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Three Dimensional Second Grade Fluid Flow Between Two Parallel Horizontal Plates with Periodic Suction/Injection in Slip Flow Regime

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Abstract. Three dimensional incompressible second grade fluid flow between two parallel horizontal permeable plates is developed and analyzed theoretically. The flow turns into three dimensional due to the application of sinusoidal normal injection at the stationary plate lying downside and its resultant removal by suction over the higher plate in moving with uniform velocity. The equations of motion are solved through regular perturbation method. The dependence of velocity components along main flow direction through parallel porous plates on flow parameters such as slip parameter, Reynolds number, suction/injection parameter and non-Newtonian parameter are discussed graphically. It is noted that slip enhances the main flow component of velocity.

AMS (MOS) Subject Classification Codes: 76A05

Key Words: Second grade fluid; Periodic injection/suction; Slip parameter; Three dimensional flow; Perturbation method.

1. INTRODUCTION

In current years, the laminar flow control problem has achieved considerable position due to its importance in the reduction of drag and hence to develop the vehicle power by a considerable amount. To stable the boundary layer artificially, numerous methods have been offered. Boundary layer suction method is an effective approach of decreasing the drag coefficient which causes huge energy losses. Ahmad and Sarma [1] considered three dimensional free convected viscous fluid flow and heat transfer passing through a porous medium. Chaudhary et al. [2] analyzed three dimensional Couette flow of viscous fluid along transpiration cooling and noted the effects of suction/injection velocity on the flow field, skin friction and heat transfer. Chauhan and Kumar [3] examined heat transfer effects in a three dimensional Couette flow passing through a channel partly filled by a porous substance. Natural convection Couette flow with radiative mass and heat transfer effects passing through a porous medium in the slip flow regime was presented by Das et al. [4]. Gersten and Gross [5] studied the effect of transverse sinusoidal suction velocity on viscous fluid flow and heat transfer above a porous plane. Guria and Jana [6] investigated unsteady three dimensional fluctuating Couette flow with heat transfer and found that the main flow velocity component decreases with increase in physical parameter, however, the cross flow velocity component increases with increase in physical parameter. Also Guria and Jana [7] discussed Hydrodynamics effect on the three-dimensional flow past a vertical porous plate. Jain and Gupta [8] investigated free convection three dimensional Couette flow along transpiration cooling in slip flow regime under the influence of heat source.

In the recent years channel flows in slip flow regime have received the attention of many investigator because of their applications in technology and engineering. It is well known that the laminarization of boundary layer over a profile decreases the drag and therefore the motor power requirements by a very substantial amount. Slowed fluid elements with the boundary layer are detached over the slits and holes in the plane inside the body and, therefore, the alteration from laminar to blustery flow initiating increase of drag may be prevented or deferred [9]. Numerous researchers [10, 13] also considered three dimensional viscous fluid flow past a porous plate taking different physical conditions. Sharma et al. [11] investigated effect of radiation on heat transfer in three dimensional Couette flow with suction or injection. It is noted that Prandtl number has a greater effect on the temperature dissemination than the injection or suction parameter. Singh [12] had taken the problem of transpiration cooling with the application of the transverse periodic suction/injection velocity. Sumathi et al. [14] have studied three-dimensional fluctuating Couette slip flow past porous plates with the existence of magnetic field in transverse direction. Above investigations have been examined in viscous fluid. Even though the Navier-Stokes equations can handle the viscous fluids flows, but these are inadequate to describe the non-Newtonian fluids flow properties.

The application of normal periodic suction/injection velocity for the flow of a second grade fluid between parallel plates in slip flow regime have not discussed in the literature. Therefore, in the present work, slip effect on three dimensional flow of a second grade fluid between two horizontal parallel porous plates with periodic suction/injection is analyzed. A uniform suction velocity at the plane leads to two dimensional flow [5]; however, due to changing of suction velocity in normal direction on plane wall, the problem becomes three dimensional. The problem is solved by using regular perturbation method. The final results are examined for various non-dimensional parameters such as suction/injection parameter α, non-Newtonian elastic parameter K, Reynolds number Re and slip parameter γ. The paper is arranged as follows: Section 2 gives problem construction, Section 3 gives approximates solutions, Section 4 incorporates results, while Section 5 comprises conclusions.

2. PROBLEM CONSTRUCTION

The suction/injection velocity [5] is of the form

$$v^*(z^*) = V_0\left(1+\varepsilon\cos\pi\frac{z^*}{h}\right), \tag{2.1}$$

the equation for model of second grade fluid is

$$\tilde{T} = -p\tilde{I} + \mu\widetilde{A_1} + \alpha_1\widetilde{A_2} + \alpha_2\widetilde{A_1}^2, \tag{2.2}$$

in which p, μ, $\tilde{I}$ and $\alpha_i(i=1,2)$ denote the pressure, the dynamic viscosity, the identity tensor and material constants respectively. The Rivlin-Ericksen tensors $\widetilde{A_1}$ and $\widetilde{A_2}$ respectively are defined as,

$$\left.\begin{aligned} \widetilde{A_1} &= grad\vec{V} + \left(grad\vec{V}\right)^T, \\ \widetilde{A_2} &= \frac{d\widetilde{A_1}}{dt^*} + \widetilde{A_1}grad\vec{V} + \left(grad\vec{V}\right)^T\widetilde{A_1}, \end{aligned}\right\} \tag{2.3}$$

where "T" denotes the transpose. The material parameters meet the following conditions.

$$\alpha_1 \ge 0,\ \alpha_1+\alpha_2 = 0,\ \mu \ge 0. \tag{2.4}$$

The equations of continuity and momentum are defined by

$$div\vec{V} = 0, \tag{2.5}$$

$$\rho\frac{d\vec{V}}{dt^*} = div\tilde{T} \tag{2.6}$$

Thus, the following system of equations governed the given problem:

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0, \tag{2.7}$$

$$\rho\left(v^*\frac{\partial u^*}{\partial y^*} + w^*\frac{\partial u^*}{\partial z^*}\right) = \mu\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right) + \alpha_1\left(\begin{array}{c} v^*\frac{\partial^3 u^*}{\partial y^{*3}} + w^*\frac{\partial^3 u^*}{\partial z^*\partial y^{*2}} \\ +v^*\frac{\partial^3 u^*}{\partial y^*\partial z^{*2}} + w^*\frac{\partial^3 u^*}{\partial z^{*3}} \end{array}\right), \tag{2.8}$$

$$\begin{aligned} &\rho\left(v^*\frac{\partial v^*}{\partial y^*} + w^*\frac{\partial v^*}{\partial z^*}\right) \\ &= -\frac{\partial p^*}{\partial y^*} + \mu\left(\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right) + \alpha_1\left(\begin{array}{c} v^*\frac{\partial^3 v^*}{\partial y^{*3}} + w^*\frac{\partial^3 v^*}{\partial z^*\partial y^{*2}} + \frac{\partial^3 v^*}{\partial y^*\partial z^{*2}} + w^*\frac{\partial^3 v^*}{\partial z^{*3}} + \\ 5\frac{\partial v^*}{\partial y^*}\frac{\partial^2 v^*}{\partial y^{*2}} + 2\frac{\partial u^*}{\partial y^*}\frac{\partial^2 u^*}{\partial y^{*2}} + 2\frac{\partial w^*}{\partial y^*}\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial v^*}{\partial z^*}\frac{\partial^2 w^*}{\partial y^{*2}} \\ +\frac{\partial u^*}{\partial z^*}\frac{\partial^2 u^*}{\partial z^*\partial y^*} + \frac{\partial u^*}{\partial y^*}\frac{\partial^2 u^*}{\partial z^{*2}} + \frac{\partial v^*}{\partial y^*}\frac{\partial^2 v^*}{\partial z^{*2}} + \frac{\partial v^*}{\partial z^*}\frac{\partial^2 v^*}{\partial z^*\partial y^*} \end{array}\right), \end{aligned} \tag{2.9}$$

$$\begin{aligned} &\rho\left(v^*\frac{\partial w^*}{\partial y^*} + w^*\frac{\partial w^*}{\partial z^*}\right) \\ &= -\frac{\partial p^*}{\partial z^*} + \mu\left(\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right) + \alpha_1\left(\begin{array}{c} w^*\frac{\partial^3 v^*}{\partial y^{*3}} + v^*\frac{\partial^3 w^*}{\partial z^*\partial y^{*2}} + v^*\frac{\partial^3 w^*}{\partial z^*\partial y^{*2}} + w^*\frac{\partial^3 w^*}{\partial z^{*3}} \\ +\frac{\partial w^*}{\partial y^*}\frac{\partial^2 w^*}{\partial y^*\partial z^*} + \frac{\partial u^*}{\partial y^*}\frac{\partial^2 u^*}{\partial y^*\partial z^*} + 5\frac{\partial w^*}{\partial z^*}\frac{\partial^2 w^*}{\partial z^{*2}} + \frac{\partial w^*}{\partial y^*}\frac{\partial^2 v^*}{\partial z^{*2}} \\ +2\frac{\partial u^*}{\partial z^*}\frac{\partial^2 u^*}{\partial z^{*2}} + 2\frac{\partial v^*}{\partial z^*}\frac{\partial^2 v^*}{\partial z^{*2}} + \frac{\partial u^*}{\partial z^*}\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial w^*}{\partial z^*}\frac{\partial^2 w^*}{\partial y^{*2}} \end{array}\right), \end{aligned} \tag{2.10}$$

subject to the conditions

$$\left.\begin{aligned} u^*(0,\ z^*) = \gamma^*\frac{\partial u^*}{\partial y^*},\ v^*(0,\ z^*) = V_0\left(1+\varepsilon\cos\pi\frac{z^*}{h}\right),\ w^*(0,\ z^*) = 0, \\ u^*(h,\ z^*) = U,\ v^*(h,\ z^*) = V_0\left(1+\varepsilon\cos\pi\frac{z^*}{h}\right),\ w^*(h,\ z^*) = 0. \end{aligned}\right\} \tag{2.11}$$

Introducing the following dimensionless parameters:

$$y=\frac{y^*}{h},\ z=\frac{z^*}{h},\ \gamma=\frac{\gamma^*}{h}\ u=\frac{u^*}{U},\ v=\frac{v^*}{U},\ w=\frac{w^*}{U},\ \alpha=\frac{V_o}{U},\ \mathrm{Re}=\frac{hU}{\upsilon},\ K=\frac{\alpha_1}{\rho h^2},\ p=\frac{p^*}{\rho U^2}, \tag{2.12}$$

where α, γ, Re and K denote suction/injection parameter, slip parameter, Reynolds number and elastic parameter respectively. Then the Eqs. $(2.7)-(2.11)$ become

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{2.13}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+K\left(v\frac{\partial^3 u}{\partial y^3}+w\frac{\partial^3 u}{\partial z\partial y^2}+v\frac{\partial^3 u}{\partial y\partial z^2}+w\frac{\partial^3 u}{\partial z^3}\right), \tag{2.14}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}$$

$$=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+K\left(\begin{array}{c} v\frac{\partial^3 v}{\partial y^3}+w\frac{\partial^3 v}{\partial z\partial y^2}+v\frac{\partial^3 v}{\partial y\partial z^2}+w\frac{\partial^3 v}{\partial z^3}+\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial z\partial y}+\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z\partial y}+ \\ 5\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2}+\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial y^2}+2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}+2\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}+\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial z^2}\end{array}\right), \tag{2.15}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}$$

$$=-\frac{\partial p}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+K\left(\begin{array}{c} w\frac{\partial^3 v}{\partial y^3}+v\frac{\partial^3 w}{\partial y^3}+v\frac{\partial^3 w}{\partial y\partial z^2}+w\frac{\partial^3 w}{\partial z^3}+\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y\partial z}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y\partial z}+ \\ 5\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial z^2}+\frac{\partial w}{\partial y}\frac{\partial^2 v}{\partial z^2}+2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}+2\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial z^2}+\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial y^2}+\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial y^2}\end{array}\right), \tag{2.16}$$

subject to non-dimensional boundary conditions

$$\left.\begin{array}{l} u(0,\ z)=\gamma\frac{\partial u}{\partial y},\ v(0,\ z)=\alpha(1+\varepsilon\cos\pi z),\ w(0,\ z)=0, \\ u(1,\ z)=1,\ v(1,\ z)=\alpha(1+\varepsilon\cos\pi z),\ w(1,\ z)=0. \end{array}\right\} \tag{2.17}$$

The velocities in the $x-$, $y-$ and $z-$ directions are represented by u, v and w respectively.

3. SOLUTION

In this part the regular perturbation solution for the velocity field is obtained.

3.1 TRANSVERSE SOLUTION

Since $0\le\varepsilon<1$, hence we consider the solution of the type

$$g(y,\ z)=g_0(y)+\varepsilon g_1(y,\ z)+\varepsilon^2 g_2(y,\ z)+\cdots, \tag{3.18}$$

here g represents one of u, v, w and p. The cross flow solutions $v_1(y,\ z)$, $w_1(y,\ z)$ and $p_1(y,\ z)$ are independent of u, the component of main flow velocity. The equations of motion will be of the form

$$\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}=0, \tag{3.19}$$

$$\alpha\frac{\partial v_1}{\partial y}=-\frac{\partial p_1}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_1}{\partial y^2}+\frac{\partial^2 v_1}{\partial z^2}\right)+K\alpha\left(\frac{\partial^3 v_1}{\partial y^3}+\frac{\partial^3 v_1}{\partial y\partial z^2}\right), \tag{3.20}$$

$$\alpha\frac{\partial w_1}{\partial y}=-\frac{\partial p_1}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w_1}{\partial y^2}+\frac{\partial^2 w_1}{\partial z^2}\right)+K\alpha\left(\frac{\partial^3 w_1}{\partial y^3}+\frac{\partial^3 w_1}{\partial y\partial z^2}\right). \tag{3.21}$$

subjected to the boundary conditions

$$v_1(0,\ z)=\alpha\cos\pi z,\ w_1(0,\ z)=0,\ v_1(1,\ z)=\alpha\cos\pi z,\ w_1(1,\ z)=0. \tag{3.22}$$

The suction/injection velocity comprises of basic constant distribution v_o along a super imposed weak periodic distribution $\varepsilon v_o\cos\pi z$, hence the normal components of velocity $v_1(y,z)$, $w_1(y,z)$ and pressure component $p_1(y,z)$ are also separated into small and main periodic components. Therefore we consider that

$$v_1(y,\ z)=v_{11}(y)\cos\pi z, \tag{3.23}$$

$$w_1(y,\ z)=-\frac{1}{\pi}v_{11}'\sin\pi z, \tag{3.24}$$

$$p_1(y,\ z)=p_{11}(y)\cos\pi z. \tag{3.25}$$

Where "$'$" represents the derivative with respect to "y". It is noted that the components of velocity $(3.23)-(3.24)$ satisfy the equation of continuity (3.19). Substituting Eqs. $(3.23)-(3.25)$ into Eqs. (3.20) and (3.21) to get

$$K\alpha\left(v_{11}'''-\pi^2 v_{11}'\right)+\frac{1}{\mathrm{Re}}\left(v_{11}''-\pi^2 v_{11}\right)-\alpha v_{11}'=p_{11}', \tag{3.26}$$

$$K\alpha\left(v_{11}^{iv}-\pi^2 v_{11}''\right)+\frac{1}{\mathrm{Re}}\left(v_{11}'''-\pi^2 v_{11}'\right)-\alpha v_{11}''=\pi^2 p_{11}. \tag{3.27}$$

Eliminating the pressure p_{11} from Eqs. (3.26) and (3.27) to get

$$KR\left(v_{11}^{v}-2\pi^2 v_{11}'''+\pi^4 v_{11}'\right)+v_{11}^{iv}+\pi^4 v_{11}-2\pi^2 v_{11}''-R\left(v_{11}'''-\pi^2 v_{11}'\right)=0, \tag{3.28}$$

where $\alpha\,\mathrm{Re}=R$. Assuming $K\ll 1$, and taking

$$v_{11}=v_{110}+Kv_{111}+O\left(K^2\right), \tag{3.29}$$

then solution of Eq. (3.28) becomes

$$v_{11}=S_3e^{-\pi y}+S_4e^{\pi y}+S_5e^{S_1y}+S_6e^{S_2y}+K\left(\begin{array}{c}S_7e^{-\pi y}+S_8e^{\pi y}+S_9e^{S_1y}+\\ S_{10}e^{S_2y}+y\left(S_{11}e^{S_1y}+S_{12}e^{S_2y}\right)\end{array}\right), \tag{3.30}$$

$$v_1(y,z)=\left(\begin{array}{c}S_3e^{-\pi y}+S_4e^{\pi y}+S_5e^{S_1y}+S_6e^{S_2y}+\\ K\left(\begin{array}{c}S_7e^{-\pi y}+S_8e^{\pi y}+S_9e^{S_1y}+\\ S_{10}e^{S_2y}+y\left(S_{11}e^{S_1y}+S_{12}e^{S_2y}\right)\end{array}\right)\end{array}\right)\cos\pi z, \tag{3.31}$$

$$w_1(y,z)=-\frac{1}{\pi}\left(\begin{array}{c}-S_3\pi e^{-\pi y}+S_4\pi e^{\pi y}+S_5S_1e^{S_1y}+S_6S_2e^{S_2y}\\ +K\left(\begin{array}{c}-S_7\pi e^{-\pi y}+S_8\pi e^{\pi y}+\\ S_9S_1e^{S_1y}+S_{10}S_2e^{S_2y}\\ +y\left(S_{11}S_1e^{S_1y}+S_{12}S_2e^{S_2y}\right)\\ +\left(S_{11}e^{S_1y}+S_{12}e^{S_2y}\right)\end{array}\right)\end{array}\right)\sin\pi z, \tag{3.32}$$

the constants $S_i\left(i=1,2,3,\cdots,12\right)$ are defined in Appendix A.

3.2 SOLUTION OF MAIN FLOW

In the case when $\varepsilon=0$, the problem become two dimensional flow, and therefore

$$KR\frac{d^3u_0}{dy^3}+\frac{d^2u_0}{dy^2}-R\frac{du_0}{dy}=0, \tag{3.33}$$

subjected to the boundary conditions

$$u_0(0)=\gamma\frac{\partial u_0(0)}{\partial y},\ u_0(1)=1. \tag{3.34}$$

Since $K \ll 1$, so assuming

$$u_0(y)=u_{00}(y)+Ku_{01}(y)+O(K^2). \tag{3.35}$$

The solution of the problem $(3.33)-(3.34)$ is

$$u_0(y)=1+\frac{e^{Ry}-e^R}{e^R+\gamma R-1}+K\left(S_{13}+S_{14}e^{Ry}+yS_{15}e^{Ry}\right), \tag{3.36}$$

where the constants S_{13}, S_{14} and S_{15} are defined in Appendix A. When $\varepsilon \neq 0$, the equations of motion $(2.14)-(2.16)$ governing the flow and boundary conditions (2.17) are perturbed taking

$$u(y,z)=u_0(y)+\varepsilon u_1(y,z)+O(\varepsilon^2), \tag{3.37}$$

$$v(y,z)=v_0(y)+\varepsilon v_1(y,z)+O(\varepsilon^2). \tag{3.38}$$

The first order equation subjected to the boundary conditions are

$$\alpha\frac{\partial u_1}{\partial y}+v_1\frac{\partial u_0}{\partial y}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial^2 u_1}{\partial z^2}\right)+K\left(\alpha\frac{\partial^3 u_1}{\partial y^3}+\alpha\frac{\partial^3 u_1}{\partial y\partial z^2}+v_1\frac{\partial^3 u_0}{\partial y^3}\right), \tag{3.39}$$

$$u_1(0,z)=0=u_1(1,z). \tag{3.40}$$

The solution of Eq. (3.39) can be expressed as $u_1(y,z)=u_{11}(y)\cos\pi z$. Then

$$\frac{\partial^2 u_{11}}{\partial y^2}-R\frac{\partial u_{11}}{\partial y}-\pi^2 u_{11}=\mathrm{Re}\,v_{11}\left(\frac{\partial u_0}{\partial y}-K\frac{\partial^3 u_0}{\partial y^3}\right)+KR\left(\pi^2\frac{\partial u_{11}}{\partial y}-\frac{\partial^3 u_{11}}{\partial y^3}\right), \tag{3.41}$$

The corresponding boundary conditions (3.40) become

$$u_{11}(0)=0=u_{11}(1). \tag{3.42}$$

Since the Eq. (3.41) is third-order having only two boundary conditions. Therefore, we define the solution of Eq. (3.41) as follows:

$$u_{11}(y)=u_{110}(y)+Ku_{111}(y)+O(K^2). \tag{3.43}$$

Then the solution of the problem will be

$$u(y,z)=1+\frac{e^{Ry}-e^{R}}{e^{R}+\gamma R-1}+K\left(S_{13}+S_{14}e^{Ry}+yS_{15}e^{Ry}\right)+$$

$$\varepsilon\left(\begin{array}{c} S_{16}e^{S_1 y}+S_{17}e^{S_2 y}+S_{18}e^{(R-\pi)y}+ \\ S_{19}e^{(R+\pi)y}+S_{20}e^{(R+S_1)y}+S_{21}e^{(R+S_2)y}+ \\ K\left(\begin{array}{c} S_{26}e^{S_1 y}+S_{27}e^{S_1 y}+\frac{S_{28}}{S_1-S_2}ye^{S_1 y}+\frac{S_{29}}{S_2-S_1}ye^{S_2 y}+ \\ \frac{S_{30}}{S_{22}}e^{(R-\pi)y}+\frac{S_{31}}{S_{23}}e^{(R+\pi)y}+\frac{S_{32}}{S_{24}}e^{(R+S_1)y}+\frac{S_{33}}{S_{24}}e^{(R+S_2)y} \\ +\frac{S_{34}}{S_{22}}\left(y-\frac{R-2\pi}{S_{22}}\right)e^{(R-\pi)y}+\frac{S_{35}}{S_{23}}\left(y-\frac{R+2\pi}{S_{23}}\right)e^{(R+\pi)y}+ \\ \frac{S_{36}}{S_{24}}\left(y-\frac{R+2S_1}{S_{24}}\right)e^{(R+S_1)y}+\frac{S_{37}}{S_{25}}\left(y-\frac{R+2S_2}{S_{25}}\right)e^{(R+S_2)y} \end{array}\right) \end{array}\right)\cos\pi z. \quad (3.44)$$

The constants $S_i\,(i=16,17,18,\cdots,37)$ are defined in Appendix A.

4. RESULTS AND DISCUSSION

In this study, fully developed steady laminar flow of an incompressible second grade fluid through two parallel horizontal porous plates with sinusoidal suction/injection in slip flow regime is modeled and examined analytically. The higher plate is moving with constant velocity U with the positive x-axis while other plate is kept fixed. The effects of several dimensionless parameters on main flow velocity are shown graphically (Figs. 1-4).

The effects of suction/injection parameter α and Reynolds number Re are shown in Fig. 1 and Fig. 3 respectively. It is noted that the velocity declines exponentially along growing suction/injection parameter or Reynolds number. For greater value of Reynolds number or suction/injection there is more decline. The maximum and minimum velocities arise on the plates, which are actually the velocities of the plates. The Fig. 2 gives the effect of non-Newtonian parameter K on the main flow velocity component u. This figure reveals that the main flow velocity component increases exponentially with the increase of K. Effect of slip parameter γ on velocity component u is depicted in Fig. 4. It is noted that the slip parameter enhances the velocity.

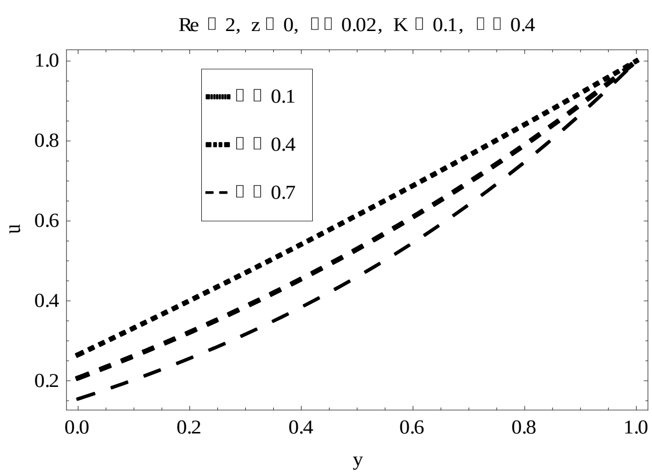

Fig. 1. Main flow velocity *u* along *y* for different values of α.

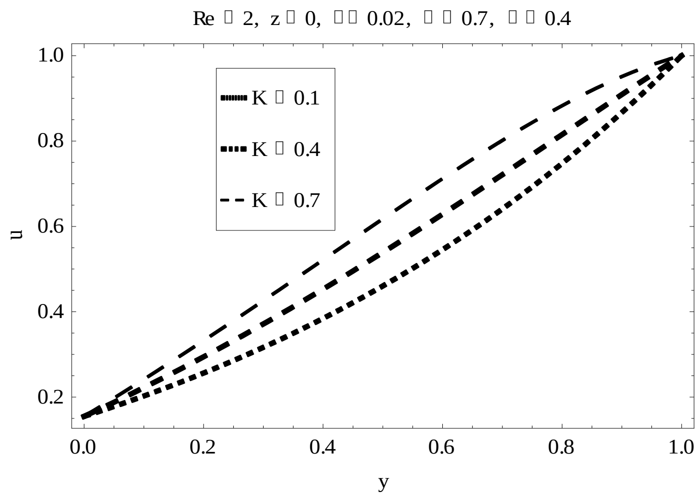

Fig. 2. Main flow velocity *u* along *y* for different values of *K*.

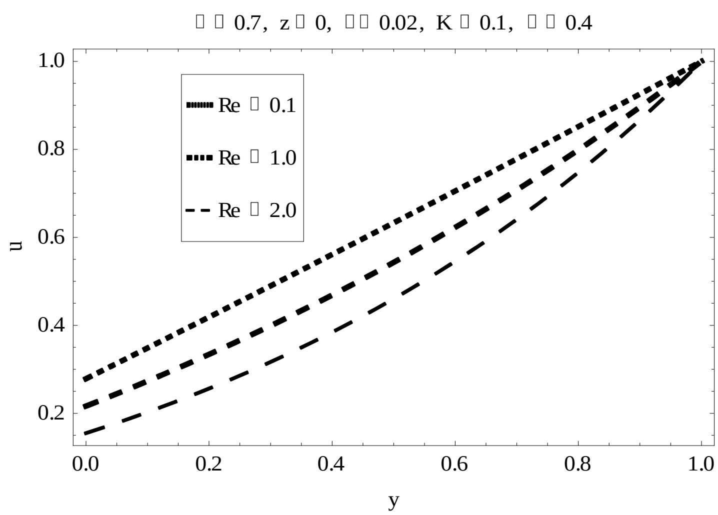

Fig. 3. Main flow velocity *u* along *y* for different values of Re.

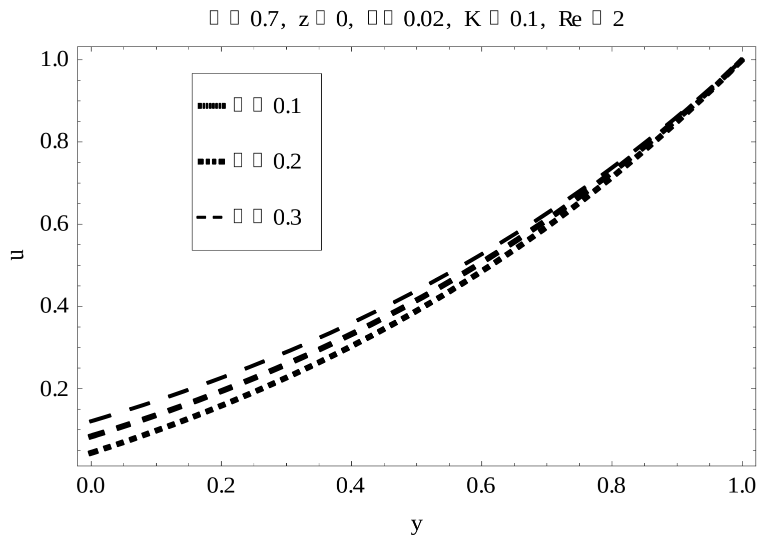

Fig. 4. Main flow velocity u along y for different values of γ.

5. FINAL REMARKS

In the light of above conversations following deductions are prepared:

1. The velocity component u decreases with increasing either suction/injection parameter or Reynolds number. However it decreases with an increase in non-Newtonian parameter.
2. Slip parameter enhances the main flow velocity.
3. Reynolds number provides a mechanism to control the main flow velocity.
4. No-slip results are recovered when $\gamma = 0$.

Appendix A

The constants elaborate in this paper are,

$$S_1 = \frac{R - \sqrt{R^2 + 4\pi^2}}{2}, \; S_2 = \frac{R + \sqrt{R^2 + 4\pi^2}}{2},$$

$$\begin{aligned} S_3 = {} & (e^{\pi}(-e^{S_1} S_1 S_2 + e^{S_2} S_1 S_2 + e^{S_1+\pi} S_1 S_2 - e^{S_2+\pi} S_1 S_2 + e^{S_1} S_1 \pi - \\ & e^{S_1+S_2} S_1 \pi - e^{\pi} S_1 \pi + e^{S_2+\pi} S_1 \pi - e^{S_2} S_2 \pi + e^{S_1+S_2} S_2 \pi + e^{\pi} S_2 \pi - \\ & e^{S_1+\pi} S_2 \pi)\alpha / (-e^{S_1} S_1 S_2 + e^{S_2} S_1 S_2 + e^{S_1+2\pi} S_1 S_2 - e^{S_2+2\pi} S_1 S_2 + \\ & e^{S_1} S_1 \pi + e^{S_2} S_1 \pi - 2e^{\pi} S_1 \pi - 2e^{S_1+S_2+\pi} S_1 \pi + e^{S_1+2\pi} S_1 \pi + e^{S_2+2\pi} S_1 \pi \\ & -e^{S_1} S_2 \pi - e^{S_2} S_2 \pi + 2e^{\pi} S_2 \pi + 2e^{S_1+S_2+\pi} S_2 \pi - e^{S_1+2\pi} S_2 \pi - \\ & e^{S_2+2\pi} S_2 \pi + e^{S_1} \pi^2 - e^{S_2} \pi^2 - e^{S_1+2\pi} \pi^2 + e^{S_2+2\pi} \pi^2), \end{aligned}$$

$$S_4 = -\left(\left(\begin{matrix} e^{S_1}S_1S_2 - e^{S_2}S_1S_2 - e^{S_1+\pi}S_1S_2 + e^{S_2+\pi}S_1S_2 \\ + S_1\pi - e^{S_2}S_1\pi - e^{S_1+\pi}S_1\pi + e^{S_1+S_2+\pi}S_1\pi - \\ S_2\pi + e^{S_1}S_2\pi + e^{S_2+\pi}S_2\pi - e^{S_1+S_2+\pi}S_2\pi \end{matrix}\right)\alpha\right) /$$

$$\left(\begin{matrix} -e^{S_1}S_1S_2 + e^{S_2}S_1S_2 + e^{S_1+2\pi}S_1S_2 - e^{S_2+2\pi}S_1S_2 + e^{S_1}S_1\pi \\ + e^{S_2}S_1\pi - 2e^{\pi}S_1\pi - 2e^{S_1+S_2+\pi}S_1\pi + e^{S_1+2\pi}S_1\pi + e^{S_2+2\pi}S_1\pi \\ - e^{S_1}S_2\pi - e^{S_2}S_2\pi + 2e^{\pi}S_2\pi + 2e^{S_1+S_2+\pi}S_2\pi \\ - e^{S_1+2\pi}S_2\pi - e^{S_2+2\pi}S_2\pi + e^{S_1}\pi^2 - e^{S_2}\pi^2 - e^{S_1+2\pi}\pi^2 + e^{S_2+2\pi}\pi^2 \end{matrix}\right),$$

$$S_5 = -\left(\left(\begin{matrix} -S_2\pi - e^{S_2}S_2\pi + 2e^{\pi}S_2\pi - e^{2\pi}S_2\pi + \\ 2e^{S_2+\pi}S_2\pi - e^{S_2+2\pi}S_2\pi + \pi^2 - e^{S_2}\pi^2 - e^{2\pi}\pi^2 + e^{S_2+2\pi}\pi^2 \end{matrix}\right)\alpha\right) /$$

$$\left(\begin{matrix} e^{S_1}S_1S_2 - e^{S_2}S_1S_2 - e^{S_1+2\pi}S_1S_2 + e^{S_2+2\pi}S_1S_2 - e^{S_1}S_1\pi - \\ e^{S_2}S_1\pi + 2e^{\pi}S_1\pi + 2e^{S_1+S_2+\pi}S_1\pi - e^{S_1+2\pi}S_1\pi - e^{S_2+2\pi}S_1\pi + \\ e^{S_1}S_2\pi + e^{S_2}S_2\pi - 2e^{\pi}S_2\pi - 2e^{S_1+S_2+\pi}S_2\pi + e^{S_1+2\pi}S_2\pi \\ + e^{S_2+2\pi}S_2\pi - e^{S_1}\pi^2 + e^{S_2}\pi^2 + e^{S_1+2\pi}\pi^2 - e^{S_2+2\pi}\pi^2 \end{matrix}\right)$$

$$S_6 = -\left(\pi\left(\begin{matrix} -S_1 - e^{S_1}S_1 + 2e^{\pi}S_1 - e^{2\pi}S_1 + 2e^{S_1+\pi}S_1 - \\ e^{S_1+2\pi}S_1 + \pi - e^{S_1}\pi - e^{2\pi}\pi + e^{S_1+2\pi}\pi \end{matrix}\right)\alpha\right) /$$

$$\left(\begin{matrix} -e^{S_1}S_1S_2 + e^{S_2}S_1S_2 + e^{S_1+2\pi}S_1S_2 - e^{S_2+2\pi}S_1S_2 + e^{S_1}S_1\pi + \\ e^{S_2}S_1\pi - 2e^{\pi}S_1\pi - 2e^{S_1+S_2+\pi}S_1\pi + e^{S_1+2\pi}S_1\pi + e^{S_2+2\pi}S_1\pi \\ - e^{S_1}S_2\pi - e^{S_2}S_2\pi + 2e^{\pi}S_2\pi + 2e^{S_1+S_2+\pi}S_2\pi - e^{S_1+2\pi}S_2\pi - \\ e^{S_2+2\pi}S_2\pi + e^{S_1}\pi^2 - e^{S_2}\pi^2 - e^{S_1+2\pi}\pi^2 + e^{S_2+2\pi}\pi^2 \end{matrix}\right)$$

$$\begin{aligned} S_7 = &-(e^{\pi}(e^{S_1}S_{11} - e^{\pi}S_{11} - e^{S_1}S_1S_{11} + e^{S_1}S_{12} - e^{\pi}S_{12} - e^{S_2}S_1S_{12} + \\ &e^{S_1}S_{11}\pi + e^{S_2}S_{12}\pi)) / (-S_1 + e^{2\pi}S_1 + \pi + e^{2\pi}\pi - 2e^{S_1+\pi}\pi) - \\ &(e^{\pi}(e^{S_2}S_1 - e^{\pi}S_1 - e^{S_1}S_2 + e^{\pi}S_2 + e^{S_1}\pi - e^{S_2}\pi)((2\left(e^{S_1} - e^{-\pi}\right)\pi \\ &-\left(-e^{-\pi} + e^{\pi}\right)\left(S_1 + \pi\right))(2\left(-e^{S_1}\left(1 + S_1\right)S_{11} - e^{S_2}\left(1 + S_2\right)S_{12}\right)\pi - \\ &\left(-S_{11} - S_{12}\right)\left(e^{-\pi}\pi + e^{\pi}\pi\right)) - \\ &\left(-\left(-e^{-\pi} + e^{\pi}\right)\left(-S_{11} - S_{12}\right) + 2\left(-e^{S_1}S_{11} - e^{S_2}S_{12}\right)\pi\right) \\ &\left(2\pi\left(e^{S_1}S_1 + e^{-\pi}\pi\right) - \left(S_1 + \pi\right)\left(e^{-\pi}\pi + e^{\pi}\pi\right)\right))) / ((S_1 - e^{2\pi}S_1 - \pi - \\ &e^{2\pi}\pi + 2e^{S_1+\pi}\pi)(-(2(e^{S_2} - e^{-\pi})\pi - \left(-e^{-\pi} + e^{\pi}\right)\left(S_2 + \pi\right)) \\ &(2\pi(e^{S_1}S_1 + e^{-\pi}\pi) - \left(S_1 + \pi\right)\left(e^{-\pi}\pi + e^{\pi}\pi\right)) + (2(e^{S_1} - e^{-\pi})\pi - \\ &\left(-e^{-\pi} + e^{\pi}\right)\left(S_1 + \pi\right))(2\pi(e^{S_2}S_2 + e^{-\pi}\pi) - \left(S_2 + \pi\right)\left(e^{-\pi}\pi + e^{\pi}\pi\right)))), \end{aligned}$$

$$S_8 = -(S_{11} - e^{S_1+\pi}S_{11} + e^{S_1+\pi}S_1S_{11} + S_{12} - e^{S_1+\pi}S_{12} + e^{S_2+\pi}S_1S_{12}$$
$$+e^{S_1+\pi}S_{11}\pi + e^{S_2+\pi}S_{12}\pi)/(-S_1 + e^{2\pi}S_1 + \pi + e^{2\pi}\pi - 2e^{S_1+\pi}\pi) -$$
$$((S_1 - e^{S_2+\pi}S_1 - S_2 + e^{S_1+\pi}S_2 + e^{S_1+\pi}\pi - e^{S_2+\pi}\pi)((2(e^{S_1} - e^{-\pi})\pi$$
$$-(-e^{-\pi} + e^{\pi})(S_1+\pi))(2(-e^{S_1}(1+S_1)S_{11} - e^{S_2}(1+S_2)S_{12})\pi -$$
$$(-S_{11} - S_{12})(e^{-\pi}\pi + e^{\pi}\pi)) -$$
$$(-(-e^{-\pi} + e^{\pi})(-S_{11} - S_{12}) + 2(-e^{S_1}S_{11} - e^{S_2}S_{12})\pi)$$
$$(2\pi(e^{S_1}S_1 + e^{-\pi}\pi) - (S_1+\pi)(e^{-\pi}\pi + e^{\pi}\pi)))/((S_1 - e^{2\pi}S_1 -$$
$$\pi - e^{2\pi}\pi + 2e^{S_1+\pi}\pi)(-(2(e^{S_2} - e^{-\pi})\pi - (-e^{-\pi} + e^{\pi})(S_2+\pi))(2\pi$$
$$(e^{S_1}S_1 + e^{-\pi}\pi) - (S_1+\pi)(e^{-\pi}\pi + e^{\pi}\pi)) + (2(e^{S_1} - e^{-\pi})\pi -$$
$$(-e^{-\pi} + e^{\pi})(S_1+\pi))(2\pi(e^{S_2}S_2 + e^{-\pi}\pi) - (S_2+\pi)(e^{-\pi}\pi + e^{\pi}\pi)))),$$

$$S_9 = -(-S_{11} + e^{2\pi}S_{11} - S_{12} + e^{2\pi}S_{12} - 2e^{S_1+\pi}S_{11}\pi - 2e^{S_2+\pi}S_{12}\pi)/$$
$$(-S_1 + e^{2\pi}S_1 + \pi + e^{2\pi}\pi - 2e^{S_1+\pi}\pi) - ((2(e^{S_2} - e^{-\pi})\pi -$$
$$(-e^{-\pi} + e^{\pi})(S_2+\pi))((2(e^{S_1} - e^{-\pi})\pi - (-e^{-\pi} + e^{\pi})(S_1+\pi))$$
$$(2(-e^{S_1}(1+S_1)S_{11} - e^{S_2}(1+S_2)S_{12})\pi - (-S_{11} - S_{12})(e^{-\pi}\pi + e^{\pi}\pi))$$
$$-(-(-e^{-\pi} + e^{\pi})(-S_{11} - S_{12}) + 2(-e^{S_1}S_{11} - e^{S_2}S_{12})\pi)$$
$$(2\pi(e^{S_1}S_1 + e^{-\pi}\pi) - (S_1+\pi)(e^{-\pi}\pi + e^{\pi}\pi)))/((2(e^{S_1} - e^{-\pi})\pi$$
$$-(-e^{-\pi} + e^{\pi})(S_1+\pi))(-(2(e^{S_2} - e^{-\pi})\pi - (-e^{-\pi} + e^{\pi})(S_2+\pi))$$
$$(2\pi(e^{S_1}S_1 + e^{-\pi}\pi) - (S_1+\pi)(e^{-\pi}\pi + e^{\pi}\pi)) + (2(e^{S_1} - e^{-\pi})\pi -$$
$$(-e^{-\pi} + e^{\pi})(S_1+\pi))(2\pi(e^{S_2}S_2 + e^{-\pi}\pi) - (S_2+\pi)(e^{-\pi}\pi + e^{\pi}\pi)))),$$

$$S_{10} = ((2(e^{S_1} - e^{-\pi})\pi - (-e^{-\pi} + e^{\pi})(S_1+\pi))$$
$$(2(-e^{S_1}(1+S_1)S_{11} - e^{S_2}(1+S_2)S_{12})\pi - (-S_{11} - S_{12})(e^{-\pi}\pi + e^{\pi}\pi))$$
$$-(-(-e^{-\pi} + e^{\pi})(-S_{11} - S_{12}) + 2(-e^{S_1}S_{11} - e^{S_2}S_{12})\pi)$$
$$(2\pi(e^{S_1}S_1 + e^{-\pi}\pi) - (S_1+\pi)(e^{-\pi}\pi + e^{\pi}\pi)))/(-(2(e^{S_2} - e^{-\pi})\pi -$$
$$(-e^{-\pi} + e^{\pi})(S_2+\pi))(2\pi(e^{S_1}S_1 + e^{-\pi}\pi) - (S_1+\pi)(e^{-\pi}\pi + e^{\pi}\pi))$$
$$+(2(e^{S_1} - e^{-\pi})\pi - (-e^{-\pi} + e^{\pi})(S_1+\pi))$$
$$(2\pi(e^{S_2}S_2 + e^{-\pi}\pi) - (S_2+\pi)(e^{-\pi}\pi + e^{\pi}\pi))),$$

$$S_{11} = \frac{-RS_1S_5(S_1 - \pi^2)}{(S_1 - S_2)},\ S_{12} = \frac{-RS_2S_6(S_2 - \pi^2)}{(S_2 - S_1)},$$

$$S_{13} = \frac{S_{15}e^R}{e^R - 1},\ S_{14} = \frac{-S_{15}e^R}{e^R - 1},\ S_{15} = \frac{-R^3}{e^R + \gamma R - 1},$$

$$S_{16} = \frac{1}{e^{S_1} - e^{S_2}}\begin{pmatrix} S_{18}(e^{S_2} - e^{R-\pi}) + S_{19}(e^{S_2} - e^{R+\pi}) + \\ S_{20}(e^{S_2} - e^{R+S_1}) + S_{21}(e^{S_2} - e^{R+S_2}) \end{pmatrix},$$

$$S_{17} = \frac{1}{e^{S_2} - e^{S_1}}\begin{pmatrix} S_{18}(e^{S_1} - e^{R-\pi}) + S_{19}(e^{S_1} - e^{R+\pi}) + \\ S_{20}(e^{S_1} - e^{R+S_1}) + S_{21}(e^{S_1} - e^{R+S_2}) \end{pmatrix},$$

$$S_{18}=\frac{\mathrm{Re}S_3R}{\left(e^R+\gamma R-1\right)S_{22}},\; S_{19}=\frac{\mathrm{Re}S_4R}{\left(e^R+\gamma R-1\right)S_{23}},$$

$$S_{20}=\frac{\mathrm{Re}S_5R}{\left(e^R+\gamma R-1\right)S_{24}},\; S_{21}=\frac{\mathrm{Re}S_6R}{\left(e^R+\gamma R-1\right)S_{25}},$$

$$S_{22}=\left(R-\pi-S_1\right)\left(R-\pi-S_2\right),\; S_{23}=\left(R+\pi-S_1\right)\left(R+\pi-S_2\right),$$

$$S_{24}=R\left(R+S_1-S_2\right),\; S_{25}=\left(R+S_2-S_1\right)R,$$

$$S_{26}=\frac{1}{e^{S_1}-e^{S_2}}\begin{pmatrix} \frac{S_{28}}{S_2-S_1}e^{S_1}+\frac{S_{29}}{S_1-S_2}e^{S_2}+\frac{S_{30}}{S_{22}}\left(e^{S_2}-e^{R-\pi}\right)+ \\ \frac{S_{31}}{S_{23}}\left(e^{S_2}-e^{R+\pi}\right)+\frac{S_{32}}{S_{24}}\left(e^{S_2}-e^{R+S_1}\right)+\frac{S_{33}}{S_{25}}\left(e^{S_2}-e^{R+S_2}\right) \\ -\frac{S_{34}}{S_{22}}\left(\frac{2(R-\pi)-R}{S_{22}}e^{S_2}+\left(1-\frac{2(R-\pi)-R}{S_{22}}\right)e^{R-\pi}\right) \\ -\frac{S_{35}}{S_{23}}\left(\frac{2(R+\pi)-R}{S_{23}}e^{S_2}+\left(1-\frac{2(R+\pi)-R}{S_{23}}\right)e^{R+\pi}\right) \\ -\frac{S_{36}}{S_{24}}\left(\frac{2(R+S_1)-R}{S_{24}}e^{S_2}+\left(1-\frac{2(R+S_1)-R}{S_{24}}\right)e^{R+S_1}\right) \\ -\frac{S_{37}}{S_{25}}\left(\frac{2(R+S_2)-R}{S_{25}}e^{S_2}+\left(1-\frac{2(R+S_2)-R}{S_{25}}\right)e^{R+S_2}\right) \end{pmatrix},$$

$$S_{27}=\frac{1}{e^{S_2}-e^{S_1}}\begin{pmatrix} \frac{S_{28}}{S_2-S_1}e^{S_1}+\frac{S_{29}}{S_1-S_2}e^{S_2}+\frac{S_{30}}{S_{22}}\left(e^{S_1}-e^{R-\pi}\right)+ \\ \frac{S_{31}}{S_{23}}\left(e^{S_1}-e^{R+\pi}\right)+\frac{S_{32}}{S_{24}}\left(e^{S_1}-e^{R+S_1}\right)+\frac{S_{33}}{S_{25}}\left(e^{S_1}-e^{R+S_2}\right) \\ -\frac{S_{34}}{S_{22}}\left(\frac{2(R-\pi)-R}{S_{22}}e^{S_1}+\left(1-\frac{2(R-\pi)-R}{S_{22}}\right)e^{R-\pi}\right) \\ -\frac{S_{35}}{S_{23}}\left(\frac{2(R+\pi)-R}{S_{23}}e^{S_1}+\left(1-\frac{2(R+\pi)-R}{S_{23}}\right)e^{R+\pi}\right) \\ -\frac{S_{36}}{S_{24}}\left(\frac{2(R+S_1)-R}{S_{24}}e^{S_1}+\left(1-\frac{2(R+S_1)-R}{S_{24}}\right)e^{R+S_1}\right) \\ -\frac{S_{37}}{S_{25}}\left(\frac{2(R+S_2)-R}{S_{25}}e^{S_1}+\left(1-\frac{2(R+S_2)-R}{S_{25}}\right)e^{R+S_2}\right) \end{pmatrix},$$

$$S_{28}=RS_{16}S_1(\pi^2-S_1),\; S_{29}=RS_{17}S_2(\pi^2-S_2),$$

$$S_{30}=\mathrm{Re}S_3S_{14}R+RS_{18}\left(R-\pi\right)\left(\pi^2-\left(R-\pi\right)^2\right)+\mathrm{Re}\begin{pmatrix} S_3S_{15}+ \\ \frac{R}{e^R+\gamma R-1}\left(S_7-S_3R\right) \end{pmatrix},$$

$$S_{31}=\mathrm{Re}S_4S_{14}R+RS_{19}\left(R+\pi\right)\left(\pi^2-\left(R+\pi\right)^2\right)+\mathrm{Re}\begin{pmatrix} S_4S_{15}+ \\ \frac{R}{e^R+\gamma R-1}\left(S_8-S_4R\right) \end{pmatrix},$$

$$S_{32}=\mathrm{Re}S_5S_{14}R+RS_{20}\left(R+S_1\right)\left(\pi^2-\left(R+S_1\right)^2\right)+\mathrm{Re}\begin{pmatrix} S_5S_{15}+ \\ \frac{R}{e^R+\gamma R-1}\left(S_9-S_5R\right) \end{pmatrix},$$

$$S_{33}=\mathrm{Re}S_6S_{14}R+RS_{21}\left(R+S_2\right)\left(\pi^2-\left(R+S_2\right)^2\right)+\mathrm{Re}\begin{pmatrix} S_6S_{15}+ \\ \frac{R}{e^R+\gamma R-1}\left(S_{10}-S_6R\right) \end{pmatrix},$$

$$S_{34}=\mathrm{Re}RS_3S_{15},\; S_{35}=\mathrm{Re}RS_4S_{15},\; S_{36}=\mathrm{Re}R\left(S_5S_{15}+\frac{S_{11}}{e^R+\gamma R-1}\right),$$

$$S_{37}=\mathrm{Re}R\left(S_6S_{15}+\frac{S_{12}}{e^R+\gamma R-1}\right),$$

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