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Difference Operations of Soft Matrices with Applications in Decision Making

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Abstract. In this work, we give four new soft matrix operations called soft difference product, soft restricted difference product, soft extended difference product and soft weak-extended difference product and obtain their related properties. We then define soft max-row function, positive difference max-row soft matrix and negative difference max-row soft matrix. Using these, we construct novel efficient decision making method which determines both the optimal choice and the unlikely choice. We applied to the decision making problems based on benchmarking in order to show that our method performs well with uncertainties.

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1. Introduction

There are many mathematical tools such that the fuzzy set [32], the vague set [14], the rough set [25] and the interval mathematics [16] in order to describe uncertainty. Since these theories require the pre-specification of some parameters, Molodtsov [24] proposed the soft set theory to model uncertain, fuzzy, not clearly defined structures in 1999. There

are no limited conditions to the depiction of objects in soft set theory, therefore the researchers can choose the parameters in any form they need. This situation simplifies the process of decision making and also makes more efficient in the partial lack of information. Since introduction of the operations of soft sets [22], a rich literature on the properties and practice of the soft set theory was developed [3, 17, 21, 26, 28, 31]. The algebraic and topological structures of soft set theory were studied in details [1, 2, 4, 6, 12, 19, 27, 29].
Many authors developed decision making methods by utilizing the soft sets and also applied them to the decision making problems in several fields. Maji et al. [23] studied the soft sets based on the decision making. In [10, 20], they presented the reduction of soft set parametrization and the algorithm of parameter reduction, which can be used in various decision making. Çağman and Enginoğlu [8] described the products of soft sets and the function of $uni-int$ decision. They presented a $uni-int$ decision making procedure, which obtain a set of optimal objects from the alternatives. Feng et al. [13] constructed novel decision making methods which is called $uni-int^k$, $uni-int^t_s$ and int^m-int^n decision making. In[15, 30], the notions of bijective soft set and exclusive disjunctive soft set were introduced. Then, these structures were applied to the decision making and the information systems. Çetkin et al. [11] introduced the inverse soft sets, and also they showed that this notion is quite efficient in the decision process. Soft matrices and their related operations were defined for the first time in [9]. Subsequently, they utilize these soft structures to construct novel decision making methods. In [5], the products defined in [9] were generalized for soft matrices in different types. Kamacı et al. [18] introduced the row-products of soft matrices. With the help of the generalized products and row-products, novel decision algorithms was created. Basu et al. [7] published a study on the addition and subtraction of soft matrices.
In [8], the soft difference operation of two soft sets was defined. In this paper, we first define the operations of soft difference product, soft restricted difference product, soft extended difference product and soft weak-extended difference product for the soft matrices. These operations have several advantages in solving various decision making problems. We define soft max-row function, positive difference max-row soft matrix and negative difference max-row soft matrix. Then, we propose a new decision making model called "Soft Difference Max-Row Decision Making" using these concepts. Finally, we give three examples that one of them is a decision making problem and the others are the benchmarking problems with respect to the focused partner and topic.

2. PRELIMINARIES

Molodtsov [24] introduced the idea of soft set in the following manner:

Definition 2.1. *(*[24]*) Let U be an initial universal set, $P(U)$ be the power set of the set U, E be a set of parameters and $A \subseteq E$. A soft set (F, A) is a set of ordered pairs given by*

$$F_A = \{(x, F(x)) : x \in E, F(x) \in P(U)\}$$

where $F : E \to P(U)$ such that $x \notin A \Rightarrow F(x) = \emptyset$.
A soft set (F, A) can also be represented as F_A.

Definition 2.2. [8] *Let F_A and G_B be two soft sets over the universe set U. Then,* soft union *of F_A and G_B denoted by $F_A \tilde{\cup} G_B = H_E$ is a soft set defined by*

$$H(x) = F(x) \cup G(x)$$

for all $x \in E$.

Definition 2.3. [8] *Let* F_A *and* G_B *be two soft sets over the universe set* U. *Then,* soft intersection *of* F_A *and* G_B *denoted by* $F_A \tilde{\cap} G_B = H_E$ *is a soft set defined by*

$$H(x) = F(x) \cap G(x)$$

for all $x \in E$.

Definition 2.4. [8] *Let* F_A *and* G_B *be two soft sets over the universe set* U. *Then,* F_A *is a* soft subset *of* G_B, *denoted by* $F_A \tilde{\subseteq} G_B$, *if*

$$F(x) \subseteq G(x)$$

for all $x \in E$.

Definition 2.5. *(*[9]*) Let* F_A *be a soft sets over the universe set* U. *Then, the set*

$$R_A = \{(u, e) : e \in A, u \in F(e)\}$$

is said to be a relation form of F_A. *The characteristic function of* R_A *is described as*

$$\chi_{R_A} : U \times E \longrightarrow \{0, 1\}, \chi_{R_A}(u, e) = \begin{cases} 1, & (u, e) \in R_A \\ 0, & (u, e) \notin R_A. \end{cases}$$

If $U = \{u_1, u_2, ..., u_m\}$, $E = \{e_1, e_2, ..., e_n\}$ *and* $A \subseteq E$, *then* R_A *can be represented as a table in the following form:*

R_A	e_1	e_2	$. \; . \; .$	e_n
u_1	$\chi_{R_A}(u_1, e_1)$	$\chi_{R_A}(u_1, e_2)$	$. \; . \; .$	$\chi_{R_A}(u_1, e_n)$
u_2	$\chi_{R_A}(u_2, e_1)$	$\chi_{R_A}(u_2, e_2)$	$. \; . \; .$	$\chi_{R_A}(u_2, e_n)$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
u_m	$\chi_{R_A}(u_m, e_1)$	$\chi_{R_A}(u_m, e_2)$	$. \; . \; .$	$\chi_{R_A}(u_m, e_n)$

If $a_{ij} = \chi_{R_A}(u_i, e_j)$, *the matrix*

$$[a_{ij}]_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{bmatrix}.$$

is said to be an $m \times n$ *soft matrix of the soft set* F_A *over* U.

The set of all $m \times n$ soft matrices over the universe set U will be denoted as $SM_{m \times n}$. From now on, $[a_{ij}] \in SM_{m \times n}$ means that $[a_{ij}]$ is an $m \times n$ soft matrix.
By the notion of soft matrix, a soft set F_A is uniquely characterized as the matrix $[a_{ij}]$. This means that a soft set is formally equal to its corresponding soft matrix.

Example 2.6. *Let* $U = \{u_1, u_2, u_3, u_4, u_5\}$ *be a universal set and* $E = \{e_1, e_2, e_3, e_4\}$ *a parameter set. If* $A = \{e_1, e_3\}$ *and* $F : A \to P(U)$, $F(e_1) = \{u_1, u_2\}$, $F(e_3) = \{u_2, u_3, u_4\}$, *then we write a soft set*

$$F_A = \{(e_1, \{u_1, u_2\}), (e_3, \{u_2, u_3, u_4\})\}$$

Also, the relation form of F_A is

$$R_A = \{(u_1, e_1), (u_2, e_1), (u_2, e_3), (u_3, e_3), (u_4, e_3)\}.$$

Therefore, the soft matrix $[a_{ij}] \in SM_{5\times4}$ of F_A is

$$[a_{ij}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Definition 2.7. *([9]) Let $[a_{ij}] \in SM_{m\times n}$.*

(1) If $a_{ij} = 0$ for all i, j, then the soft matrix $[a_{ij}]$ is called a zero soft matrix and this is denoted as $[0]$.
(2) If $a_{ij} = 1$ for all i, j, then the soft matrix $[a_{ij}]$ is called a universal soft matrix and this is denoted as $[1]$.
(3) If $a_{ij} = 1$ for all i and $j \in I_A = \{j : e_j \in A\}$, then the soft matrix $[a_{ij}]$ is called an A-universal soft matrix and this is denoted as $[1_A]$.

Definition 2.8. *([9]) Let $[a_{ij}], [b_{ij}] \in SM_{m\times n}$. Then,*

(1) $[a_{ij}]$ is a soft submatrix of $[b_{ij}]$ if $a_{ij} \leq b_{ij}$ for all i, j. This is denoted as $[a_{ij}]\tilde{\subseteq}[b_{ij}]$.
(2) the soft matrix $[c_{ij}]$ is said to be a union of $[a_{ij}]$ and $[b_{ij}]$ if $c_{ij} = max\{a_{ij}, b_{ij}\}$ for all i, j. This is denoted as $[c_{ij}] = [a_{ij}]\tilde{\cup}[b_{ij}]$.
(3) the soft matrix $[c_{ij}]$ is said to be a intersection of $[a_{ij}]$ and $[b_{ij}]$ if $c_{ij} = min\{a_{ij}, b_{ij}\}$ for all i, j. This is denoted as $[c_{ij}] = [a_{ij}]\tilde{\cap}[b_{ij}]$.

3. Complements and Difference Products of Soft Matrices

Before we introduce difference products in four different types, we define A-complement of a soft matrix which will allow us to benefit.
From now on, the soft matrix corresponding to F_A which is a soft set over U will be denoted by $[F_A] = [a_{ij}]$.

Complements of Soft Matrices

In this section, we introduce two type complements of soft matrices and their concerned properties.

Definition 3.1. [8] *Let $F_A = (f_A, E)$ be a soft set over U. Then the soft set $F_A^c = (f_A, E)^c = (f_A^c, E)$ is called a* complement *of F_A, where $f_A^c : E \to P(U)$ is a mapping such that $f_A^c(x) = U \setminus f_A(x)$ for all $x \in E$.*

Definition 3.2. *Let $F_A = (f_A, E)$ be a soft set over U. Then the soft set $F_A^{c_A} = (f_A, E)^{c_A} = (f_A^{c_A}, E)$ is called a A-*complement *of F_A, where $f_A^{c_A} : E \to P(U)$ is a mapping such that $f_A^{c_A}(x) = U \setminus f_A(x)$ for all $x \in A$. ($x \notin A \Rightarrow f_A^{c_A}(x) = \emptyset$).*

Example 3.3. *Assume that the set of alternatives is* $U = \{u_1, u_2, u_3, u_4\}$ *and the set of parameters is* $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. *Let* $A = \{e_1, e_3, e_4, e_5\}$, *and then the soft set* $F_A = \{(e_1, \{u_2, u_3, u_4\}), (e_3, \emptyset), (e_4, \{u_1\}), (e_5, U)\}$. *So, we obtain the complement of this soft set as* $F_A^c = \{(e_1, \{u_1\}), (e_2, U), (e_3, U), (e_4, \{u_2, u_3, u_4\}), (e_5, \emptyset), (e_6, U)\}$ *and the* A*-complement of this soft set as* $F_A^{c_A} = \{(e_1, \{u_1\}), (e_3, U), (e_4, \{u_2, u_3, u_4\}), (e_5, \emptyset)\}$.

Definition 3.4. [9] *Let* $[F_A] = [a_{ij}] \in SM_{m\times n}$. *The soft matrix* $[c_{ij}]$ *is said to be a complement of* $[a_{ij}]$ *if* $c_{ij} = 1 - a_{ij}$ *for all* $j \in 1, 2, ..., n$. *It is denoted by* $[c_{ij}] = [a_{ij}]^c$.

Definition 3.5. *Let* $A \subseteq E = \{e_j : 1 \le j \le n\}$, $I_A = \{j : e_j \in A\}$ *and* $[F_A] = [a_{ij}] \in SM_{m\times n}$. *If*

$$c_{ij} = \begin{cases} 1 - a_{ij}, & \text{if } j \in I_A \\ 0, & \text{if } j \notin I_A \end{cases}$$

then the soft matrix $[c_{ij}]$ *is said to be an* A*-complement of* $[a_{ij}]$. *It is denoted by* $[c_{ij}] = [a_{ij}]^{c_A}$.

Example 3.6. *Consider soft set* F_A *given in Example 3.3. The soft matrix corresponding to* F_A *is*

$$[F_A] = [a_{ij}] = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Then we obtain

$$[F_A]^c = [a_{ij}]^c = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix},$$

$$[F_A]^{c_A} = [a_{ij}]^{c_A} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

Proposition 3.7. [9] *Let* $[F_A] \in SM_{m\times n}$ *and* $[G_B] \in SM_{m\times n}$. *Then*

i) $([F_A]^c)^c = [F_A]$
ii) $[0]^c = [1]$
iii) $[F_A]\widetilde{\cap}[F_A]^c = [0]$
iv) $[F_A]\widetilde{\cup}[F_A]^c = [1]$
v) $([F_A]\widetilde{\cup}[G_B])^c = [F_A]^c\widetilde{\cap}[G_B]^c$
vi) $([F_A]\widetilde{\cap}[G_B])^c = [F_A]^c\widetilde{\cup}[G_B]^c$

Proposition 3.8. *Let* $[F_A] \in SM_{m\times n}$ *and* $[G_B] \in SM_{m\times n}$. *Then*

i) $([F_A]^{c_A})^{c_A} = [F_A]$
ii) $[0]^{c_A} = [1_A]$
iii) $[F_A]\widetilde{\cap}[F_A]^{c_A} = [0]$
iv) $[F_A]\widetilde{\cup}[F_A]^{c_A} = [1_A]$
v) $[F_A]^{c_A}\widetilde{\cap}[G_B]^{c_B} = ([F_A]\widetilde{\cup}[G_B])^{c_{A\cap B}}$
vi) $[F_A]^{c_A}\widetilde{\subseteq}[F_A]^c$

Proof. Consider $[F_A] = [a_{ij}] \in SM_{m\times n}$ and $[G_B] = [b_{ij}] \in SM_{m\times n}$.

i) Let $([F_A]^{c_A})^{c_A} = [c_{ij}]$. From Definition 3.5, we write

$$\begin{aligned} c_{ij} &= \begin{cases} 1-(1-a_{ij}), & \text{if } j\in I_A \\ 0, & \text{if } j\notin I_A \end{cases} \\ &= \begin{cases} a_{ij}, & \text{if } j\in I_A \\ 0, & \text{if } j\notin I_A \end{cases} \end{aligned}$$

for all $1 \le i \le m$. Therefore $[c_{ij}] = [F_A]$.

ii) Let $[0]^{c_A} = [c_{ij}]$. From Definition 3.5, we write

$$c_{ij} = \begin{cases} 1-0, & \text{if } j\in I_A \\ 0, & \text{if } j\notin I_A \end{cases}$$

for all $1 \le i \le m$. Therefore $[c_{ij}] = [1_A]$ from Definition 2.7.

iii) Let $[F_A]\widetilde{\cap}[F_A]^{c_A} = [c_{ij}]$. From Definitions 2.8 and 3.5, we write

$$\begin{aligned} c_{ij} &= \begin{cases} min\{a_{ij}, 1-a_{ij}\}, & \text{if } j\in I_A \\ min\{a_{ij}, 0\}, & \text{if } j\notin I_A \end{cases} \\ &= \begin{cases} 0, & \text{if } j\in I_A \\ 0, & \text{if } j\notin I_A \end{cases} \\ &= 0 \end{aligned}$$

for all i and j. Therefore $[c_{ij}] = [0]$.

iv) Let $[F_A]\widetilde{\cup}[F_A]^{c_A} = [c_{ij}]$. From Definitions 2.8 and 3.5, we write

$$\begin{aligned} c_{ij} &= \begin{cases} max\{a_{ij}, 1-a_{ij}\}, & \text{if } j\in I_A \\ max\{0, 0\}, & \text{if } j\notin I_A \end{cases} \\ &= \begin{cases} 1, & \text{if } j\in I_A \\ 0, & \text{if } j\notin I_A \end{cases} \end{aligned}$$

for all $1 \le i \le m$. Therefore $[c_{ij}] = [1_A]$ from Definition 2.7.

v) Let $[F_A]^{c_A}\widetilde{\cap}[G_B]^{c_B} = [c_{ij}]$ and $([F_A]\widetilde{\cup}[G_B])^{c_{A\cap B}} = [d_{ij}]$. From Definitions 2.8 and 3.5, we write

$$\begin{aligned} c_{ij} &= \begin{cases} min\{1-a_{ij}, 1-b_{ij}\}, & \text{if } j\in I_{A\cap B} \\ min\{1-a_{ij}, 0\}, & \text{if } j\in I_{A\setminus B} \\ min\{0, 1-b_{ij}\}, & \text{if } j\in I_{B\setminus A} \\ 0, & \text{if } j\notin I_{A\cup B} \end{cases} \\ &= \begin{cases} min\{1-a_{ij}, 1-b_{ij}\}, & \text{if } j\in I_{A\cap B} \\ 0, & \text{if } j\notin I_{A\cap B} \end{cases} \end{aligned} \tag{3. 1}$$

and

$$\begin{aligned} d_{ij} &= \begin{cases} 1-max\{a_{ij}, b_{ij}\}, & \text{if } j\in I_{A\cap B} \\ 0, & \text{if } j\notin I_{A\cap B} \end{cases} \\ &= \begin{cases} min\{1-a_{ij}, 1-b_{ij}\}, & \text{if } j\in I_{A\cap B} \\ 0, & \text{if } j\notin I_{A\cap B} \end{cases} \end{aligned} \tag{3. 2}$$

for all $1 \leq i \leq m$. Therefore $[c_{ij}] = [d_{ij}]$ from (3. 1) and (3. 2).

vi) Let $[F_A]^{c_A} = [c_{ij}]$ and $[F_A]^c = [d_{ij}]$. From Definitions 3.4 and 3.5, we write

$$c_{ij} = \begin{cases} 1 - a_{ij}, & \text{if } j \in I_A \\ 0, & \text{if } j \notin I_A \end{cases}$$

and $d_{ij} = 1 - a_{ij}$ for all $1 \leq j \leq n$. Here if $j \notin I_A$, then $c_{ij} = 0$ and $d_{ij} = 1$ since $a_{ij} = 0$. Therefore $[c_{ij}] \tilde{\subseteq} [d_{ij}]$ from Definition 2.8.

Difference Products of Soft Matrices

In this section, four new soft matrix operations called soft difference product, soft restricted difference product, soft extended difference product and soft weak-extended difference product are defined.

Definition 3.9. [8] *Let F_A and G_B be soft sets over the universe set U. Then, soft difference of F_A and G_B denoted by $F_A \tilde{\setminus} G_B = H_E$ is a soft set defined as*

$$H(x) = F(x) \setminus G(x)$$

for all $x \in E$.

Proposition 3.10. [8] *Let F_A and G_B be soft sets over the universe set U. Then,*

$$F_A \tilde{\setminus} G_B = F_A \tilde{\cap} G_B^c.$$

Definition 3.11. *Let $[F_A] = [a_{ij}], [G_B] = [b_{ij}] \in SM_{m \times n}$. Soft difference product of $[F_A]$ and $[G_B]$ denoted by $[F_A] \tilde{\setminus} [G_B] = [c_{ij}] \in SM_{m \times n}$ is defined as $c_{ij} = min\{a_{ij}, 1 - b_{ij}\}$.*

A soft set F_A is uniquely characterized by the soft matrix $[F_A]$ and vice versa. The following theorem shows that this is valid for the difference operation of soft sets and the difference product of soft matrices.

Theorem 3.12. *Let F_A and G_B be two soft sets on the universe set U. Then,*

$$[F_A] \tilde{\setminus} [G_B] = [H_C] \Leftrightarrow H_C = F_A \tilde{\setminus} G_B.$$

Proof. Let $[F_A] = [a_{ij}]$, $[G_B] = [b_{ij}]$. By Definition 3.11, $[a_{ij}] \tilde{\setminus} [b_{ij}] = [c_{ij}]$ where $c_{ij} = min\{a_{ij}, 1 - b_{ij}\}$. It is clear that $[a_{ij}] \tilde{\cap} [b_{ij}]^c = [c_{ij}]$. Then, we have $[H_C] = [c_{ij}] \Leftrightarrow H_C = F_A \tilde{\cap} G_B^c = F_A \tilde{\setminus} G_B$ by Proposition 3.10.

Theorem 3.13. *Let F_A, G_B and H_C be three soft sets on the universe set U. Then,*

i) $[F_A] \tilde{\setminus} [F_A] = [0]$
ii) $[F_A] \tilde{\setminus} [0] = [F_A]$
iii) $[F_A] \tilde{\setminus} [1] = [0]$
iv) $[1] \tilde{\setminus} [F_A] = [F_A]^c$
v) $[0] \tilde{\setminus} [F_A] = [0]$
vi) $[F_A] \tilde{\setminus} [F_A]^c = [F_A]$
vii) $[F_A] \tilde{\setminus} [G_B] = [G_B] \tilde{\setminus} [F_A] \Leftrightarrow [F_A] = [G_B]$
viii) $([F_A] \tilde{\setminus} [G_B]) \tilde{\cap} [H_C] = ([F_A] \tilde{\cap} [H_C]) \tilde{\setminus} ([G_B] \tilde{\cap} [H_C])$

ix) $[F_A]\widetilde{\cap}([G_B]\widetilde{\setminus}[H_C]) = ([F_A]\widetilde{\cap}[G_B])\widetilde{\setminus}([F_A]\widetilde{\cap}[H_C])$
x) $([F_A]\widetilde{\setminus}[G_B])\widetilde{\setminus}[H_C] = ([F_A]\widetilde{\setminus}[H_C])\widetilde{\setminus}([G_B]\widetilde{\setminus}[H_C])$

Proof. Let $[F_A] = [a_{ij}]$, $[G_B] = [b_{ij}]$ and $[H_C] = [c_{ij}]$.

i) Let $[F_A]\widetilde{\setminus}[F_A] = [d_{ij}]$. Then, $d_{ij} = min\{a_{ij}, 1 - a_{ij}\} = 0$ for all i, j. Therefore $[d_{ij}] = [0]$.
ii) Let $[F_A]\widetilde{\setminus}[0] = [d_{ij}]$. Then, $d_{ij} = min\{a_{ij}, 1\} = a_{ij}$ for all i, j. Therefore $[d_{ij}] = [F_A]$.
iii) Let $[F_A]\widetilde{\setminus}[1] = [d_{ij}]$. Then, $d_{ij} = min\{a_{ij}, 0\} = 0$ for all i, j. Therefore $[d_{ij}] = [0]$.
iv) Let $[1]\widetilde{\setminus}[F_A] = [d_{ij}]$. Then, $d_{ij} = min\{1, 1 - a_{ij}\} = 1 - a_{ij}$ for all i, j. Therefore $[d_{ij}] = [F_A]^c$.
v) Let $[0]\widetilde{\setminus}[F_A] = [d_{ij}]$. Then, $d_{ij} = min\{0, 1 - a_{ij}\} = 0$ for all i, j. Therefore $[d_{ij}] = [0]$.
vi) Let $[F_A]\widetilde{\setminus}[F_A]^c = [d_{ij}]$. Then, $d_{ij} = min\{a_{ij}, a_{ij}\} = a_{ij}$ for all i, j. Therefore $[d_{ij}] = [F_A]$.
vii) $\Rightarrow$: Let $[d_{ij}] = [F_A]\widetilde{\setminus}[G_B] = [G_B]\widetilde{\setminus}[F_A] = [e_{ij}]$. Then, $d_{ij} = min\{a_{ij}, 1 - b_{ij}\} = min\{b_{ij}, 1 - a_{ij}\} = e_{ij}$ for all i and j. Therefore $a_{ij} = b_{ij}$ for all i and j, so $[F_A] = [G_B]$.
$\Leftarrow$: It is clear.
viii) Let $([F_A]\widetilde{\setminus}[G_B])\widetilde{\cap}[H_C] = [d_{ij}]$ and $([F_A]\widetilde{\cap}[H_C])\widetilde{\setminus}([G_B]\widetilde{\cap}[H_C]) = [e_{ij}]$.
For all i, j

$$\begin{aligned} d_{ij} &= min\{min\{a_{ij}, 1 - b_{ij}\}, c_{ij}\} \\ &= min\{min\{a_{ij}, c_{ij}\}, 1 - min\{b_{ij}, c_{ij}\}\} \\ &= e_{ij} \end{aligned}$$

Therefore $[d_{ij}] = [e_{ij}]$.
ix) The proof can be proved similarly to proof of (viii).
x) Let $([F_A]\widetilde{\setminus}[G_B])\widetilde{\setminus}[H_C] = [d_{ij}]$ and $([F_A]\widetilde{\setminus}[H_C])\widetilde{\setminus}([G_B]\widetilde{\setminus}[H_C]) = [e_{ij}]$.
For all i, j

$$\begin{aligned} d_{ij} &= min\{min\{a_{ij}, 1 - b_{ij}\}, 1 - c_{ij}\} \\ &= min\{min\{a_{ij}, 1 - c_{ij}\}, 1 - min\{b_{ij}, 1 - c_{ij}\}\} \\ &= e_{ij} \end{aligned}$$

Therefore, $[d_{ij}] = [e_{ij}]$.

Definition 3.14. *Let F_A and G_B be soft sets on the universe set U. Then, soft restricted difference of F_A and G_B denoted by $F_A \widetilde{\setminus}_r G_B = H_E$ is a soft set defined as*

$$H(x) = \begin{cases} F(x) \cap (U \setminus G(x)), & \text{if } x \in B \\ \emptyset, & \text{if } x \notin B \end{cases}$$

Proposition 3.15. *Let F_A and G_B be soft sets on the universe set U. Then,*

$$F_A \widetilde{\setminus}_r G_B = F_A \widetilde{\cap} G_B^{c_B}.$$

Proof. Let $F_A \widetilde{\setminus}_r G_B = H_E$. Then, we can write

$$H(x) = \begin{cases} F(x) \cap (U \setminus G(x)), & \text{if } x \in B \\ \emptyset, & \text{if } x \notin B \end{cases}$$

Also, let $F_A \widetilde{\cap} G_B^{c_B} = I_E$. From Definition 3.2, we can write

$$I(x) = \begin{cases} F(x) \cap (U \setminus G(x)), & \text{if } x \in B \\ \emptyset, & \text{if } x \notin B \end{cases}$$

Therefore $H(x) = I(x)$ for all $x \in E$.

Definition 3.16. *Let $[F_A] = [a_{ij}], [G_B] = [b_{ij}] \in SM_{m\times n}$ and let $I_B = \{j : e_j \in B\}$. Soft restricted difference product of $[F_A]$ and $[G_B]$ denoted by $[F_A] \widetilde{\setminus}_r [G_B] = [c_{ij}] \in SM_{m\times n}$ is defined as*

$$c_{ij} = \begin{cases} min\{a_{ij}, 1 - b_{ij}\}, & \text{if } j \in I_B \\ 0, & \text{if } j \notin I_B \end{cases}$$

Theorem 3.17. *Let F_A and G_B be two soft sets on the universe set U. Then,*

$$[F_A] \widetilde{\setminus}_r [G_B] = [H_C] \Leftrightarrow H_C = F_A \widetilde{\setminus}_r G_B.$$

Proof. Using Definition 3.14, Proposition 3.15 and Definition 3.16, it can be proved similar to Theorem 3.12.

Theorem 3.18. *Let F_A, G_B and H_C be three soft sets on the universe set U. Then,*

i) $[F_A] \widetilde{\setminus}_r [F_A] = [0]$
ii) $[F_A] \widetilde{\setminus}_r [1_A] = [0]$
iii) $[1] \widetilde{\setminus}_r [F_A] = [F_A]^{c_A}$
iv) $[0] \widetilde{\setminus}_r [F_A] = [0]$
v) $[F_A] \widetilde{\setminus}_r [F_A]^{c_A} = [F_A]$
vi) $[F_A] \widetilde{\setminus}_r [G_B] = [G_B] \widetilde{\setminus}_r [F_A] \Leftrightarrow [F_A] = [G_B]$
vii) $([F_A] \widetilde{\setminus}_r [G_B]) \widetilde{\cap} [H_C] = ([F_A] \widetilde{\cap} [H_C]) \widetilde{\setminus}_r ([G_B] \widetilde{\cap} [H_C])$
viii) $[F_A] \widetilde{\cap} ([G_B] \widetilde{\setminus}_r [H_C]) = ([F_A] \widetilde{\cap} [G_B]) \widetilde{\setminus}_r ([F_A] \widetilde{\cap} [H_C])$
ix) $[F_A] \widetilde{\setminus}_r [G_B] \widetilde{\subseteq} [F_A] \widetilde{\setminus} [G_B]$

Proof. Let $[F_A] = [a_{ij}]$, $[G_B] = [b_{ij}]$ and $[H_C] = [c_{ij}]$.

i) Let $[F_A] \widetilde{\setminus}_r [F_A] = [d_{ij}]$. Then, from Definition 3.16 we write

$$\begin{aligned} d_{ij} &= \begin{cases} min\{a_{ij}, 1 - a_{ij}\}, & \text{if } j \in I_A \\ 0, & \text{if } j \notin I_A \end{cases} \\ &= 0 \end{aligned}$$

for all i and j. Therefore $[d_{ij}] = [0]$.

ii) Let $[F_A]\widetilde{\setminus}_r[1_A]=[d_{ij}]$. Then, from Definition 3.16 we write

$$\begin{aligned} d_{ij} &= \begin{cases} min\{a_{ij}, 1-1\}, & \text{if } j\in I_A \\ 0, & \text{if } j\notin I_A \end{cases} \\ &= \begin{cases} 0, & \text{if } j\in I_A \\ 0, & \text{if } j\notin I_A \end{cases} \\ &= 0 \end{aligned}$$

for all i and j. Therefore $[d_{ij}]=[0]$.

iii) Let $[1]\widetilde{\setminus}_r[F_A]=[d_{ij}]$. Then,

$$\begin{aligned} d_{ij} &= \begin{cases} min\{1, 1-a_{ij}\}, & \text{if } j\in I_A \\ 0, & \text{if } j\notin I_A \end{cases} \\ &= \begin{cases} 1-a_{ij}, & \text{if } j\in I_A \\ 0, & \text{if } j\notin I_A \end{cases} \end{aligned}$$

for all $1\leq i\leq m$. Therefore $[d_{ij}]=[F_A]^{c_A}$.

iv) Let $[0]\widetilde{\setminus}_r[F_A]=[d_{ij}]$. Then,

$$\begin{aligned} d_{ij} &= \begin{cases} min\{0, 1-a_{ij}\}, & \text{if } j\in I_A \\ 0, & \text{if } j\notin I_A \end{cases} \\ &= 0 \end{aligned}$$

for all i and j. Therefore $[d_{ij}]=[0]$.

v) Let $[F_A]\widetilde{\setminus}_r[F_A]^{c_A}=[d_{ij}]$. Then,

$$\begin{aligned} d_{ij} &= \begin{cases} min\{a_{ij}, a_{ij}\}, & \text{if } j\in I_A \\ 0, & \text{if } j\notin I_A \end{cases} \\ &= \begin{cases} a_{ij}, & \text{if } j\in I_A \\ 0, & \text{if } j\notin I_A \end{cases} \end{aligned}$$

for all $1\leq i\leq m$. Therefore $[d_{ij}]=[F_A]$.

vi) $\Rightarrow$: Let $[d_{ij}]=[F_A]\widetilde{\setminus}_r[G_B]=[G_B]\widetilde{\setminus}_r[F_A]=[e_{ij}]$. Then,

$$d_{ij} = \begin{cases} min\{a_{ij}, 1-b_{ij}\}, & \text{if } j\in I_B \\ 0, & \text{if } j\notin I_B \end{cases} = \begin{cases} min\{1-a_{ij}, b_{ij}\}, & \text{if } j\in I_A \\ 0, & \text{if } j\notin I_A \end{cases} = e_{ij}$$

for all for all $1\leq i\leq m$. It is possible if $I_A=I_B$ and $a_{ij}=b_{ij}$ for all i,j, so $[F_A]=[G_B]$.

$\Leftarrow$: It is clear.

vii) Let $([F_A]\widetilde{\setminus}_r[G_B])\widetilde{\cap}[H_C] = [d_{ij}]$ and $([F_A]\widetilde{\cap}[H_C])\widetilde{\setminus}_r([G_B]\widetilde{\cap}[H_C]) = [e_{ij}]$. For all $1 \leq i \leq m$,

$$\begin{aligned} d_{ij} &= \begin{cases} min\{min\{a_{ij}, 1-b_{ij}\}, c_{ij}\}, & \text{if } j \in I_{B\cap C} \\ min\{min\{a_{ij}, 1-b_{ij}\}, 0\}, & \text{if } j \in I_{B\setminus C} \\ min\{0, c_{ij}\}, & \text{if } j \in I_{C\setminus B} \\ 0, & \text{if } j \notin I_{B\cup C} \end{cases} \\ &= \begin{cases} min\{min\{a_{ij}, 1-b_{ij}\}, c_{ij}\}, & \text{if } j \in I_{B\cap C} \\ 0, & \text{if } j \notin I_{B\cap C} \end{cases} \\ &= \begin{cases} min\{min\{a_{ij}, c_{ij}\}, 1-min\{b_{ij}, c_{ij}\}\}, & \text{if } j \in I_{B\cap C} \\ 0, & \text{if } j \notin I_{B\cap C} \end{cases} \end{aligned} \tag{3. 3}$$

and

$$e_{ij} = \begin{cases} min\{min\{a_{ij}, c_{ij}\}, 1-min\{b_{ij}, c_{ij}\}\}, & \text{if } j \in I_{B\cap C} \\ 0, & \text{if } j \notin I_{B\cap C} \end{cases} \tag{3. 4}$$

We obtain $[d_{ij}] = [e_{ij}]$ from (3. 3) and (3. 4).

viii) Let $[F_A]\widetilde{\cap}([G_B]\widetilde{\setminus}_r[H_C]) = [d_{ij}]$ and $([F_A]\widetilde{\cap}[G_B])\widetilde{\setminus}_r([F_A]\widetilde{\cap}[H_C]) = [e_{ij}]$. For all $1 \leq i \leq m$,

$$\begin{aligned} d_{ij} &= \begin{cases} min\{a_{ij}, min\{b_{ij}, 1-c_{ij}\}\}, & \text{if } j \in I_C \\ 0, & \text{if } j \notin I_C \end{cases} \\ &= \begin{cases} min\{min\{a_{ij}, b_{ij}\}, min\{a_{ij}, 1-c_{ij}\}\}, & \text{if } j \in I_C \\ 0, & \text{if } j \notin I_C \end{cases} \\ &= \begin{cases} min\{min\{a_{ij}, b_{ij}\}, 1-min\{a_{ij}, c_{ij}\}\}, & \text{if } j \in I_C \\ 0, & \text{if } j \notin I_C \end{cases} \end{aligned} \tag{3. 5}$$

and

$$e_{ij} = \begin{cases} min\{min\{a_{ij}, b_{ij}\}, 1-min\{a_{ij}, c_{ij}\}\}, & \text{if } j \in I_C \\ 0, & \text{if } j \notin I_C \end{cases} \tag{3. 6}$$

We obtain $[d_{ij}] = [e_{ij}]$ from (3. 5) and (3. 6).

ix) Let $[F_A]\widetilde{\setminus}_r[G_B] = [d_{ij}]$ and $[F_A]\widetilde{\setminus}[G_B] = [e_{ij}]$. Then for all $1 \leq i \leq m$,

$$d_{ij} = \begin{cases} min\{a_{ij}, 1-b_{ij}\}, & \text{if } j \in I_B \\ 0, & \text{if } j \notin I_B \end{cases}$$

and $e_{ij} = min\{a_{ij}, 1-b_{ij}\}$ for all j. If $j \notin I_B$, then $d_{ij} = 0$ but $e_{ij} = 1$ in case of $a_{ij} = 1$. Therefore $[d_{ij}]\widetilde{\subseteq}[e_{ij}]$ from Definition 2.8.

Definition 3.19. *Let F_A and G_B be soft sets on the universe set U. Then, soft extended difference of F_A and G_B denoted by $F_A\widetilde{\setminus}_\epsilon G_B = H_E$ is a soft set defined as*

$$H(x) = F(x) \cup (U \setminus G(x))$$

for all $x \in E$.

Proposition 3.20. *Let* F_A *and* G_B *be two soft sets on the universe set* U*. Then,*

$$F_A \widetilde{\setminus}_\epsilon G_B = F_A \widetilde{\cup} G_B^c.$$

Proof. Let $F_A \widetilde{\setminus}_\epsilon G_B = H_E$. Then, we can write

$$H(x) = F(x) \cup (U \setminus G(x))$$

Also, let $F_A \widetilde{\cup} G_B^{c_B} = I_E$. From Definition 3.2, we can write

$$I(x) = F(x) \cup (U \setminus G(x))$$

Therefore $H(x) = I(x)$ for all $x \in E$.

Definition 3.21. *Let* $[F_A] = [a_{ij}], [G_B] = [b_{ij}] \in SM_{m\times n}$*. Soft extended difference product of* $[F_A]$ *and* $[G_B]$ *denoted by* $[F_A]\widetilde{\setminus}_\epsilon[G_B] = [c_{ij}]$ *where* $[c_{ij}] \in SM_{m\times n}$ *is defined as* $c_{ij} = max\{a_{ij}, 1 - b_{ij}\}$.

Theorem 3.22. *Let* F_A *and* G_B *be two soft sets on the universe set* U*. Then,*

$$[F_A]\widetilde{\setminus}_\epsilon[G_B] = [H_C] \Leftrightarrow H_C = F_A \widetilde{\setminus}_\epsilon G_B.$$

Proof. Using Definition 3.19, Proposition 3.20 and Definition 3.21, it can be proved similar to Theorem 3.12.

Theorem 3.23. *Let* F_A, G_B *and* H_C *be three soft sets on the universe set* U*. Then,*

i) $[F_A]\widetilde{\setminus}_\epsilon[F_A] = [1]$

ii) $[F_A]\widetilde{\setminus}_\epsilon[0] = [1]$

iii) $[F_A]\widetilde{\setminus}_\epsilon[1] = [F_A]$

iv) $[1]\widetilde{\setminus}_\epsilon[F_A] = [1]$

v) $[0]\widetilde{\setminus}_\epsilon[F_A] = [F_A]^c$

vi) $[F_A]\widetilde{\setminus}_\epsilon[F_A]^c = [F_A]$

vii) $([F_A]\widetilde{\setminus}_\epsilon[G_B])^c = [G_B]\widetilde{\setminus}[F_A]$

viii) $[F_A]\widetilde{\setminus}_\epsilon[G_B] = [G_B]\widetilde{\setminus}_\epsilon[F_A] \Leftrightarrow [F_A] = [G_B]$

ix) $([F_A]\widetilde{\setminus}_\epsilon[G_B])\widetilde{\cup}[H_C] = ([F_A]\widetilde{\cup}[H_C])\widetilde{\setminus}_\epsilon([G_B]\widetilde{\cup}[H_C])$

x) $[F_A]\widetilde{\cup}([G_B]\widetilde{\setminus}_\epsilon[H_C]) = ([F_A]\widetilde{\cup}[G_B])\widetilde{\setminus}_\epsilon([F_A]\widetilde{\cup}[H_C])$

xi) $([F_A]\widetilde{\setminus}_\epsilon[G_B])\widetilde{\setminus}_\epsilon[H_C] = ([F_A]\widetilde{\setminus}_\epsilon[H_C])\widetilde{\setminus}_\epsilon([G_B]\widetilde{\setminus}_\epsilon[H_C])$

Proof. The proof can be seen similarly to proof of the Theorem 3.13.

Definition 3.24. *Let* F_A *and* G_B *be soft sets on the universe set* U*. Then, soft weak-extended difference of* F_A *and* G_B *denoted by* $F_A \widetilde{\setminus}_w G_B = H_E$ *is a soft set defined as*

$$H(x) = \begin{cases} F(x) \cup (U \setminus G(x)), & \text{if } x \in B \\ F(x), & \text{if } x \notin B \end{cases}$$

Proposition 3.25. *Let* F_A *and* G_B *be two soft sets on the universe set* U*. Then,*

$$F_A \widetilde{\setminus}_w G_B = F_A \widetilde{\cup} G_B^{c_B}.$$

Proof. Using Definitions 3.2 and 3.24 , it can be proved similar to Proposition 3.15.

Definition 3.26. *Let $[F_A] = [a_{ij}], [G_B] = [b_{ij}] \in SM_{m\times n}$. Soft weak-extended difference product of $[F_A]$ and $[G_B]$ denoted by $[a_{ij}]\widetilde{\setminus}_w[b_{ij}] = [c_{ij}] \in SM_{m\times n}$ is defined as*

$$c_{ij} = \begin{cases} max\{a_{ij}, 1 - b_{ij}\}, & \text{if } j \in I_B \\ a_{ij}, & \text{if } j \notin I_B \end{cases}$$

Theorem 3.27. *Let F_A and G_B be two soft sets on the universe set U. Then,*

$$[F_A]\widetilde{\setminus}_w[G_B] = [H_C] \Leftrightarrow H_C = F_A\widetilde{\setminus}_w G_B.$$

Proof. Using Definition 3.24, Proposition 3.25 and Definition 3.26, it can be proved similar to Theorems 3.12.

Theorem 3.28. *Let F_A, G_B and H_C be three soft sets on the universe set U. Then,*

i) $[F_A]\widetilde{\setminus}_w[F_A] = [1_A]$
ii) $[F_A]\widetilde{\setminus}_w[1_A] = [F_A]$
iii) $[1]\widetilde{\setminus}_w[F_A] = [1]$
iv) $[0]\widetilde{\setminus}_w[F_A] = [F_A]^{c_A}$
v) $[F_A]\widetilde{\setminus}_w[F_A]^{c_A} = [F_A]$
vi) $[F_A]\widetilde{\setminus}_w[G_B] = [G_B]\widetilde{\setminus}_w[F_A] \Leftrightarrow [F_A] = [G_B]$
vii) $([F_A]\widetilde{\setminus}_w[G_B])\widetilde{\cup}[H_C] = ([F_A]\widetilde{\cup}[H_C])\widetilde{\setminus}_w([G_B]\widetilde{\cup}[H_C])$
viii) $[F_A]\widetilde{\cup}([G_B]\widetilde{\setminus}_w[H_C]) = ([F_A]\widetilde{\cup}[G_B])\widetilde{\setminus}_w([F_A]\widetilde{\cup}[H_C])$
ix) $[F_A]\widetilde{\setminus}_w[G_B]\widetilde{\subseteq}[F_A]\widetilde{\setminus}_\epsilon[G_B]$

Proof. The proof can be seen similarly to proof of the Theorems 3.13 and 3.18.

4. Soft Difference Max-Row Decision Making

To construct an effective decision making method, firstly we need to define max-row function and max-row matrix for a soft matrix.

Definition 4.1. *Let $[a_{ij}] \in SM_{m\times n}$ be a soft matrix. Then* soft max-row function M_r *is defined as below:*

$$M_r : SM_{m\times n} \longrightarrow SM_{m\times 1}, M_r([a_{ij}]) = [f_{i1}]$$

where

$$f_{i1} = \max_{j\in\{1,2,\ldots,m\}}\{a_{ij}\}.$$

The soft matrix $M_r([a_{ij}])$ is called a max-row soft matrix *of the soft matrix $[a_{ij}]$.*

Example 4.2. *Consider the soft matrix $[a_{ij}]$ in Example 2.6, max-row soft matrix of $[a_{ij}]$ is*

$$M_r([a_{ij}]) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

By the following definitions, we introduce favorable soft matrix, unfavorable soft matrix, decision soft matrices and then optimum set, suboptimum set and non-optimum set.

Definition 4.3. *Let $[a_{ij}] \in SM_{m\times n}$ be a soft matrix. Then $[a_{ij}]$ is called*

(1) favorable soft matrix if $a_{ij} = 1$ implies that the alternative u_i satisfies the parameter e_j,

(2) unfavorable soft matrix if $a_{ij} = 1$ implies that the alternative u_i doesn't satisfy the parameter e_j.

Definition 4.4. *Let $[c_{ij}] \in SM_{m\times n}$ be intersection or union of the favorable soft matrices and $[d_{ij}] \in SM_{m\times n}$ be intersection or union of the unfavorable soft matrices. Then,*

(1) $[e_{i1}] = M_r([c_{ij}]\widetilde{\setminus}[d_{ij}])$ is called positive difference max-row soft matrix and $[f_{i1}] = M_r([d_{ij}]\widetilde{\setminus}[c_{ij}])$ is called negative difference max-row soft matrix.
Here, it can be taken $\widetilde{\setminus}_r, \widetilde{\setminus}_\epsilon$ and $\widetilde{\setminus}_w$ instead of $\widetilde{\setminus}$.

(2) $[g_{i1}] = [e_{i1}]\widetilde{\setminus}[f_{i1}]$ is called positive decision soft matrix and $[h_{i1}] = [f_{i1}]\widetilde{\setminus}[e_{i1}]$ is called negative decision soft matrix.

Definition 4.5. *Let $[g_{i1}] \in SM_{m\times 1}$ be the positive decision soft matrix and $[f_{i1}] \in SM_{m\times 1}$ be the negative decision soft matrix. Then, the set*

(1) $opt(U) = \{u_i : u_i \in U,\ g_{i1} = 1\}$ is said to be an optimum set of U.

(2) $sub - opt(U) = \{u_i : u_i \in U,\ g_{i1} = h_{i1} = 0\}$ is said to be a suboptimum set of U.

(3) $non - opt(U) = \{u_i : u_i \in U,\ h_{i1} = 1\}$ is said to be non-optimum set of U.

Soft difference max-row decision making method selects both optimum alternative(s) and unlikely alternative(s) from the set of all alternatives. This method is organized as in the following algorithm:

Step 1: Determine feasible subsets from the parameter set.

Step 2: Create the soft matrix for each of parameter subsets.

Step 3: Using one of the convenient operations such as $\widetilde{\cap}$ or $\widetilde{\cup}$ according to the problem, find favorable soft matrices and unfavorable soft matrices, if possible.

Step 4: Using one of the convenient operations such as $\widetilde{\setminus}, \widetilde{\setminus}_r, \widetilde{\setminus}_\epsilon$ or $\widetilde{\setminus}_w$ according to the problem, find positive difference max-row soft matrix and negative difference max-row soft matrix.

Step 5: Find positive decision soft matrix and negative decision soft matrix.

Step 6: Obtain an optimum set of U (suboptimum set of U if the optimum set is empty) and non-optimum set of U.

Now, we apply this algorithm for a decision problem by using the soft weak-extended difference product $\widetilde{\setminus}_w$.

Example 4.6. *Assume that there are six candidates who apply for the empty position of a company. And there are two business partner of this company who are decision makers. Let the set of candidates be $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ which is characterized by a set of parameters $E = \{e_1, e_2, e_3, e_4, e_5\}$. For $i = 1, 2, 3, 4, 5$, the parameters e_i stand for "young age", "experience", "having good references", "computer knowledge" and "knowing foreign languages".*
The first decision marker investigates eligible candidates and the second one investigates noneligible candidates.

When each partner selects the parameters from the parameter set E, we can apply the soft difference max-row decision making algorithm as follows:
Step 1: *To evaluate the candidates, the partners consider set of parameters $A = \{e_1, e_2, e_3, e_4\}$ and $B = \{e_2, e_3, e_4\}$, respectively. The partners generate the following soft sets over the universe set U with respect to their parameters, respectively.*
$F_A = \{(e_1, \{u_2, u_5, u_6\}), (e_2, \{u_1, u_4, u_6\}), (e_3, \{u_2, u_3, u_6\}), (e_4, \{u_2, u_4, u_6\})\}$,
$G_B = \{(e_2, \{u_2, u_3, u_4, u_5\}), (e_3, \{u_1, u_5\}), (e_4, \{u_1, u_3, u_5\})\}$.
Step 2: *For the soft sets F_A and G_B, the following soft matrices are created respectively.*

$$[F_A] = [a_{ij}] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \textit{ and } [G_B] = [b_{ij}] = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Step 3: *The matrix of the first decision maker $[a_{ij}]$ is a favorable soft matrix and the matrix of the second decision maker $[b_{ij}]$ is an unfavorable soft matrix since the first decision marker investigates eligible candidates and the second one investigates noneligible candidates.*
Step 4: *Using the soft weak-extended difference product, we find the positive difference max-row soft matrix and the negative difference max-row soft matrix as*

$$[e_{i1}] = M_r([a_{ij}]\widetilde{\setminus}_w[b_{ij}]) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \textit{ and } [f_{i1}] = M_r([b_{ij}]\widetilde{\setminus}_w[a_{ij}]) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

Step 5: *We obtain the positive decision soft matrix and the negative decision soft matrix as*

$$[g_{i1}] = [e_{i1}]\widetilde{\setminus}[f_{i1}] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \textit{ and } [h_{i1}] = [f_{i1}]\widetilde{\setminus}[e_{i1}] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

Step 6: *Now we can find an optimum set and a non-optimum set of U.*

$$opt(U) = \{u_6\}$$

which u_6 is an optimum candidate for the empty position of the company. Also,

$$non - opt(U) = \{u_5\}$$

which u_5 is an unlikely candidate for the empty position of the company.

In this example, if we use soft restricted difference product instead of soft weak-extended difference product, then we obtain $opt(U) = \{u_3, u_6\}$. This doesn't give a definite result, because only one candidate will be preferred.

5. Applications on Benchmarking Problem

Benchmarking means that a company ,in order to increase competitive power, examines other companies that have superior performance and these companies' techniques of doing business, as well as compares their own techniques and applies in their own companies the knowledge gained from this comparison. The benchmarking agreement is also an agreement to ensure the continuation of this benchmarking. Thus, the benchmarking companies can constantly monitor developments in the topic of benchmarking in other companies.
In order to better understanding of the following examples, let us first give the benchmarking types and their benchmarking processes.

Benchmarking Types	
(i) Benchmarking according to the focused partner(s)	(ii) Benchmarking according to the focused topic(s)
Benchmarking Process: 1. Determine partner(s) for benchmarking 2. Construct benchmarking team by specifying tasks 3. Determine topic(s) of benchmarking 4. Obtain necessary data and analyze 5. Apply method and evaluate results	*Bechmarking Process:* 1. Determine topic(s) of benchmarking 2. Construct benchmarking team by specifying tasks 3. Determine partner(s) for benchmarking 4. Obtain necessary data and analyze 5. Apply method and evaluate results

5.1. An example on benchmarking problem according to the focused partner

Assume that a company Y operates in six different business areas. The set of these business areas is $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$, where u_i $(1 \leq i \leq 6)$ stand for "textile raw material production", "electricity distribution services", "designing of safety systems", "automotive spare parts production", "manufacture of cleaning articles" and "production of electrical household appliances", respectively. Another company X, which operates in the same business areas, appoints three benchmarker to make benchmarking agreement with the company Y.
First benchmarker investigates *positive* aspects in terms of production of company Y.
Second benchmarker investigates *positive* aspects in terms of staff of company Y.
Third benchmarker investigates all of the *negative* aspects of company Y.
For this case, let $E = \{e_1, e_2, ..., e_{24}\}$ be a set of all parameters which is determined by benchmarkers. These parameters; e_1=raw material cost, e_2=raw material procurement, e_3=advertising revenues of product, e_4=profitability, e_5=inventory control, e_6=collection, e_7=optimum number of staff, e_8=relationship between objectives and results, e_9=staff motivation, e_{10}=market share, e_{11}=production rate, e_{12}=product design, e_{13}=storage control, e_{14}=production cost, e_{15}=production of multiple choice , e_{16}=quality management, e_{17}=management style, e_{18}=amount of daily production, e_{19}=customer delight, e_{20}=distri bution of products, e_{21}=technology systems, e_{22}=internal control mechanisms, e_{23}=positive competition of staff, e_{24}=staff efficiency.
When each benchmarker selects the parameters from the parameter set E, we can apply the soft difference max-row decision making algorithm as follows:
Step 1: The benchmarkers determine the parameter subsets as follows, respectively
$A = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_{10}, e_{11}, e_{12}, e_{15}, e_{18}, e_{19}, e_{20}, e_{21}\}$,
$B = \{e_4, e_7, e_8, e_9, e_{11}, e_{14}, e_{16}, e_{17}, e_{19}, e_{21}, e_{24}\}$ and

$C = \{e_1, e_2, e_3, e_5, e_6, e_7, e_9, e_{10}, e_{11}, e_{12}, e_{14}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}, e_{21}, e_{23}, e_{24}\}$.
As a result of research, they obtain their soft sets as follows:
$F_A = \{(e_1, \{u_1, u_2, u_5, u_6\}), (e_2, \{u_2, u_3, u_4, u_5, u_6\}), (e_3, \{u_5, u_6\}), (e_4, \{u_1, u_3, u_5\}),$
$(e_5, \{u_2, u_3, u_5, u_6\}), (e_6, \{u_1, u_4, u_6\}), (e_7, \{u_1, u_3, u_4, u_5\}), (e_8, \{u_2, u_3\}),$
$(e_{10}, \{u_1, u_2, u_4, u_5, u_6\}), (e_{11}, \{u_2, u_4, u_5\}), (e_{12}, \{u_1, u_3, u_6\}), (e_{15}, \{u_1, u_3, u_4, u_6\}),$
$(e_{18}, \{u_1, u_4, u_6\}), (e_{19}, \{u_1, u_2, u_5, u_6\}), (e_{20}, \{u_3, u_5, u_6\}), (e_{21}, \{u_1, u_4\})\}$,
$G_B = \{(e_4, \{u_1, u_2, u_5\}), (e_7, \{u_1, u_4, u_5, u_6\}), (e_8, \{u_2, u_3\}), (e_9, \{u_1, u_2, u_4, u_6\}),$
$(e_{11}, \{u_1, u_4, u_5\}), (e_{14}, \{u_1, u_2, u_4, u_6\}), (e_{16}, U), (e_{17}, \{u_1, u_2, u_4, u_5\}),$
$(e_{19}, \{u_1, u_2, u_4, u_5\}), (e_{21}, \{u_2, u_4\})(e_{24}, \{u_1, u_5\})\}$,
$H_C = \{(e_1, \{u_2, u_4, u_5\}), (e_2, \{u_1, u_6\})\}, (e_3, \{u_2, u_3\}), (e_4, \{u_2, u_5, u_6\}),$
$(e_5, \{u_1, u_4, u_6\}), (e_6, \{u_2, u_3, u_4, u_5\}), (e_7, \{u_2, u_3, u_4, u_5\}), (e_9, \{u_1, u_3\}),$
$(e_{10}, \{u_1, u_3\}), (e_{11}, \{u_1, u_4, u_5\}), (e_{12}, \{u_1, u_4, u_5\}), (e_{14}, \{u_5, u_6\}), (e_{16}, \{u_2, u_6\}),$
$(e_{17}, \{u_2, u_3, u_5\}), (e_{18}, \{u_1, u_3, u_6\}), (e_{19}, \{u_1, u_2, u_4, u_5, u_6\}), (e_{20}, \{u_1, u_4\}),$
$(e_{21}, \{u_1, u_3, u_4\}), (e_{23}, \emptyset), (e_{24}, \{u_1, u_4, u_6\})\}$.

Step 2: For the soft sets F_A, G_B and H_C, the following soft matrices are created respectively.

$[F_A] = [a_{ij}]$=

$$\begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$[G_B] = [b_{ij}]$=

$$\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

and

$[H_C] = [c_{ij}]$=

$$\begin{bmatrix}
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}.$$

Step 3: Using the soft intersection, we obtain the favorable soft matrix

$[d_{ij}] = [a_{ij}]\widetilde{\cap}[b_{ij}]$=

$$\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

and the unfavorable soft matrix

$[c_{ij}]$=

$$\begin{bmatrix} 0&1&0&0&1&0&0&0&1&1&1&1&0&0&0&0&0&1&1&1&1&0&0&1 \\ 1&0&1&1&0&1&1&0&0&0&0&0&0&0&0&1&1&0&1&0&0&0&0&0 \\ 0&0&1&0&0&1&1&0&1&1&0&0&0&0&0&0&1&1&0&0&1&0&0&0 \\ 1&0&0&0&1&1&1&0&0&0&1&1&0&0&0&0&0&0&1&1&1&0&0&1 \\ 1&0&0&1&0&1&1&0&0&0&1&1&0&1&0&0&1&0&1&0&0&0&0&0 \\ 0&1&0&1&1&0&0&0&0&0&0&0&0&1&0&1&0&1&1&0&0&0&0&1 \end{bmatrix}.$$

Step 4: Using the soft restricted difference product, we find the positive difference max-row soft matrix and the negative difference max-row soft matrix as

$$[e_{i1}] = M_r([d_{ij}]\widetilde{\setminus}_r[c_{ij}]) = M_r([d_{ij}]\widetilde{\cap}[c_{ij}]^{c_C}) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and

$$[f_{i1}] = M_r([c_{ij}]\widetilde{\setminus}_r[d_{ij}]) = M_r([c_{ij}]\widetilde{\cap}[d_{ij}]^{c_{A\cap B}}) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

Step 5: We obtain the positive decision soft matrix and the negative decision soft matrix as, respectively

$$[g_{i1}] = [e_{i1}]\widetilde{\setminus}[f_{i1}] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } [h_{i1}] = [f_{i1}]\widetilde{\setminus}[e_{i1}] = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

Step 6: Since the optimum set of U is $opt(U) = \emptyset$, we take suboptimum set of U instead of the optimum set of U. Then, we obtain

$$sub - opt(U) = \{u_1, u_5\}$$

which u_1 and u_5 are optimum business areas that can make benchmarking agreement of the companies X and Y.

Benchmarking Process:

1. Determine partner for benchmarking: the company Y
2. Construct benchmarking team by specifying tasks: three benchmarkers
3. Determine topics of benchmarking: the business areas u_1, u_2, u_3, u_4, u_5 and u_6
4. Obtain necessary data and analyze: the soft sets F_A, G_B and H_C
5. Apply method and evaluate results: soft difference max-row decision making method

5.2. An example on benchmarking problem according to the focused topic

Suppose that a company X, which manufactures electronic devices, wants to benchmark company's latest version computer with the latest version computers of five companies which are sold cheaper. $U = \{u_1, u_2, u_3, u_4, u_5\}$ is the set of the latest version computers of five companies. The company determines the set of benchmarking parameters as $E = \{e_1 = quality\ of\ material, e_2 = product\ design, e_3 = product\ advertising, e_4 = technological\ specifications, e_5 = service\ facilities, e_6 = product\ distribution\}$. Also, the company appoints four benchmarkers to make benchmarking agreement with one of these companies.

First and second benchmarkers investigate the latest version computers of five companies that have features *better* than the latest version computer of company X.

Third and fourth benchmarkers investigate the latest version computers of five companies that have features *worse* than the latest version computer of company X.

When each benchmarker selects the parameters from the parameter set E, we can apply the soft difference max-row decision making algorithm as follows:

Step 1: The benchmarkers determine the parameter subsets as $A = B = C = D = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, respectively. As a result of research, they obtain the soft sets as

$F_A = \{(e_1, \{u_2, u_3\}), (e_2, \{u_1, u_5\}), (e_3, \{u_2, u_4, u_5\}), (e_4, \{u_1, u_3, u_4\}), (e_5, \{u_1, u_3, u_5\}), (e_6, \{u_2, u_4\})\}$,

$G_B = \{(e_1, \{u_2, u_3\}), (e_2, \{u_1, u_3, u_5\}), (e_3, \{u_1, u_2, u_5\}), (e_4, \{u_1, u_3, u_4\}), (e_5, \{u_1, u_3, u_5\}), (e_6, \{u_2, u_5\})\}$,

$H_C = \{(e_1, \{u_1, u_4, u_5\}), (e_2, \{u_3\}), (e_3, \{u_1, u_3\}), (e_4, \{u_2, u_4, u_5\}), (e_5, \{u_3, u_4\}), (e_6, \{u_1, u_5\})\}$,

$I_D = \{(e_1, \{u_1, u_4, u_5\}), (e_2, \{u_2, u_4\}), (e_3, \{u_1, u_4\}), (e_4, \{u_2, u_4\}), (e_5, \{u_3, u_4\}), (e_6, \{u_1, u_5\})\}$.

Step 2: For the soft sets F_A, G_B, H_C, I_D, the soft matrices are obtained respectively.

$$[F_A] = [a_{ij}] = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}, [G_B] = [b_{ij}] = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$[H_C] = [c_{ij}] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}, [I_D] = [d_{ij}] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Step 3: Using the soft intersection, we obtain the favorable soft matrix and the unfavorable soft matrix as follows, respectively.

$$[a_{ij}]\widetilde{\cap}[b_{ij}] = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \text{ and } [c_{ij}]\widetilde{\cap}[d_{ij}] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Step 4: Using the soft difference product, we find the positive and negative difference max-row soft matrices as

$$[e_{i1}] = M_r(([a_{ij}]\widetilde{\cap}[b_{ij}])\widetilde{\setminus}([c_{ij}]\widetilde{\cap}[d_{ij}])) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix},$$

and

$$[f_{i1}] = M_r(([c_{ij}]\widetilde{\cap}[d_{ij}])\widetilde{\setminus}([a_{ij}]\widetilde{\cap}[b_{ij}])) = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$

Step 5: We obtain the positive decision soft matrix and the negative decision soft matrix as

$$[g_{i1}] = [e_{i1}]\widetilde{\setminus}[f_{i1}] = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } [h_{i1}] = [f_{i1}]\widetilde{\setminus}[e_{i1}] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

Step 6: Consequently, we have an optimum set and a non-optimum set of U as follows:

$$opt(U) = \{u_3\}$$

which u_3 is an optimum company that can be made benchmarking agreement.

$$non - opt(U) = \{u_4\}$$

which u_4 is an unlikely company that should not be made benchmarking agreement.
Benchmarking Process:
1. Determine topic of benchmarking: the latest version computer
2. Construct benchmarking team by specifying tasks: assigned four benchmarkers
3. Determine partners for benchmarking: the companies u_1, u_2, u_3, u_4 and u_5
4. Obtain necessary data and analyze: the soft sets F_A, G_B, H_C and I_D
5. Apply method and evaluate results: soft difference max-row decision making method

6. Conclusion

Soft set theoretical structures are made more practical and efficient through the soft matrices. This shows itself in applications, as well. Relatedly, we introduced novel soft matrix operations called soft difference product, soft restricted difference product, soft extended difference product and soft weak-extended difference product. We constructed a practical novel decision making algorithm via these soft operations. Then, we applied this algorithm to solve three different problems. In the first example, a company determines a candidate which is an optimal for empty position, the other examples are benchmarking problems according to the focused partner and topic. Eventually, our method can successfully be applied to solve problems involving uncertainties.

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