\varpi_1, \{0.6/\vartheta_1, 0.5/\vartheta_2, 0.8/\vartheta_3\}), (0.8/\varpi_4, \{0.7/\vartheta_1, 0.8/\vartheta_2, 0.9/\vartheta_3\})\}$ respectively,

then it is clear that $\varpi(F_{A_1})$ is quasi-coincident with F_{A_2}.
As $\mu_{F_{A_1}}(\varpi_1) + \mu_{F_{A_2}}(\varpi_1) > 1; \varpi_1 \in R$ and $\gamma^{\varpi_1}_{F_{A_1}}(\vartheta) + \gamma^{\varpi_1}_{F_{A_2}}(\vartheta) > 1; \vartheta \in X$.

Definition 2.9. [34, 41] Let F_{A_1} and $F_{A_2} \in \mathfrak{fpfs}(F_{\widetilde{R}})$. Then F_{A_1} is called $\mathfrak{fpfs}$ quasi-coincident with F_{A_2} written as $F_{A_1} q F_{A_2}$, if
$\mu_{F_{A_1}}(\varpi) + \mu_{F_{A_2}}(\varpi) > 1; \varpi \in A_1 \widetilde{\cap} A_2$ and $\gamma^{\varpi}_{F_{A_1}}(\vartheta) + \gamma^{\varpi}_{F_{A_2}}(\vartheta) > 1; \vartheta \in X$.
If F_{A_1} is not $\mathfrak{fpfs}$ quasi-coincident with F_{A_2}, then we write $F_{A_1} \overline{q} F_{A_2}$.

Definition 2.10. [34] An $\mathfrak{fpfs}$-set F_{A_1} is called Q-neighborhood of $\varpi(F_{A_2})$ if and only if there exists $F_B \widetilde{\in} \tau$ such that
$\varpi(F_{A_2}) q F_B$ and $F_B \widetilde{\subseteq} F_{A_1}$.

Definition 2.11. [34] An $\mathfrak{fpfs}$-point $\varpi(F_A)$ is known as an adherence point of $\mathfrak{fpfs}$-set F_B iff every $\mathfrak{fpfs}$ Q-neighborhood of $\varpi(F_A)$ is a Q-coincident with F_B.

Definition 2.12. [41] Let $(X, \widetilde{\tau})$ be an $\mathfrak{fpfs}$-topological space and $\mathfrak{fpfs}(F_{\widetilde{R}}) = \{F_{A_\alpha} : \alpha \in \Omega\}$ be a collection of $\mathfrak{fpfs}$-subsets of $F_{\widetilde{R}}$. Then the collection $\mathfrak{fpfs}(F_{\widetilde{R}})$ is said to satisfy the finite intersection property if every finite $\mathfrak{fpfs}$-sub-collection of $\mathfrak{fpfs}(F_{\widetilde{R}})$ has non-empty $\mathfrak{fpfs}$-intersection. That is, for any finite $\mathfrak{fpfs}$-subset Ω_1 of Ω,

$$\widetilde{\bigcap_{\beta \in \Omega_1}} F_{A_\beta} \neq F_\phi$$

3. Some Results of $\mathfrak{fpfs}$-Compact space

In this section, we introduce some properties of $\mathfrak{fpfs}$-compact spaces. We establish various concepts for $\mathfrak{fpfs}$-compact space including, $\mathfrak{fpfs}$ neighborhood germ, $\mathfrak{fpfs}$ S-neighborhood, dual $\mathfrak{fpfs}$-point and countability of $\mathfrak{fpfs}$-space. Moreover, we define $\mathfrak{fpfs}$-Ω-accumulation point, $\mathfrak{fpfs}$-Lindelöf space and Bolzano Weierstrass property for $\mathfrak{fpfs}$-space.

Definition 3.1. An $\mathfrak{fpfs}$-set F_A in $(X, \widetilde{\tau})$ is called $\mathfrak{fpfs}$ neighborhood of an $\mathfrak{fpfs}$-point $\varpi(F_B)$ if and only if there exists a $F_C \widetilde{\in} (X, \widetilde{\tau})$ such that $\varpi(F_B) \widetilde{\in} F_C \widetilde{\subseteq} F_A$; an $\mathfrak{fpfs}$ neighborhood F_A is said to be $\mathfrak{fpfs}$-open neighborhood if and only if F_A is $\mathfrak{fpfs}$-open set. The collection of all the $\mathfrak{fpfs}$ neighborhoods of $\varpi(F_B)$ is said to be a system of $\mathfrak{fpfs}$ neighborhoods of $\varpi(F_B)$.

Example 3.2. Let $X = \{\vartheta_1, \vartheta_2, \vartheta_3\}$ be the universal set and $R = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$ be the set of parameters. If $A_1 = \{0.2/\varpi_1, 0.4/\varpi_2, 0.3/\varpi_3\}$ and $A_2 = \{0.2/\varpi_2, 0.1/\varpi_3\}$ are the fuzzy subsets of R then
$F_{A_1} = \{(0.2/\varpi_1, \{0.4/\vartheta_1, 0.2/\vartheta_2, 0.1/\vartheta_3\}), (0.4/\varpi_2, \{0.6/\vartheta_1, 0.7/\vartheta_2, 0.2/\vartheta_3\}), (0.3/\varpi_3, \{0.3/\vartheta_1, 0.2/\vartheta_2, 0.4/\vartheta_3\})\}$ and
$F_{A_2} = \{(0.2/\varpi_2, \{0.4/\vartheta_1, 0.5/\vartheta_2, 0.1/\vartheta_3\}), (0.1/\varpi_3, \{0.3/\vartheta_1, 0.2/\vartheta_2, 0.2/\vartheta_3\})\}$ are $\mathfrak{fpfs}$-sets with the set of parameters A_1 and A_2 respectively.
Then $\widetilde{\tau} = \{F_\phi, F_{\widetilde{R}}, F_{A_1}, F_{A_2}\}$ is an $\mathfrak{fpfs}$-topology.
Let $\varpi(F_B) = \{(0.1/\varpi_2, \{0.3/\vartheta_1, 0.4/\vartheta_2, 0.1/\vartheta_3\})\}$ be an $\mathfrak{fpfs}$-point. For $\varpi(F_B)$ there exists $F_{A_2} \widetilde{\in} \widetilde{\tau}$ such that $\varpi(F_B) \widetilde{\in} F_{A_2} \widetilde{\subseteq} F_{A_1}$.
This implies that F_{A_1} is $\mathfrak{fpfs}$ neighborhood of $\varpi(F_B)$. As F_{A_1} is $\mathfrak{fpfs}$-open set so F_{A_1} is called $\mathfrak{fpfs}$-open neighborhood of $\mathfrak{fpfs}$-point $\varpi(F_B)$.

Proposition 3.3. [41] *Let $\widetilde{\tau}$ be an $\mathfrak{fpfs}$-topology on X. An $\mathfrak{fpfs}$-subfamily $\mathbb{B}$ of $\widetilde{\tau}$ is an $\mathfrak{fpfs}$-base for $\widetilde{\tau}$ if and only if for each $\mathfrak{fpfs}$-point $\varpi(F_A)$ in $(X, \widetilde{\tau})$ and for each $\mathfrak{fpfs}$-open Q-neighborhood F_B of $\varpi(F_A)$, there exists $B\widetilde{\in}\mathbb{B}$ such that $\varpi(F_A)qB\widetilde{\in}F_B$.*

Definition 3.4. Let $\varpi(F_A)$ be an $\mathfrak{fpfs}$-point and $\varpi(N_B)$ a fundamental $\mathfrak{fpfs}$-set in $\mathfrak{fpfs}$-topological space $(X, \widetilde{\tau})$. If $\varpi(F_A)\widetilde{\in}\varpi(N_B)$, then $\varpi(N_B)$ is called an $\mathfrak{fpfs}$ neighborhood germ of $\varpi(F_A)$.

Theorem 3.5. *An $\mathfrak{fpfs}$-set F_A is an $\mathfrak{fpfs}$ neighborhood of an $\mathfrak{fpfs}$-point $\varpi(F_B)$ in $\mathfrak{fpfs}$-topological space $(X, \widetilde{\tau})$ if and only if there exist an open $\mathfrak{fpfs}$-set $F_C\widetilde{\in}\widetilde{\tau}$ and an $\mathfrak{fpfs}$ neighborhood germ $\varpi(N_B)$ of $\varpi(F_B)$ such that*
$\varpi(F_B)\widetilde{\in}\varpi(N_B)\widetilde{\subset}F_C\widetilde{\subset}F_A$.

Proof. Consider an $\mathfrak{fpfs}$ neighborhood F_A of an $\mathfrak{fpfs}$-point $\varpi(F_B)$ in $(X, \widetilde{\tau})$ then by definition there exists an $\mathfrak{fpfs}$-open set $F_C\widetilde{\in}\widetilde{\tau}$

$$\varpi(F_B)\widetilde{\in}F_C\widetilde{\subset}F_A \tag{3. 1}$$

For any $\varpi(N_B)$ $\mathfrak{fpfs}$ neighborhood germ of $\varpi(F_B)$

$$\varpi(F_B)\widetilde{\in}\varpi(N_B) \tag{3. 2}$$

Combining above two equations (3. 1),(3. 2) we get
$\varpi(F_B)\widetilde{\in}\varpi(N_B)\widetilde{\subset}F_C\widetilde{\subset}F_A$.
Conversely, suppose that there exists an $\mathfrak{fpfs}$-open set$F_C\widetilde{\in}\widetilde{\tau}$ and an $\mathfrak{fpfs}$ neighborhood germ $\varpi(N_B)$ of $\varpi(F_B)$ such that $\varpi(F_B)\widetilde{\in}\varpi(N_B)\widetilde{\subset}F_C\widetilde{\subset}F_A$. Then from given relation we can write that $\varpi(F_B)\widetilde{\subset}F_C\widetilde{\subset}F_A$, which clearly shows that F_A is an $\mathfrak{fpfs}$ neighborhood of an $\mathfrak{fpfs}$-point $\varpi(F_B)$ in $\mathfrak{fpfs}$-topological space $(X, \widetilde{\tau})$. □

Definition 3.6. An $\mathfrak{fpfs}$-set F_A is called $\mathfrak{fpfs}$-S-neighborhood of $\mathfrak{fpfs}$-point $\varpi(F_B)$ in $\mathfrak{fpfs}$-topological space $(X, \widetilde{\tau})$ if and only if there is an $\mathfrak{fpfs}$ neighborhood germ $\varpi(N_B)$ of $\varpi(F_B)$ and an $\mathfrak{fpfs}$-open set F_C such that $\varpi(F_B)\widetilde{\in}\varpi(N_B)\widetilde{\subset}F_C\widetilde{\subset}F_A$.

Definition 3.7. Let $\varpi(F_B)$ be an $\mathfrak{fpfs}$-point, then $[\varpi(F_B)]^c$ is said to be dual $\mathfrak{fpfs}$-point of $\varpi(F_B)$ and denoted as $\varpi(F_B^d)$.
Remark: The $\mathfrak{fpfs}$-Q-neighborhood of an $\mathfrak{fpfs}$-point is the $\mathfrak{fpfs}$ neighborhood of its dual-$\mathfrak{fpfs}$-point.

Proposition 3.8. *Let F_A be an $\mathfrak{fpfs}$-set in $\mathfrak{fpfs}$-topological space $(X, \widetilde{\tau})$ and $\varpi(F_B)$ be an $\mathfrak{fpfs}$-point. Every $\mathfrak{fpfs}$ neighborhood of its dual $\mathfrak{fpfs}$-point $\varpi(F_B^d)$ is $\mathfrak{fpfs}$-quasi-coincident with F_A if and only if $\varpi(F_B)\widetilde{\in}\overline{F_A}$.*

Proof. The Proof is similar to 4.15 of [34]. □

Theorem 3.9. *The $\mathfrak{fpfs}$-point $\varpi(F_B)\widetilde{\in}F_A^o$ if and only if its dual $\mathfrak{fpfs}$-point $\varpi(F_B^d)\widetilde{\nsubseteq}\overline{(F_A^c)}$.*

Proof. If $\varpi(F_B^d)\widetilde{\nsubseteq}\overline{(F_A^c)}$, then there is an $\mathfrak{fpfs}$ neighborhood F_C of $\varpi(F_B)$ $\mathfrak{fpfs}$-quasi coincident with $(F_A)^c$, i.e, $F_C\widetilde{\subset}F_A$ and so $\varpi(F_B)\widetilde{\in}F_C\widetilde{\subset}F_A$, hence $\varpi(F_B)\widetilde{\in}F_A^o$.
Conversely, if $\varpi(F_B)\widetilde{\in}F_A^o$ then there is $F_C\widetilde{\in}\widetilde{\tau}$ such that
$\varpi(F_B)\widetilde{\in}F_C\widetilde{\subset}F_A$; i.e F_C is not $\mathfrak{fpfs}$-quasi-coincident with F_A^c (or F_C and F_A^c are $\mathfrak{fpfs}$-quasi-incoincident), hence $\varpi(F_B^d)\widetilde{\nsubseteq}\overline{(F_A^c)}$. □

Definition 3.10. Let Ω_Q be an $\mathfrak{fpfs}$-Q-neighborhood system of a $\mathfrak{fpfs}$-point $\varpi(F_A)$ in $(X, \widetilde{\tau})$. A $\mathfrak{fpfs}$-subfamily B_Q of Ω_Q is called a $\mathfrak{fpfs}$-Q-neighborhood base of Ω_Q if and only if for each $F_B \widetilde{\in} \Omega_Q$ there exists $F_C \widetilde{\in} B_Q$ such that $F_C \widetilde{\subset} F_B$.

Definition 3.11. An $\mathfrak{fpfs}$-topological space $(X, \widetilde{\tau})$ satisfy the $\mathfrak{fpfs}$-Q-first axiom of countability or to be $\mathfrak{fpfs}$-Q-C_1 if and only if every $\mathfrak{fpfs}$-point in $(X, \widetilde{\tau})$ has an $\mathfrak{fpfs}$-Q-neighborhood base which is countable. Otherwise $(X, \widetilde{\tau})$ is $\mathfrak{fpfs}$-C_2-space.

Remark: If $(X, \widetilde{\tau})$ is $\mathfrak{fpfs}$-C_2-space, then it is also $\mathfrak{fpfs}$-Q-C_1-space but may or may not be $\mathfrak{fpfs}$-C_1-space.

Proposition 3.12. *An $\mathfrak{fpfs}$-point $\varpi(F_A) \widetilde{\in} (F_B)^o$ if and only if there exists an $\mathfrak{fpfs}$ neighborhood of $\varpi(F_A)$ which contained in $(F_B)^o$.*

Proof. The proof is straight forward. □

Theorem 3.13. (a) $F_A^o = [\overline{(F_A^c)}]^c$,
(b) $\overline{F_A} = [(F_A^c)^o]^c$,
(c) $(\overline{F_A})^c = (F_A^c)^o$,
(d) $\overline{F_A^c} = (F_A^o)^c$.

Proof. (a) Let ${}^oF_A = \{F_{A_\alpha} : \alpha \in \Omega$ and $F_{A_\alpha} \widetilde{\subseteq} F_A\}$ be the collection of $\mathfrak{fpfs}$-open sets; then $F_A^o = \widetilde{\cup}\, {}^oF_A$. Evidently, ${}^o\overline{F_A^c} = \{F_A^c : F_{A_\alpha} \widetilde{\in}\, {}^oF_A\}$ is the collection of all $\mathfrak{fpfs}$-closed sets containing F_A^c and hence $\overline{F_A^c} = \widetilde{\cap}\, {}^oF_A^c$. By using De Morgan's law, we can write that

$$\widetilde{F_A^c} = [\widetilde{\cap}(F_A^c)]^c = \widetilde{\cup}[(F_A^c)^c] = \widetilde{\cup}(F_A^o) = F_A^o.$$

The proof is similar for the remaining parts of the theorem. □

Proposition 3.14. *The derived set of each $\mathfrak{fpfs}$-set is $\mathfrak{fpfs}$-closed if and only if the derived set of every $\mathfrak{fpfs}$-point is $\mathfrak{fpfs}$-closed.*

Definition 3.15. An $\mathfrak{fpfs}$-point $\varpi(F_A)$ is called a $\mathfrak{fpfs}$-Ω-accumulation point of F_B if and only if the $\mathfrak{fpfs}$-set consisting of all the $\mathfrak{fpfs}$-points at each of which every $\mathfrak{fpfs}$-Q-neighborhood of $\varpi(F_A)$ and F_B are $\mathfrak{fpfs}$-quasi-coincident is uncountable.

Definition 3.16. Let F_A be an $\mathfrak{fpfs}$-set in $(X, \widetilde{\tau})$. It satisfy $\mathfrak{fpfs}$-Lindelöf property if and only if every $\mathfrak{fpfs}$-open cover of F_A has a subcover which is countable.
$(X, \widetilde{\tau})$ is said to be hereditarily $\mathfrak{fpfs}$-Lindelöf space if and only if every $\mathfrak{fpfs}$-set in X has the $\mathfrak{fpfs}$-Lindelöf property.

Proposition 3.17. *If $(X, \widetilde{\tau})$ is $\mathfrak{fpfs}$-C_2-space then it is hereditarily $\mathfrak{fpfs}$-Lindelöf space.*

Definition 3.18. [41] The family of $\mathfrak{fpfs}$-sets ${}^oF_A$ is an $\mathfrak{fpfs}$-cover for a $\mathfrak{fpfs}$-set F_B if and only if $F_B \widetilde{\subseteq} \widetilde{\cup}\{F_A : F_A \widetilde{\in}\, {}^oF_A\}$. If each member of ${}^oF_A$ is $\mathfrak{fpfs}$-open set then it is an $\mathfrak{fpfs}$-open cover. An $\mathfrak{fpfs}$-subcover of ${}^oF_A$ also an $\mathfrak{fpfs}$-cover.

Definition 3.19. [41] An $\mathfrak{fpfs}$-topological space X is $\mathfrak{fpfs}$-compact if and only if each $\mathfrak{fpfs}$-open cover of $F_{\widetilde{R}}$ has a finite $\mathfrak{fpfs}$-subcover. An $\mathfrak{fpfs}$-subspace (Y, E) of $(X, \widetilde{\tau})$ is said to be $\mathfrak{fpfs}$-compact if Y with relative $\mathfrak{fpfs}$-topology is compact.

Example 3.20. (i) [41] Let $X = \{\vartheta_1, \vartheta_2, ...\}$ be the universal set and $R = \{\varpi_1, \varpi_2, ...\}$ be the set of parameters. We define an $\mathfrak{fpfs}$-set

$$F_{A_n} = \{(\frac{1}{\varpi_1}, \{\frac{1}{\vartheta_1}\}), (\frac{1/2}{\varpi_2}, \{\frac{1}{\vartheta_1}, \frac{1/2}{\vartheta_2}\}), ..., (\frac{1/n}{\varpi_n}, \{\frac{1}{\vartheta_1}, \frac{1/2}{\vartheta_2}, ..., \frac{1/n}{\vartheta_n}\}) : n = 1, 2, 3, ...\}$$

Then $\tau = \{F_{A_n} : n = 1, 2, 3, ...\}\widetilde{\cup}\{F_\phi, F_{\widetilde{R}}\}$ is an $\mathfrak{fpfs}$-topology on X and $(X, \widetilde{\tau})$ is $\mathfrak{fpfs}$-compact.
(ii) Any $\mathfrak{fpfs}$-topological space $(X, \widetilde{\tau})$ where either $F_{\widetilde{R}}$ is finite or $\widetilde{\tau}$ consists of a finite number of elements is $\mathfrak{fpfs}$-compact. This is so because, in this case, every $\mathfrak{fpfs}$-open cover of $F_{\widetilde{R}}$ is finite. Every $\mathfrak{fpfs}$-subspace of such a space is also $\mathfrak{fpfs}$-compact.

Proposition 3.21. [41] *Let $(X, \widetilde{\tau})$ and $(Y, \widetilde{\tau}')$ be $\mathfrak{fpfs}$-topological spaces and $f_{up} : (X, \widetilde{\tau}) \to (Y, \widetilde{\tau}')$ be an $\mathfrak{fpfs}$-mapping. If $(X, \widetilde{\tau})$ is $\mathfrak{fpfs}$-compact and f_{up} is $\mathfrak{fpfs}$-continuous surjection, then $(Y, \widetilde{\tau}')$ is $\mathfrak{fpfs}$-compact.*

Theorem 3.22. *$\mathfrak{fpfs}$-compact subset of $\mathfrak{fpfs}$-hausdorff space is $\mathfrak{fpfs}$-closed.*

Proof. Let $(X, \widetilde{\tau})$ be an $\mathfrak{fpfs}$-T_2-space and F_A be a $\mathfrak{fpfs}$-compact subset of $(X, \widetilde{\tau})$. We will show that F_A^c is $\mathfrak{fpfs}$-open. For this let $\varpi(F_B)\widetilde{\in}F_A^c$. Let $\varpi(F_C)$ be any arbitrary $\mathfrak{fpfs}$-element of F_A, then $\varpi(F_B) \neq \varpi(F_C)$. Since $(X, \widetilde{\tau})$ is $\mathfrak{fpfs}$-hausdorff space, there are $\mathfrak{fpfs}$-open sets F_D and F_E in $(X, \widetilde{\tau})$ containing $\varpi(F_B)$ and $\varpi(F_C)$ respectively such that

$$F_D\widetilde{\cap}F_E = F_\phi$$

The collection $\{F_E\widetilde{\cap}F_A : \varpi(F_C)\widetilde{\in}F_A\}$ is a $\mathfrak{fpfs}$-open cover of F_A. Since F_A is $\mathfrak{fpfs}$-compact , then there exists an $\mathfrak{fpfs}$-open cover for F_A given as

$$\{F_{E_i}\widetilde{\cap}F_A : i = 1, 2, 3, ..., n\}$$

Now corresponding to each E_i, let F_{DE_i} be the $\mathfrak{fpfs}$-open set containing $\varpi(F_B)$.

Then $U_D = \widetilde{\bigcap_{i=1}^{n}} F_{DE_i}$ is $\mathfrak{fpfs}$-open, contains $\varpi(F_B)$ and

$$U_D\widetilde{\cap}F_A = U_D\widetilde{\cap}\widetilde{\bigcup_{i=1}^{n}}(F_{E_i}\widetilde{\cap}F_A)$$

$$\widetilde{\subseteq}U_D\widetilde{\cap}\widetilde{\bigcup_{i=1}^{n}}(F_{E_i})$$

$$\widetilde{\subseteq}\widetilde{\bigcup_{i=1}^{n}}(U_D\widetilde{\cap}F_{E_i}) = F_\phi \because F_{DE_i}\widetilde{\cap}F_{E_i} = F_\phi$$

Hence $\varpi(F_B)\widetilde{\in}U_D\widetilde{\subseteq}F_A^c$. This implies that F_A^c is $\mathfrak{fpfs}$-open. Thus F_A is $\mathfrak{fpfs}$-closed. □

Theorem 3.23. *Let $(X, \widetilde{\tau})$ be an $\mathfrak{fpfs}$-Hausdorff space F_A a $\mathfrak{fpfs}$-compact subset of $(X, \widetilde{\tau})$ and $\varpi(F_B)$ be a $\mathfrak{fpfs}$-element of $(X, \widetilde{\tau})$ and $\varpi(F_B)\overline{q}F_A$. Then there are disjoint $\mathfrak{fpfs}$-open sets F_C and F_D in $(X, \widetilde{\tau})$ such that*

$$\varpi(F_B)\widetilde{\in}F_C and F_A\widetilde{\subseteq}F_D$$

Proof. Suppose that F_A is an $\mathfrak{fpfs}$-compact subset of $(X, \widetilde{\tau})$ and $\varpi(F_B)\widetilde{\in}F_A^c$. For each $\varpi(F_G)\widetilde{\in}F_A$, $\varpi(F_G) \neq \varpi(F_B)$, since $(X, \widetilde{\tau})$ is $\mathfrak{fpfs}$-Hausdorff, there are $\mathfrak{fpfs}$-open sets F_{GB} and F_G such that

$$\varpi(F_B)\widetilde{\in}F_{GB},\ \varpi(F_G)\widetilde{\in}F_G \text{ and } F_{GB}\widetilde{\cap}F_G = F_\phi$$

Now $\{F_G\widetilde{\cap}F_A : \varpi(F_G)\widetilde{\in}F_A\}$ is a $\mathfrak{fpfs}$-open cover for F_A. Since F_A is a $\mathfrak{fpfs}$-compact, this $\mathfrak{fpfs}$-open cover has a finite $\mathfrak{fpfs}$-sub-cover

$$F_{G_1}\widetilde{\cap}F_A\ ,\ F_{G_2}\widetilde{\cap}F_A\ , ..., F_{G_n}\widetilde{\cap}F_A$$

Let $F_{GB_1},\ F_{GB_2},\ F_{GB_3}, ...,\ F_{GB_n}$ be the corresponding $\mathfrak{fpfs}$-open sets in $(X, \widetilde{\tau})$ containing $\varpi(F_B)$.
Take

$$F_C = \widetilde{\bigcap_{i=1}^{n}} F_{GB_i} \text{ and } F_D = \widetilde{\bigcup_{i=1}^{n}} F_{G_i}$$

Then

$$\varpi(F_B)\widetilde{\in}F_C\ ,\ F_A\widetilde{\subseteq}F_D$$

and

$$F_C\widetilde{\cap}F_D = F_C\widetilde{\cap}(\widetilde{\bigcup_{i=1}^{n}} F_{G_i})$$

$$= \widetilde{\bigcup_{i=1}^{n}}(F_C\widetilde{\cap}F_{G_i}) = F_\phi.$$

□

Remark: Any arbitrary $\mathfrak{fpfs}$-subset of an $\mathfrak{fpfs}$-compact space may not be an $\mathfrak{fpfs}$-compact.

Theorem 3.24. *Every* $\mathfrak{fpfs}$*-closed subset of an* $\mathfrak{fpfs}$*-compact space is* $\mathfrak{fpfs}$*-compact.*

Remark: $\mathfrak{fpfs}$-closed subset of a $\mathfrak{fpfs}$-compact Hausdorff space is itself $\mathfrak{fpfs}$-compact and $\mathfrak{fpfs}$-Hausdorff.

Theorem 3.25. *Every* $\mathfrak{fpfs}$*-compact Hausdorff space is* $\mathfrak{fpfs}$*-normal.*

Proof. Let $(X, \widetilde{\tau})$ be an $\mathfrak{fpfs}$-compact Hausdorff space and F_{A_1}, F_{A_2} be arbitrary two disjoint $\mathfrak{fpfs}$-closed subsets of $(X, \widetilde{\tau})$. By Theorem 3.23 for any $\varpi(F_B)\widetilde{\in}F_{A_1}$ then there are $\mathfrak{fpfs}$-open sets F_B and F_D in $(X, \widetilde{\tau})$ such that

$$\varpi(F_B)\widetilde{\in}F_B,\ F_{A_2}\widetilde{\subseteq}F_D,\ F_B\widetilde{\cap}F_D = F_\phi$$

The sets $\{F_B : \varpi(F_B)\widetilde{\in}F_{A_1}\}$ form a $\mathfrak{fpfs}$-open cover for F_{A_1}. Since F_{A_1} is $\mathfrak{fpfs}$-closed, F_{A_1} is $\mathfrak{fpfs}$-compact, so there are $\mathfrak{fpfs}$-elements $\varpi(F_{B_1}),\ \varpi(F_{B_2}), ..., \varpi(F_{B_n})$ such that

$$F_{A_1}\widetilde{\subseteq}\widetilde{\bigcup_{i=1}^{n}} F_{B_i}$$

Let

$$F_U = \widetilde{\bigcup_{i=1}^{n}} F_{B_i} \ , \ F_V = \widetilde{\bigcap_{i=1}^{n}} F_{D_i}$$

Then

$$F_{A_1} \widetilde{\subseteq} F_U \ , \ F_{A_2} \widetilde{\subseteq} F_V \text{ and } F_U \widetilde{\cap} F_V = F_\phi$$

Hence $(X, \widetilde{\tau})$ is $\mathfrak{fpfs}$-normal. □

Definition 3.26. An $\mathfrak{fpfs}$-topological space $(X, \widetilde{\tau})$ is countably $\mathfrak{fpfs}$-compact if and only if every countable $\mathfrak{fpfs}$-open cover has an $\mathfrak{fpfs}$-subcover which is finite.
Remark: Clearly, every $\mathfrak{fpfs}$-compact space is countably $\mathfrak{fpfs}$-compact space. This is so because if every $\mathfrak{fpfs}$-open cover of an $\mathfrak{fpfs}$-space $(X, \widetilde{\tau})$ has an $\mathfrak{fpfs}$-subcover which is finite then every countable $\mathfrak{fpfs}$-open cover also has a finite $\mathfrak{fpfs}$-subcover.

Theorem 3.27. *Let $(X, \widetilde{\tau})$ be an $\mathfrak{fpfs}$-topological space. Then every $\mathfrak{fpfs}$-subset of X which is countably infinite has an $\mathfrak{fpfs}$-limit point if and only if any infinite $\mathfrak{fpfs}$-subset of $(X, \widetilde{\tau})$ has an $\mathfrak{fpfs}$-limit point.*

Proof. If every infinite $\mathfrak{fpfs}$-subset of a $\mathfrak{fpfs}$-topological space has an $\mathfrak{fpfs}$-limit point in $(X, \widetilde{\tau})$ then it immediately follows that every countably infinite $\mathfrak{fpfs}$-subset of $(X, \widetilde{\tau})$also has an $\mathfrak{fpfs}$-limit point in $(X, \widetilde{\tau})$.
Conversely, suppose that every $\mathfrak{fpfs}$-subset of X which is countably infinite has an $\mathfrak{fpfs}$-limit point in $(X, \widetilde{\tau})$. Let F_A be any infinite $\mathfrak{fpfs}$-subset of $(X, \widetilde{\tau})$. Then F_A contains a countably infinite $\mathfrak{fpfs}$-subset F_B

$$F_B = \{(\frac{\mu_{F_B}(\varpi_i)}{\varpi_i}, \{\frac{\gamma_{F_B}^{\varpi_i}(\vartheta_i)}{\vartheta_i}\}) : i, j = 1, 2, 3, ..., n, ...\}$$

By our assumption F_B has an $\mathfrak{fpfs}$-limit point $\varpi(F_C)$ in $(X, \widetilde{\tau})$. But then $\varpi(F_C)$ is also a $\mathfrak{fpfs}$-limit point of F_A. □

Theorem 3.28. *If $(X, \widetilde{\tau})$ is a countably $\mathfrak{fpfs}$-compact space, then every $\mathfrak{fpfs}$-subset of $(X, \widetilde{\tau})$ which is infinite has an $\mathfrak{fpfs}$-limit point in $(X, \widetilde{\tau})$.*

Proof. Suppose that F_A is any infinite $\mathfrak{fpfs}$-subset of a countably $\mathfrak{fpfs}$-compact space $(X, \widetilde{\tau})$. Then F_A has a countably infinite $\mathfrak{fpfs}$-subset

$$F_B = \{(\frac{\mu_{F_B}(\varpi_i)}{\varpi_i}, \{\frac{\gamma_{F_B}^{\varpi_i}(\vartheta_i)}{\vartheta_i}\}) : i, j = 1, 2, 3, ..., n, ...\}$$

Suppose that F_A and therefore also F_B, has no $\mathfrak{fpfs}$-limit point in $(X, \widetilde{\tau})$. Consider the $\mathfrak{fpfs}$-subsets

$$F_{C_P} = \{(\frac{\mu_{F_{C_P}}(\varpi_n)}{\varpi_n}, \{\frac{\gamma_{F_{C_P}}^{\varpi_n}(\vartheta_m)}{\vartheta_m}\}) : p, q = 1, 2, 3, ... ; \ n = p, p{+}1, ... ; \ m = q, q{+}1, ...\}$$

Since the $\mathfrak{fpfs}$-derived set of every $F_{C_P}, P = 1, 2, 3, ...$ is $\mathfrak{fpfs}$-empty, so F_{C_P} are all $\mathfrak{fpfs}$-closed. Also $\{F_{C_P}, P = 1, 2, 3, ...\}$ satisfies the finite intersection property because

$$F_{C_{P_1}} \widetilde{\cap} F_{C_{P_2}} \widetilde{\cap} \ldots \widetilde{\cap} F_{C_{P_k}} = F_{C_{P_0}} \neq F_\phi$$

where $P_0 = \max(P_1.P_2, ..., P_k)$.
Now,

$$\widetilde{\bigcap_{p=1}^{\infty}} F_{C_P} = F_\phi$$

for if an $\mathfrak{fpfs}$-point $\varpi(F_{x_P})$ of F_B is in $\widetilde{\bigcap\limits_{p=1}^{\infty}} F_{C_P}$, then $\varpi(F_{x_P}) \widetilde{\notin} F_{C_{p-1}}$. But then $(X, \widetilde{\tau})$ is not countably $\mathfrak{fpfs}$-compact, a contradiction. Hence F_B and therefore also F_A, has an $\mathfrak{fpfs}$-limit point in $(X, \widetilde{\tau})$. □

Definition 3.29. An $\mathfrak{fpfs}$-space $(X, \widetilde{\tau})$ is said to satisfy the Bolzano-Weierstrass property if and only if every $\mathfrak{fpfs}$-subset of $(X, \widetilde{\tau})$ which is infinite has a $\mathfrak{fpfs}$-limit point in $(X, \widetilde{\tau})$. Thus by Theorem 3.28 we can say that every countably $\mathfrak{fpfs}$-compact space satisfy the Bolzano-Weierstrass property.

Proposition 3.30. *Let $(X, \widetilde{\tau})$ be an $\mathfrak{fpfs}$-T_1-space. Then $(X, \widetilde{\tau})$ is countably $\mathfrak{fpfs}$-compact if and only if $(X, \widetilde{\tau})$ satisfy Bolzano-Weierstrass property.*

4. AN ALGORITHM FOR $\mathfrak{fpfs}$-COMPACT SPACE TO DECISION-MAKING

Despite shooting excellent pictures using a quality digital camera, at times it's necessary to edit the pictures taken to suit your photography business. This is when driven by a desire to turn your image into an art or transforming it into a completely new image so it can seek attention. It is necessary to edit photos to increase their attractiveness and quality, hence improving their value. Photo editing may be applied so as to obscure or take out unwanted details that deprive focus away from the subject you wanted to underline. It is thus very significant to both professional and amateur photographers to learn photo editing skills and deliver the software itself to be able to contend with what is seen as the standard of photography now a days.
We have modified the algorithm used in [18] for $\mathfrak{fpfs}$-compact space.
Algorithm:
The algorithm which we will use in the given application is based on the following steps.
step 1: Construct an $\mathfrak{fpfs}$-compact space using universal set X and selected set of parameters .
step 2: Select some $\mathfrak{fpfs}$-open sets from the $\mathfrak{fpfs}$-compact space.
step 3: Find the fuzzy decision sets from all selected $\mathfrak{fpfs}$-open sets by using the formula given as

$$F_A^d = \{\gamma_{F_A^d}(\vartheta)/\vartheta : \vartheta \in X\}$$

where

$$\mu_{F_A^d}(\vartheta) = \frac{1}{|supp(A)|} \sum_{\varpi \in supp(A)} \mu_{F_A}(\varpi) \gamma_{F_A}^{\varpi}(\vartheta)$$

step 4: Add the all fuzzy decision sets by using fuzzy addition.
step 5: Find the largest choice value from the resultant set after fuzzy addition.

Example 4.1. A good graphic designer signed a contract for a marriage ceremony. After capturing the photographs, family assigned him to make a photo album of beautiful family pictures with better graphics. For editing the images and to make them graphically better he needs to use an image editing software. We constitute a new algorithm which helps him choose the best software that makes his task extremely easy and advance.
The set $X = \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5, \vartheta_6, \vartheta_7, \vartheta_8, \vartheta_9, \vartheta_{10}, \vartheta_{11}, \vartheta_{12}, \vartheta_{13}, \vartheta_{14}, \vartheta_{15}\}$ represent the list of some image editing softwares, where
ϑ_1 = CorelDRAW Graphics Suite X5,
ϑ_2 = Adobe Indesign,
ϑ_3 = Adobe Flash,
ϑ_4 = Art Rage 3.5,
ϑ_5 = Adobe photoshop,
ϑ_6 = Wacom Tablets,
ϑ_7 = Gimp,
ϑ_8 = Paint.NET,
ϑ_9 = Pixelmator,
ϑ_{10} = 5DFLY,
ϑ_{11} = Skitch,
ϑ_{12} = Adobe Illuster,
ϑ_{13} = Picnik,
ϑ_{14} = Fat Paint,
ϑ_{15} = AutoCAD.
The set of parameters,

$$R = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5, \varpi_6, \varpi_7, \varpi_8, \varpi_9, \varpi_{10}, \varpi_{11}\}$$

is the set of characteristics or qualities of softwares, where
ϖ_1 = capability,
ϖ_2 = security,
ϖ_3 = performance,
ϖ_4 = compatibility,
ϖ_5 = integrity,
ϖ_6 = flexibility,
ϖ_7 = modularity,
ϖ_8 = portability,
ϖ_9 = reusability,
ϖ_{10} = correctness,
ϖ_{11} = IT-bility.
"There are many parameters which tell us about the importance and quality of image editing softwares but here we consider only eleven parameters because they are most important for image editing and enough to identify the quality of softwares. We can increase the number of attributes according to our choice or according to the given data for any decision-making problem.
He needs only six parameters "capability", "security", "flexibility", "portability", "reusability" and "IT-bility", which constitute a subset E of R given by
$E = \{\varpi_1, \varpi_2, \varpi_6, \varpi_8, \varpi_9, \varpi_{11}\}$.

We consider an $\mathfrak{fpfs}$-compact space on given set of parameters E and on X. We choose some $\mathfrak{fpfs}$-sets from $\mathfrak{fpfs}$-compact space according to the photographer's choice through which we can make the decision set. Suppose that (F_A) is the selected $\mathfrak{fpfs}$-set where $A = \{0.8/\varpi_1, 0.6/\varpi_2, 0.7/\varpi_6, 0.5/\varpi_8, 0.4/\varpi_9\}$ is fuzzy subset of E. All the attributes are important, but here the attributes are chosen with the desire of photographer, because he knows well that how to make an album according to the customer's choice. All the values presented in the table are according to a survey, opinion and experience of some professional photographers".
The tabular form of $\mathfrak{fpfs}$-set F_A can be represented as

F_A	$0.8/\varpi_1$	$0.6/\varpi_2$	$0.7/\varpi_6$	$0.5/\varpi_8$	$0.4/\varpi_9$
ϑ_1	0.2	0.3	0.2	0.1	0.1
ϑ_2	0.4	0.2	0.4	0.3	0.3
ϑ_3	0.1	0.4	0.1	0.2	0.2
ϑ_4	0.2	0.1	0.2	0.3	0.3
ϑ_5	0.9	0.9	0.9	0.9	0.9
ϑ_6	0.4	0.4	0.4	0.3	0.3
ϑ_7	0.3	0.4	0.3	0.3	0.3
ϑ_8	0.1	0.2	0.1	0.2	0.2
ϑ_9	0.6	0.6	0.6	0.5	0.6
ϑ_{10}	0.2	0.3	0.2	0.2	0.2
ϑ_{11}	0.4	0.1	0.2	0.3	0.5
ϑ_{12}	0.8	0.8	0.8	0.8	0.8
ϑ_{13}	0.1	0.2	0.1	0.1	0.1
ϑ_{14}	0.7	0.7	0.7	0.7	0.7
ϑ_{15}	0.1	0.1	0.1	0.1	0.1

Now we calculate the support of a fuzzy set A given as

$$\text{supp}(A) = \{\varpi : \mu(\varpi) \neq 0\}$$

where $\mu(\varpi)$ denotes the degree of membership of the element ϖ. Clearly, Supp(A) is a crisp set. We can see that in above $\mathfrak{fpfs}$-set F_A, $\text{supp}(A) = \{\varpi_1, \varpi_2, \varpi_6, \varpi_8, \varpi_9\}$ which implies that $|supp(A)| = 5$.
The fuzzy decision set F_A^d of F_A can be calculated by using the given formulas

$$F_A^d = \{\gamma_{F_A^d}(\vartheta)/\vartheta : \vartheta \in X\}$$

where

$$\mu_{F_A^d}(\vartheta) = \frac{1}{|supp(A)|} \sum_{\varpi \in supp(A)} \mu_{F_A}(\varpi)\gamma_{F_A}^{\varpi}(\vartheta)$$

The above formulas are modified forms of the formulas used in [18] for $\mathfrak{fps}$-set. We modify these formulas for $\mathfrak{fpfs}$-set.
$F_A^d = \{0.114/\vartheta_1, 0.198/\vartheta_2, 0.114/\vartheta_3, 0.126/\vartheta_4, 0.54/\vartheta_5, 0.222/\vartheta_6, 0.192/\vartheta_7,$
$0.09/\vartheta_8, 0.35/\vartheta_9, 0.132/\vartheta_{10}, 0.174/\vartheta_{11}, 0.48/\vartheta_{12}, 0.072/\vartheta_{13}, 0.42/\vartheta_{14}, 0.06/\vartheta_{15}\}$.
Suppose that (F_B) is the other $\mathfrak{fpfs}$-set which is selected,
here $B = \{0.5/\varpi_1, 0.7/\varpi_2, 0.3/\varpi_6, 0.7/\varpi_8, 0.8/\varpi_9, 0.3/\varpi_{11}\}$ is fuzzy subset of E.

Thus the tabular form of $\mathfrak{fpfs}$-set F_B can be represented as

F_B	$0.5/\varpi_1$	$0.7/\varpi_2$	$0.3/\varpi_6$	$0.7/\varpi_8$	$0.8/\varpi_9$	$0.3/\varpi_{11}$
ϑ_1	0.2	0.3	0.2	0.1	0.1	0.1
ϑ_2	0.4	0.2	0.4	0.3	0.3	0.3
ϑ_3	0.1	0.4	0.1	0.2	0.2	0.2
ϑ_4	0.2	0.1	0.2	0.3	0.3	0.3
ϑ_5	0.9	0.9	0.9	0.9	0.9	0.9
ϑ_6	0.4	0.4	0.4	0.3	0.3	0.3
ϑ_7	0.3	0.4	0.3	0.3	0.3	0.3
ϑ_8	0.1	0.2	0.1	0.2	0.2	0.2
ϑ_9	0.6	0.6	0.6	0.5	0.6	0.6
ϑ_{10}	0.2	0.3	0.2	0.2	0.2	0.2
ϑ_{11}	0.4	0.1	0.2	0.3	0.5	0.5
ϑ_{12}	0.8	0.8	0.8	0.8	0.8	0.8
ϑ_{13}	0.1	0.2	0.1	0.1	0.1	0.1
ϑ_{14}	0.7	0.7	0.7	0.7	0.7	0.7
ϑ_{15}	0.1	0.1	0.1	0.1	0.1	0.1

As we can see that supp$(B) = \{\varpi_1, \varpi_2, \varpi_6, \varpi_8, \varpi_9, \varpi_{11}\}$ which implies that $|supp(B)| = 6$.
Similarly by using the above method we can find out the fuzzy decision set F_B^d for F_B.
$F_B^d = \{0.091/\vartheta_1, 0.166/\vartheta_2, 0.12/\vartheta_3, 0.1283/\vartheta_4, 0.495/\vartheta_5, 0.19/\vartheta_6, 0.176/\vartheta_7, 0.0966/\vartheta_8, 0.318/\vartheta_9, 0.121/\vartheta_{10}, 0.181/\vartheta_{11}, 0.44/\vartheta_{12}, 0.066/\vartheta_{13}, 0.385/\vartheta_{14}, 0.055/\vartheta_{15}\}$.
Now we take sum of both fuzzy decision sets F_A^d and F_B^d according to fuzzy rules by using the given formula which is basically the addition of two fuzzy sets and was defined by Zadeh.

$$\gamma_{F_A^d + F_B^d}(\vartheta) = \gamma_{F_A^d}(\vartheta) + \gamma_{F_B^d}(\vartheta) - [\gamma_{F_A^d}(\vartheta) * \gamma_{F_B^d}(\vartheta)] \; \forall \, \vartheta \in X.$$

Which implies that
$F_A^d + F_B^d = \{0.1947/\vartheta_1, 0.3312/\vartheta_2, 0.2204/\vartheta_3, 0.2382/\vartheta_4, 0.7677/\vartheta_5, 0.3699/\vartheta_6, 0.3343/\vartheta_7, 0.1779/\vartheta_8, 0.5567/\vartheta_9, 0.2370/\vartheta_{10}, 0.3235/\vartheta_{11}, 0.7088/\vartheta_{12}, 0.1332/\vartheta_{13}, 0.6433/\vartheta_{14}, 0.1117/\vartheta_{15}\}$.
Finally, we pick out the largest degree of membership by

$$\max \gamma_{F_A^d + F_B^d}(\vartheta) = 0.7677$$

Which shows that the photographer should select ϑ_5= Adobe photoshop for editing the images and to make them graphically better. His second choice goes to ϑ_{12}= Adobe Illuster and third choice goes to to ϑ_{14}= Fat Paint.
"This application is based on decision-making and tells us about our preferences. When the decision comes out then it does not means that some products are good or others are bad, infect it gives us first, second and third preferences based on our choice of attributes which help us to make our decisions correctly and beneficently. All the values given to the attributes tells us that how much the photographer needs that quality or parameter for his work and that values given by his own choice. Similarly in the table all the values given to the softwares tells us that how much a software has that quality or in what percentage that software contain the corresponding parameter and that values are calculated by a survey

and according to the opinion and experiences of some professional photographers. This application basically tells us how we can make our decision by using $\mathfrak{fpfs}$-sets, which was chosen from the $\mathfrak{fpfs}$-compact space".

5. Conclusion

In this paper, we introduced $\mathfrak{fpfs}$ neighborhood germ and $\mathfrak{fpfs}$-S-neighborhood. We premised some consequences on $\mathfrak{fpfs}$-compact topological space. We then presented a new algorithm for decision-making, which demonstrated that this method would be appreciated for the researchers. We can utilize the results derived from the studies on $\mathfrak{fpfs}$-compact space to meliorate the concepts. It can be spread over many areas of the problems that contain vagueness and would be valuable to offer the proposed method to subsequent written reports. We trust that the judgments in this report will be fruitful for the investigators to boost and advertise the further analysis on $\mathfrak{fpfs}$-topology to carry out a worldwide structure for their applications in virtual life.

6. Acknowledgments

The authors are highly thankful to the Editor-in-chief and the referees for their valuable comments and suggestions for improving the quality of our paper.

References

[1] S. E. Abbas, M. A. Hebeshi and I. M. Taha, *On Upper and Lower Contra-Continuous Fuzzy Multifunctions*, Punjab Univ. J. Math. Vol. **47**, No. 1 (2015) 105-117.

[2] M. Akram and S. Nawaz, *Certain types of soft graphs*, U. P. B. Sci. Bull. Series A, **78**, No. 4 (2016) 67-82.

[3] M. Akram and M. Sarwar, *novel applications of m-polar fuzzy hypergraphs*, Journal of Intelligent and Fuzzy Systems (2016) 1-16.

[4] M. Akram and M. Sitara, *Interval-Valued Neutrosophic Graph Structures*, Punjab Univ. J. Math. Vol. **50**, No. 1 (2018) 113-137.

[5] M. Akram and G. Shahzadi, *Certain Characterization of m-Polar Fuzzy Graphs by Level Graphs*, Punjab Univ. J. Math. Vol. **49**, No. 1 (2017) 1-12.

[6] M. I. Ali, F. Feng, X. Y. Liu, W. K. Min and M. Shabir, O*n some new operations in soft set theory*, Computers and Mathematics with Applications **57**, (2009) 1547-1553.

[7] B. Ali, M. Abbas and S. Costache, Fixed fuzzy point theorems for fuzzy mappings on complete metric spaces, U. P. B. Sci. Bull. Series A, **78**, No. 3 (2016) 31-42.

[8] M. Aslam Malik and M. Riaz, *G-subsets and G-orbits of under action of the Modular Group*, Punjab Univ. J. Math. Vol. **43**, (2011) 75-84.

[9] M. Aslam Malik and M. Riaz, *Orbits of under the action of the Modular Group PSL (2, Z)*, U. P. B. Sci Bull. Series A, Applied Mathematics and Physics **74**, No. 4 (2012) 109-116.

[10] K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems **20**, (1986) 87-96.

[11] S. Atmaca and I. Zorlutuna, *On fuzzy soft topological spaces*, Annals of Fuzzy Mathematics and Informatics **5**, No. 2 (2013) 377-386.

[12] A. Aygünoğlu, V. Çetkin and H. Aygün, *An Introduction to Fuzzy Soft Topological Spaces*, Hacettepe Journal of Mathematics and Statistics **43**, No. 2 (2014) 193-204.

[13] M. J. Borah and B. Hazarika, *Some results of mixed fuzzy soft topology and applications in Chemistry*, Annals of Fuzzy Mathematics and Informatics **12**, No. 1 (2016) 129-138.

[14] C. L. Chang, *Fuzzy topological spaces*, Journal of mathematical analysis and applications, **24**, (1968) 182-190.

[15] N. Çağman, F. Çitak and S. Enginoglu, *Fuzzy parameterized fuzzy soft set theory and its applications,* Turkish Journal of Fuzzy Systems **1**, No. 1 (2010) 21-35.
[16] N. Çağman, S. Karataş and S. Enginoglu, *Soft topology*, Computers and Mathematics with Applications **62**, (2011) 351-358.
[17] N. Çağman, S. Enginoglu and F. Çitak, *Fuzzy soft set theory and its applications*, Iranian Journal of Fuzzy Systems **8**, No. 8 (2011) 137-147.
[18] N. Çağman, F. Çitak and S. Enginoglu, *FP-soft set theory and its applications*, Annals of Fuzzy Mathematics and Informatics **2**, No. 2 (2011) 219-226.
[19] F. Feng, M. Akram, B. Davvaz and V. L. Fotea, *Attribute analysis of information systems based on elementary soft implications*, Knowledge-Based Systems **70**, (2014) 281-292.
[20] F. Feng, C. Li, B. Davvaz and M. I. Ali, *Soft sets combined with fuzzy sets and rough sets: a tentative approach*, Soft Computing **14**, No. 9 (2010) 899-911.
[21] M. A. Khan and Sumitra, *Common Fixed Point Theorems for Converse Commuting and OWC Maps in Fuzzy Metric Spaces*, Punjab Univ. J. Math. Vol. **44**, (2012) 57-63.
[22] A. Kharal and B. Ahmad, *Mappings on soft classes*, New Mathematics and Natural Computation **7**, No. 3 (2011) 471-482.
[23] D. Molodtsov, *Soft set theory-first results*, Computers and Mathematics with Applications **37**, (1999) 19-31.
[24] P. K. Maji, R. Biswas and A. R. Roy, *Intuitionistic fuzzy soft sets*, Journal of fuzzy Mathematics **9**, No. 3 (2001) 677-691.
[25] P. K. Maji, R. Biswas and A. R. Roy, *Fuzzy Soft sets*, Journal of fuzzy Mathematics **9**, No. 3 (2001) 589-602.
[26] P. K. Maji, R. Biswas and A. R. Roy, *An application of soft sets in decision making problem*, Computer and Mathematics with Applications **44**, (2002) 1077-1083.
[27] P. K. Maji, R. Biswas and A. R. Roy, *Soft set theory*, Computers and Mathematics with Applications **45**, (2003) 555-562.
[28] T. Mahmood, F. Mehmood and Q. Khan, *Some Generalized Aggregation Operators for Cubic Hesitant Fuzzy Sets and Their Applications to Multi Criteria Decision Making*, Punjab Univ. J. Math. Vol. **49**, No. 1 (2017) 31-49.
[29] N. Palaniappan, *Fuzzy topology*, Narosa Publishing House India, second edition (2005).
[30] H. A. Othman, *ON Fuzzy Infra-Semiopen Sets*, Punjab Univ. J. Math. Vol. **48**, No. 2 (2016) 1-10.
[31] M. Riaz, K. Naeem and M. O. Ahmad, *Novel Concepts of Soft Sets with Applications*, Annals of Fuzzy Mathematics and Informatics **13**, No. 2 (2017) 239251.
[32] M. Riaz and K. Naeem, *Measurable Soft Mappings*, Punjab Univ. J. Math. Vol. **48**, No. 2 (2016) 19-34.
[33] M. Riaz and Z. Fatima, *Certain properties of soft metric spaces*, Journal of Fuzzy Mathematics (in press) (2017).
[34] M. Riaz and M. R. Hashmi, *Fuzzy Parameterized Fuzzy Soft Topology with Applications*, Annals of Fuzzy Mathematics and Informatics **13**, No. 5 (2017) 593-613.
[35] M. Riaz and M. R. Hashmi, *Certain Applications of Fuzzy Parameterized Fuzzy Soft Sets in Decision-Making Problems*, International Journal of Algebra and Statistics **5**, No. 2 (2016) 135-146.
[36] K. P. R. Rao and K. V. Siva Parvathi, General Common Fixed Point Theorems in Fuzzy Metric Spaces, Punjab Univ. J. Math. Vol. **44**, (2012) 51-55.
[37] M. Shabir and M. Naz, *On soft topological spaces*, Computers and Mathematics with Applications **61**, (2011) 1786-1799.
[38] A. Sezgin, A. O. Atagn, *On operations of soft sets*, Computers and Mathematics with Applications **61**, No. 5 (2011) 1457-1467.
[39] A. Sezgin, A. O. Atagn and N. Çağman, *Soft intersection near-rings with applications*, Neural Computing and Applications **21**, No. 1 (2012) 221-229.
[40] S. Roy and T. K. Samanta, A note on a soft topological space, Punjab Univ. J. Math. Vol. **46**, No. 1 (2014) 19-24.
[41] I. Zorlutuna and S. Atmaca, *Fuzzy parametrized fuzzy soft topology*, New Trends in Mathematical Sciences **4**, No. 1 (2016) 142-152.
[42] L. A. Zadeh, *Fuzzy sets*, Information and Control **8**, (1965) 338-353.
[43] H. J. Zimmermann, *Fuzzy set theory and its applications*, Kluwer Academic Publishers, (1993).