Punjab University
Journal of Mathematics (ISSN 1016-2526)
Vol. 50(3)(2018) pp. 145-151

Modified Bracketing Method for Solving Nonlinear Problems With Second Order of Convergence

Umair Khalid Qureshi[1], Asif Ali Shaikh[2], Prem Kumar Malhi[3]

[1,2,3] Department of Basic Science and Related Studies,
Mehran University of Engineering and Technology, Pakistan.
[1] Email: khalidumair531@gmail.com (Corresponding Author)

Received: 21 November, 2017 / Accepted: 10 April, 2018 / Published online: 10 May, 2018

Abstract. This paper has developed a Modified Bracketing Method for solving nonlinear problems. The Modified Bracketing Method is modification of Bisection Method and Regula-Falsi Method, and it has second order of convergence. The Modified Bracketing Method converges quicker than Bisection Method, Regula-Falsi Method, Steffensen Method and Newton Raphson Method. The comparison table-1 for different test functions demonstrates the faster convergence of proposed method. EXCEL and C++ are implemented for the results and graphical representation.

AMS (MOS) Subject Classification Codes: 35S29; 40S70; 25U09
Key Words: non-linear problems, bracketing algorithms, convergence analysis, absolute percentage error, accuracy.

1. Introduction

For estimating a single root of nonlinear functions are imperative field of research in numerical analysis, which applications arises in numerous outlets of applied and pure science have deliberated in general framework of nonlinear problems (Iwetan, 2012), (Dalquist, 2008) and (Golbabai, 2007), such as non-linear equation

$$f(x) = 0 \tag{1. 1}$$

Due to prominence of (1.1), one of the elementary technique such as bisection method is used for estimating a root of nonlinear functions.

$$m = \frac{a+b}{2} \tag{1. 2}$$

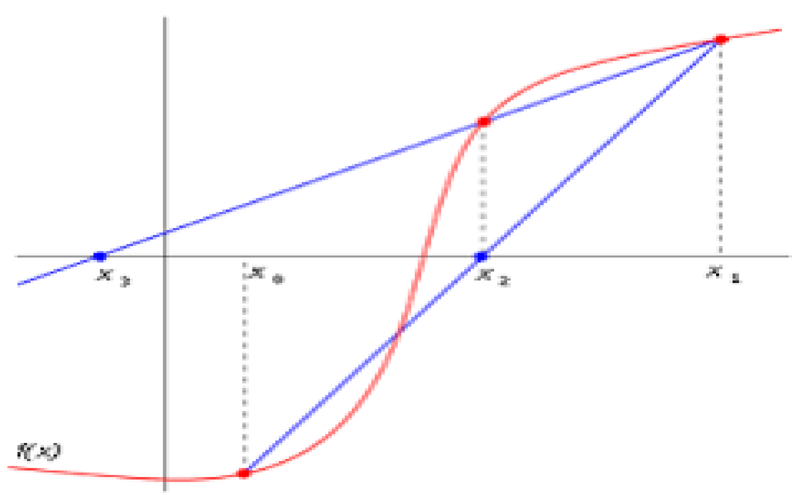
The technique (1.2) is robust and slow convergence bisection technique (Biswa, 2012). Bisection method is sure to converge for an continuous equations on interval [a,b] where f(a)f(b) less than 0. Alternative, regula-falsi method is another root finding technique for solving nonlinear problems.

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)} \tag{1. 3}$$

The technique (1.3) is fast converging regula falsil technique as the assessment of bisection method. Both techniques are a linearly convergence technique but in some cases regula-falsi method suffers due to sluggish convergence. Illinois method improves the disadvantage of a variant of regula-falsi technique in (Golbabai, 2007). Furthermore, modification of regula-falsi method was performed by Sangah and Siyal [6-7]. Additionally, Countless numerical methods had been suggested by using different techniques including quadrature formula, homotopy perturbation method and its variant forms, Taylor series, divided difference and decomposition method (Solanki, 2014), (Chun, 2005), (Frontini, 2004), (Abbasbandy, 2003) and (Babolian, 2002). Likewise, some two-pint algorithms had been designated in literatures for solving non-linear equations. In similar investigation combined the famous Bisection Method, Regula-Falsi Method and Newton Raphson method to offer a method for estimating a single root of nonlinear problems with good accuracy as well as iterated point of view by (Allame, 2012) and (Noor, 2006). Correspondingly, in this study a Modified bracketing technique have been proposed, which is mixture of classical two-point methods. The Modified bracketing method is appropriate for solving non-linear problems on the predened interval. The proposed method is fast converging to approaching the root and free from pitfall. The Modified bracketing method is simpler and easier to use.

2. Modified Bracketing Method

Let starting with initial value xo and x1, we draw a line through the points (xo, yo) and (x1, y1), as validated in the figure.



The point-slope form of this line is as follows

$$y = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0) + f(x_0) \tag{2. 4}$$

For estimating the root of this line, the value of such that through resolving the following equation for x:

$$0 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0) + f(x_0) \tag{2. 5}$$

The solution is

$$x = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0) \tag{2. 6}$$

It is somewhat called Regula-Falsi Method. By using bisection method, we are modifying (2.6), such as

$$x_1 = \frac{a + b}{2} \tag{2. 7}$$

Where (a, b) be an interval, for better approximation we convert this interval into sub-interval such as (a,(a+b)/2), if f(a)f((a+b)/2) greater than 0 exist, such as

$$x_1 = \frac{3a + b}{4} \tag{2. 8}$$

As we know that b=a+h and rather 'h' can be written as h=f(a), we get

$$b = a + f(a) \tag{2. 9}$$

Therefore, (2.9) Substitute in (2.8),

$$x_1 = a + 0.3f(a) \tag{2. 10}$$

Where 'a' is an initial guess, so we can write as a=xo

$$x_1 = x_0 + 0.3f(x_0) \tag{2. 11}$$

(2.11) substitute in (2.6), we get

$$x = x_0 - \frac{0.3f(x_0)}{f(x_0 + 0.3f(x_0)) - f(x_0)} f(x_0) \tag{2. 12}$$

In general,

$$x_{n+1} = x_n - \frac{0.3f^2(x_n)}{f(x_n + 0.3f(x_n)) - f(x_n)} \tag{2. 13}$$

Hence (2.13) is the Modified Bracketing Method.

3. Convergence Analysis

In this segment, given a key result in this paper. We will give here the Mathematical proof that the proposed method (2.13) is 2nd-order convergence.

Proof

By using Taylor series, we are expanding

$$f(x_n), f^2(x_n) \tag{3. 14}$$

and

$$f(x_n + 0.3f(x_n)) \tag{3. 15}$$

only second order term about ‘a‘ and using

$$c = \frac{f'(a)}{2f'(a)} \tag{3. 16}$$

such as

$$f(x_n) = f'(a)(e_n + ce_n^2) \tag{3. 17}$$

Or

$$f^2(x) = (e^2)_n f'^2(a)(1 + e_n c)^2 \tag{3. 18}$$

$$f^2(x) = e_n^2 f'^2(a)(1 + 2e_n c) \tag{3. 19}$$

and

$$f(x_n + 0.3f(x_n)) = f'(a)[(e_n + 0.3f(x_n)) + c(e_n + 0.3f(x_n))^2] \tag{3. 20}$$

By using(3.17)and (3.19), we get

$$f(x_n + 0.3f(x_n)) - f(x_n) = f‘(a)[(e_n + 0.3f(x_n) - e_n) + c[(e_n + 0.3f(x_n))^2 - e_n^2]] \tag{3. 21}$$

$$f(x_n + 0.3f(x_n)) - f(x_n) = 0.3f(x_n)f‘(a)[1 + (2e_n + 0.3f(x_n))c] \tag{3. 22}$$

or

$$f(x_n + 0.3f(x_n)) - f(x_n) = 0.3f'^2(a)e_n(1 + ce_n)[1 + (2e_n + 0.3f'(a)e_n(1 + ce_n))c] \tag{3. 23}$$

$$f(x_n + 0.3f(x_n)) - f(x_n) = 0.3f'^2(a)e_n[1 + 3e_n c + 0.3ce_n f'(a)] \tag{3. 24}$$

By using (3.17), (3.19) and (3.21) in (2.13), we get

$$e_{n+1} = e_n - \frac{(0.3e_n^2 f'^2(a)(1 + 2e_n c))}{(0.3f'^2(a)e_n[1 + 3e_n c + 0.3ce_n f'(a)])} \tag{3. 25}$$

$$e_{n+1} = e_n - \frac{(e_n(1 + 2e_n c))}{[1 + 3e_n c + 0.3ce_n f'(a)]} \tag{3. 26}$$

$$e_{n+1} = e_n - e_n(1 + 2e_n c)[1 + e_n(3c + 0.3cf‘(a))]^{-1} \tag{3. 27}$$

$$e_{n+1} = e_n - e_n(1 + 2e_n c)[1 - e_n(3c + 0.3cf'(a))] \tag{3. 28}$$

$$e_{n+1} = e_n - e_n[1 - e_n c - 0.3ce_n f'(a)] \tag{3. 29}$$

$$e_{n+1} = e_n - e_n + e_n[e_n c + 0.3ce_n f'(a)] \tag{3. 30}$$

Finally, we get

$$e_{n+1} = e_n^2[c + 0.3cf'(a)] \tag{3. 31}$$

Hence this is proven that the (2.13) has a Quadratically convergence.

4. NUMERICAL RESULTS

To analyze the Proposed Modified Bracketing Technique is practical on some examples including algebraic and transcendental equations and related through the famous Regula-Falsi Method,

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)} \tag{4. 32}$$

Steffensen Method,

$$x_{n+1} = x_n - \frac{f^2(x_n)}{f(x_n + f(x_n)) - f(x_n)} \tag{4. 33}$$

and Newton Raphson Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{4. 34}$$

From the above techniques, it has been observed by using C++ programing that the existing techniques keeping drawback and converge slowly in some cases but proposed method haven‘t drawback and converge fastly in these cases. By the employment of these existing techniques to solve the nonlinear functions, where not only demonstrate the method practically but also assist to check the validity of theoretic fallouts of proposed method. From the results and graphical interpretations are deceptive that the modified algorithm is working faster than other conventional existing methods. Additionally, time component and the number of iterations on modified algorithm and existing sectioning methods are justifiable from the below graphical portrayal and data representation.

TABLE-1

Functions	Methods	Iterations	X	A E %
$Cosx-x^3$ (0,1)	Regula-Falsi Method	8	0.865474	0.000001
	Steffensen method	4		0.00312357
	Newton Raphson Method	3		0.00021011
	Modified Bracketing Method	2		0.00040543
e^x-5x (0,0.5)	Regula-Falsi Method	6	0.259171	0.00000298
	Steffensen method	4		0.00000298
	Newton Raphson Method	4		0.00000298
	Modified Bracketing Method	3		0.00001788
x^3 -9x+1 (0,1)	Regula-Falsi Method	4	0.111264	0.000010431
	Steffensen method	3		0.000398085
	Newton Raphson Method	3		0.000398085
	Modified Bracketing Method	3		0.000005961
Sinx-x+1 (0,2.5)	Regula-Falsi Method	7	1.93456	0.000031012
	Steffensen method	5		0.039410001
	Newton Raphson Method	5		0.000011921
	Modified Bracketing Method	4		0.000011921
x^3 +4x-10 (1.5,3)	Regula-Falsi Method	11	1.55677	0.000012400
	Steffensen method	9		0.000011921
	Newton Raphson Method	4		0.000011921
	Modified Bracketing Method	4		0.000011921

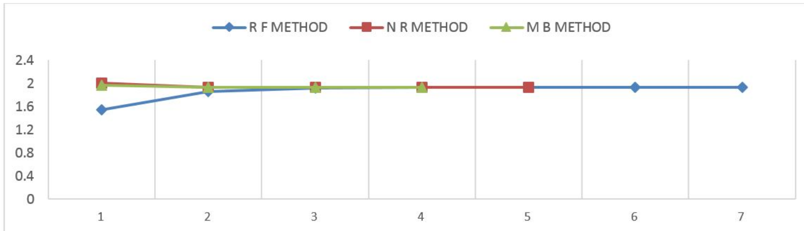


Figure-1: Comparative Illustration `$x_n$` of sinx-x+1

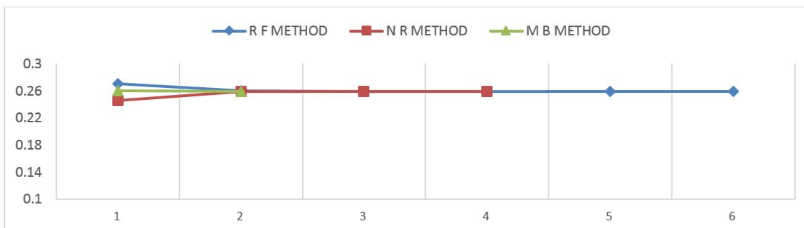


Figure-2: Comparative Illustration `$x_n$` of e^x-5x

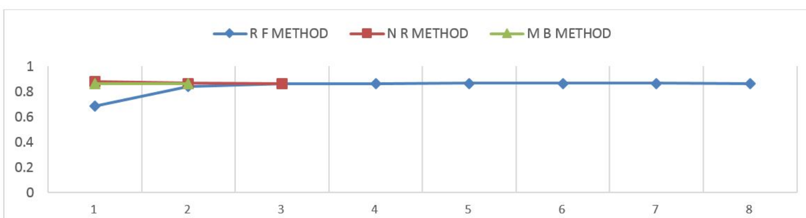


Figure-3: Comparative Illustration `$x_n$` of Cosx-x^3

5. CONCLUSION

This paper a Modified Bracketing Method has been suggested for estimating a root of nonlinear functions. The proposed technique is a second order of convergence. Throughout the study, it can be concluded that the Modified Bracketing Method is execution decent in comparison of Regula-Falsi Method, Steffensen Method and Newton Raphson Method. A Proposed Bracketing Method is fast converging to approaching the root and free from pitfall. Henceforth the Modified Bracketing Method is performing-well, more efficient and easy to employ with reliable results for solving non-linear equations.

REFERENCES

[1] S. Abbasbandy, *Improving Newton Raphson Method for Nonlinear Equations by Modified Adomian Decomposition Method*, Appl. Math. Comput. **145**, (2003) 887-893.

[2] M. Allame and N. Azad, *On Modified Newton Method for Solving a Nonlinear Algebraic Equations by Mid-Point*, World Applied Sciences Journal17 **17**, No. 12 IDOSI Publications (2012) 1546-1548.

[3] E. Babolian and J. Biazar, *Solution of Nonlinear Equations by Adomian Decomposition Method*, Appl. Math. Compute **132**, (2002) 167-172.

[4] N. D. Biswa, *Lecture Notes on Numerical Solution of root Finding Problems,* (2012).

[5] C. Chun, *Iterative Methods Improving Newtons Method by the Decomposition Method,* Comput. Math. Appl. **50**, (2005) 1559-1568.

[6] G. Dalquist and A. Bjorck, *Numerical Methods in Scientic Computing,* SIAM. **1**, (2008).

[7] M. Dowell and D. Jarratt, *A modified regula falsi method for computing the root of an equation,*, BIT. **11**, 168-174 .

[8] M. Frontini and F. Sormani, *Third-Order Methods from Quadrature Formulae for Solving Systems of Non-linear Equations*, Appl. Math. Comput. **149**, (2004) 771-782.

[9] A. Golbabai and M. Javidi, *A Third-Order Newton Type Method for Nonlinear Equations Based on Modified Homotopy Perturbation Method*, Appl. Math. And Comput. **191**, (2007) 199-205.

[10] A. Golbabai and M. Javidi, *New Iterative Methods for Nonlinear Equations by Modified Homotopy Perturbation Method*, Appl. Math. And Comput. **191**, (2007) 122-127.

[11] C. N. Iwetan, I. A. Fuwape, M. S. Olajide and R. A. Adenodi, *Comparative Study of the Bisection and Newton Methods in solving for Zero and Extremes of a Single-Variable Function*, J. of NAMP. **21**, (2012) 173-176

[12] M. A. Noor and F. Ahmad, *Applied Mathematics and Computation*, (2006) 167-172.

[13] A. A. Sangah, *Comparative study of Existing bracketing methods with modified Bracketing algorithm for Solving Non-Linear Equation in single variable*, Sindh University, Research Journal (Sci.Ser.) **48**, No. 1 (2016) 171-174 .

[14] A. A. Siyal, R. A. Memon, N. M. Katbar and F. Ahmad, *Modified Algorithm for Solving Nonlinear Equations in Single Variable,* J. Appl. Environ. Biol. Sci.**7**, No. 5 (2017) 166-171.

[15] C. Solanki, P. Thapliyal and K. Tomar, *Role of Bisection Method* International Journal of Computer Applications Technology and Research ,**3**, No. 8 (2014) 535-535.