Application of Soft Semi-Open Sets to Soft Binary Topology

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Abstract. This paper introduces an application of soft semi-open sets in soft binary topology. An important outcome of this work is a formal framework for the study of information associated with ordered pairs of soft sets. Five main results concerning binary soft topological spaces are given in this paper.

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1. INTRODUCTION


In continuation, in the present paper binary soft topological structures known as soft weak structures with respect to first coordinate as well as with respect to second coordinate are defined. Moreover some basic results related to this structures are also planted in this paper. The same structures are defined over soft points of binary soft topological structure and related results are also reflected here with respect to ordinary and soft points.

2. PRELIMINARIES

Definition 2.1. Let $X$ be an initial universe and let $E$ be a set of parameters. Let $P(X)$ denote the power set of $X$ and let $A$ be a non empty subset of $E$. A pair $(F, A)$ is called a soft set over $X$, where $F$ is a mapping given by: $A \rightarrow P(X)$. In other some words, a soft over $X$ is a parameterized family subsets of the universe $X$, for $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of $\varepsilon$- approximate elements of the soft set $(F, A)$. Clearly, a soft set is not a set. Let $U_1$, $U_2$ be two initial universe sets and $E$ be a set of parameters. Let $P(U_1)$, $P(U_2)$ denote the power set of $U_1$, $U_2$ respectively. Also, let $A, B, C \subseteq E$.

Definition 2.2. [6] A pair $(F, A)$ is said be binary soft set over $U_1, U_2$ where $F$ is defined below:

$$F: A \rightarrow P(U_1) \times P(U_2), \quad F(e) = (X, Y) \text{ for each } e \in A \text{ such that } X \subseteq U_1, Y \subseteq U_2$$

Definition 2.3. [6] A binary soft set $(F, A)$ over $U_1, U_2$ is called a binary absolute soft set denoted by $A$ if $F(e) = (U_1, U_2)$ for each $e \in A$. 
Definition 2.4. [6] The extended union of two binary soft sets of \((F, A)\) and \((G, B)\) over common \(U_1, U_2\) is the binary soft set \((H, C)\) where 
\[
h(e) = \begin{cases} 
(X_1, Y_1) & \text{if } e \in A - B \\
(X_2, Y_2) & \text{if } e \in B - A \\
(X_1 \cup X_2, Y_1 \cup Y_2) & \text{if } e \in A \cap B
\end{cases} \quad (2.1)
\]
Such that \(F(e) = (X_1, Y_1)\) for each \(e \in A\) and \(G(e) = (X_2, Y_2)\) for each \(e \in B\). We denote it \((F, A) \cup (G, B) = (H, C)\).

Definition 2.5. [6] The restricted intersection of two binary soft sets of \((F, A)\) and \((G, B)\) over common \(U_1, U_2\) is the binary soft set \((H, C)\) where 
\[
c = A \cap B
\]
and for all \(e \in C\) such that \(F(e) = (X_1, Y_1)\) for each \(e \in A\) and \(G(e) = (X_2, Y_2)\) for each \(e \in B\). We denote it \((F, A) \cap (G, B) = (H, C)\).

Definition 2.6. [6] Let \((F, A)\) and \((G, B)\) be two binary soft sets over a common \(U_1, U_2\). \((F, A)\) is called a binary soft subset of \((G, B)\) if
(i) \(A \subseteq B\)
(ii) \(X_1 \subseteq X_2\) and \(Y_1 \subseteq Y_2\) such that \(F(e) = (X_1, Y_1)\) or \((X_2, Y_2)\) for each \(e \in A\). We denote it \((F, A) \subseteq (G, B)\).

Definition 2.7. [6] A binary soft set \((F, A)\) over \(U_1, U_2\) is called a binary null soft set, denoted by \(\approx\), \((\varphi, \varphi)\) for each \(e \in A\).

Definition 2.8. [6] The difference of two binary soft sets \((F, A)\) and \((G, A)\) over the common \(U_1, U_2\) is the binary soft set \((H, A)\) where \(H(e) = (X_1 \setminus X_2, Y_1 \setminus Y_2)\) for each \(e \in A\) such that \((F, A) = (X_1, Y_1)\) and \((G, A) = (X_2, Y_2)\).

Definition 2.9. [7] Let \(\tau_\Delta\) be the collection of soft sets over \(U_1, U_2\) then \(\tau_\Delta\) is said to be a binary soft topology on \(U_1, U_2\) if
(i) \(\varnothing, X \in \tau_\Delta\).
(ii) the union of any member of any binary soft sets in \(\tau_\Delta\) belongs to \(\tau_\Delta\).
(iii) the intersection of any member of any binary soft sets in \(\tau_\Delta\) belongs to \(\tau_\Delta\). Then \((U_1, U_2, \tau_\Delta, E)\) is called a binary soft topological space over \(U_1, U_2\).

Definition 2.10. Let \((F, A)\) be any binary soft subset of a binary soft topological space \((X, Y, \tau, E)\) then \((F, A)\) will be termed soft semi open (written S.S.O.) if and only if there exists soft open set \((O, E)\) such that \(e(O, E) \subseteq (F, E) \subseteq \text{Cl}(O, E)\).

Definition 2.11. Let \((F, A)\) be any binary soft subset of a binary soft topological space \((X, Y, \tau, E)\) then \((F, A)\) will be termed soft semi open (written S.S.O.) if its relative complement is soft semi-open e.g. there exists a soft closed set \((F, E) \subseteq (G, E) \subseteq (F, E)\). The set of all soft binary semi-open soft sets is denoted by \(\text{BSOSS}(X, Y, \tau, E)\) and the set of all binary s-closed sets is denoted by \(\text{BSOSS}(X, Y, \tau, E)\).

3. BINARY SOFT SEMI-SEPARATION AXIOMS

In this section, binary soft semi-separation axioms are discussed with respect to ordinary and soft points in binary soft topological spaces.
Definition 3.1. A binary soft topological space \((\tilde{X}, \tilde{Y}, M, A)\) is called a binary soft \(S_0\) space if for any two binary points \((x_1, y_1), (x_2, y_2) \in (\tilde{X}, \tilde{Y})\) such that \(x_1 < x_2, y_1 < y_2\) there exists binary soft \(s\)-open sets \((F_1, A)\) and \((F_2, A)\) which behaves as
\[(x_1, y_1) \in (F_1, A), (x_2, y_2) \notin (F_1, A)\] or \[(x_2, y_2) \in (F_2, A)\] and \((x_1, y_1) \notin (F_2, A)\).

Definition 3.2. A binary soft topological space \((\tilde{X}, \tilde{Y}, M, A)\) is called a binary soft \(S_1\) space if for any two binary points \((x_1, y_1), (x_2, y_2) \in (\tilde{X}, \tilde{Y})\) such that \(x_1 < x_2, y_1 < y_2\) if there exists binary soft \(s\)-open sets \((F_1, A)\) and \((F_2, A)\) which behaves as \((x_1, y_1) \in (F_1, A)\) and \((x_2, y_2) \notin (F_1, A)\) or \((x_2, y_2) \in (F_2, A)\) and \((x_1, y_1) \notin (F_2, A)\).

Definition 3.3. Two binary soft \(s\)-open sets \((F, A), (G, A)\) and \((H, A), (I, A)\) are said to be is joint if \(((F, A) \cap (H, A), (G, A) \cap (I, A)) = (\Phi, \Phi)\) and \((G, A) \cap (I, A) = (\Phi, \Phi)\).

Definition 3.4. A binary soft topological space \((\tilde{X}, \tilde{Y}, M, A)\) is called a binary soft \(S_2\) space if for any two binary points \((x_1, y_1), (x_2, y_2) \in (\tilde{X}, \tilde{Y})\) such that \(x_1 < x_2, y_1 < y_2\) if there exists binary soft \(s\)-open sets \((F_1, A)\) and \((F_2, A)\) which behaves as \((x_1, y_1) \in (F_1, A)\) and \((x_2, y_2) \in (F_2, A)\) and moreover \((F_1, A)\) and \((F_2, A)\) are disjoint that is \((F_1, A) \cap (F_2, A) = (\Phi, \Phi)\).

Definition 3.5. A binary soft topological space \((\tilde{X}, \tilde{Y}, M, A)\) is called a binary soft \(S_2\) space if for any two binary points \((x_1, y_1), (x_2, y_2) \in (\tilde{X}, \tilde{Y})\) such that \(x_1 < x_2, y_1 < y_2\) if there exists binary soft \(s\)-open sets \((F_1, A)\) and \((F_2, A)\) which behaves as \((x_1, y_1) \in (F_1, A)\) and \((x_2, y_2) \in (F_2, A)\) and moreover \((F_1, A)\) and \((F_2, A)\) are disjoint that is \((F_1, A) \cap (F_2, A) = (\Phi, \Phi)\).

Definition 3.6. A binary soft topological space \((\tilde{X}, \tilde{Y}, \tau \times \sigma, A)\) is called a binary soft \(s\)-\(T_0\) with respect to the first coordinate if for every pair of binary points \((x_1, \alpha), (y_1, \alpha)\) there exist \(((F, A), (G, A)) \in \tau \times \sigma\) with \(x_1 \notin (F, A), y_1 \notin (F, A), \alpha \in (G, A)\). Where \(s\)-open \((F, A)\) in \(\tau\) and \(s\)-\(T_0\) of \((G, A)\) in \(\sigma\).

Definition 3.7. A binary soft topological space \((\tilde{X}, \tilde{Y}, \tau \times \sigma, A)\) is called a binary soft \(s\)-\(T_0\) with respect to the second coordinate if for every pair of binary points \((\beta, x_2), (\beta, y_2)\) there exist \(((F, A), (G, A)) \in \tau \times \sigma\) with \(\beta \in (F, A), x_2 \notin (G, A), y_2 \notin (G, A)\). Where \(s\)-open \((F, A)\) in \(\tau\) and \(s\)-\(T_0\) of \((G, A)\) in \(\sigma\).

Definition 3.8. A binary soft topological space \((\tilde{X}, \tilde{Y}, M, A)\) is called a binary soft \(-T_0\) space if for any two binary soft points \((e_{G1}, e_{H1}), (e_{G2}, e_{H2}) \in (\tilde{X}_A, \tilde{Y}_A)\) such that \(e_{G1} < e_{G2}, e_{H1} < e_{H2}\) there exists binary soft \(-\)open sets \((F_1, A)\) and \((F_2, A)\) which behaves as \((e_{G1}, e_{H1}) \in (F_1, A), (e_{G2}, e_{H2}) \notin (F_1, A)\) or \((e_{G2}, e_{H2}) \in (F_2, A)\) and \((e_{G1}, e_{H1}) \notin (F_2, A)\).

Definition 3.9. A binary soft topological space \((\tilde{X}, \tilde{Y}, M, A)\) is called a binary soft \(-T_1\) space if for any two binary soft points \((e_{G1}, e_{H1}), (e_{G2}, e_{H2}) \in (\tilde{X}_A, \tilde{Y}_A)\) such that
A binary soft topological space \((\tau, e)\) has as open sets \((eG_1, eG_1)\) and \((eG_2, eG_2)\) is a binary soft space \(s\) over \(X\). Then for binary soft pair \(F_1, E\) and \((eG_1, eG_1)\), \((eG_2, eG_2)\) is called a binary soft \(s\) - open set over \(X\) and hence a binary soft \(s\) - open set over \(X\).

**Definition 3.10.** A binary soft topological space \((\tilde{X}, \tilde{Y}, M, A)\) is called a binary soft \(s\) - space if for any two binary soft points \((eG_1, eG_1), (eG_2, eG_2)\) such that \(eG_1 < eG_2, eG_1 < eG_2\) if there exists binary soft \(s\) - open sets \((F_1, A)\) and \((F_2, A)\) which behaves as \((eG_1, eG_1)\) and \((eG_2, eG_2)\) over \(X\). There are two pairs of distinct binary soft points, namely, \(F_1 = \{(e_1, (x_1), \{y_1\})\}, G_1 = \{(e_1, (x_1), \{y_1\})\}\) and \(F_2 = \{(e_2, (x_2), \{y_2\})\}, G_2 = \{(e_1, (x_1), \{y_1\})\}\). Then for binary soft pair \(F_1, G_1\) of points there are binary soft \(s\) - open set \((F_1, E)\) and \((F_2, E)\) such that \(F_1 \notin \{F_2, E\}, F_2 \notin \{F_1, E\}\). This show that \((U_1, U_2, \tau_{\Delta}, E)\) is a binary soft space \(s - T_{\Delta_1}\) space and hence a binary soft \(s - T_{\Delta_2}\) space. Note that \((U_1, U_2, \tau_{\Delta}, E)\) is a binary soft \(s - T_{\Delta_2}\) space.

**Definition 3.12.** A binary soft topological space \((\tilde{X}, \tilde{Y}, \tau \times \sigma, A)\) is called binary soft \(s - T_0\) with respect to first coordinate if for every pair of binary points \((eG_1, \alpha), (eG_1, \alpha)\) \(\exists ((F, A), (G, A)) \tau \times \sigma\) with \(eG_1 \notin \{F, A\}, \alpha \notin (G, A)\). Where \(s\) - open \((F, A)\) in \(\tau\) and \(s - open\) in \((G, A)\) in \(\sigma\).

**Definition 3.13.** A binary soft topological space \((\tilde{X}, \tilde{Y}, \tau \times \sigma, A)\) is called a binary soft \(s - T_0\) with respect to the second coordinate if for every pair of binary points \((\beta, eG_2), (\beta, eG_2)\) there exists \(((F, A), (G, A)) \tau \times \sigma\) with \(\beta \notin \{F, A\}, eG_2 \notin \{G, A\}, eG_2 \notin (F, A)\). Where \(s\) - open \((F, A)\) in \(\tau\) and \(s - open\) \((G, A)\) in \(\sigma\).

4. **Binary Soft Structures with Respect to Ordinary Point**

**Theorem 4.1.** If the binary soft topological space \((\tilde{X}, \tilde{Y}, \tau \times \sigma, A)\) is a binary soft \(s - T_0\), then \((\tilde{X}, \rho, A)\) and \((\tilde{Y}, \sigma, A)\) are soft \(s - T_0\)
A binary soft topological space $(X, Y, \tau \times \sigma, A)$ is a binary soft $s$-$T_0$. Suppose $x_1, x_2 \in X$ and $y_1, y_2 \in Y$ with such that $x_1 < x_2, y_1 < y_2$. Since $(X, Y, \tau \times \sigma, A)$ is a binary soft $s$-$T_0$, accordingly there binary soft $s$-open set $(F, A), (G, A)$ such that $(x_1, y_1) \in (F, A), (G, A)$; $(x_2, y_2) \in (F^c, A), (G^c, A)$ or $(x_1, y_1) \in (F^c, A), (G^c, A)$; $(x_2, y_2) \in (F, A), (G, A))$. This implies that either $x_1 \in (F, A), x_2 \in (F^c, A), y_1 \in (G, A), y_2 \in (G^c, A)$; or $x_1 \in (F^c, A), y_1 \in (G^c, A); y_2 \in (G, A)$. This implies either $x_1 \in (F, A); x_2 \in (F^c, A)$ or $x_1 \in (F, A); x_1 \in (F, A)$ and either $y_1 \in (G, A)$;

$y_2 \in (G^c, A)$ or $y_1 \in (G, A), y_2 \in (G^c, A)$. Since $((F, A), (G, A)) \in \mathcal{S}$ and $((F, A), (G, A)) \notin \mathcal{S}$, hence $(X, \rho, A) and (Y, \sigma, A)$ are soft $s$-$T_0$.

**Theorem 4.2.** A binary soft topological space $(X, Y, \tau \times \sigma, A)$ is binary soft $s$-$T_0$ space with respect to first and second coordinates, then $(X, Y, \tau \times \sigma, A)$ is binary soft $s$-$T_0$ space.

**Proof.** Let $(X, Y, \tau \times \sigma, A)$ is binary soft $s$-$T_0$ space with respect to first and second coordinates. Let $(x_1, y_1), (x_2, y_2) \in X \times Y$ with $x_1 < x_2, y_1 < y_2$. Take $\alpha \in Y$ and $\beta \in X$. Then $(x_1, \alpha), (x_2, \alpha) \in X \times Y$. Since $(X, Y, \tau \times \sigma, A)$ is a binary soft $s$-$T_0$ space with respect to first and second coordinates, by using definition, there exists $s$-open sets $(F, A)$ such that $(G, A) \in \mathcal{S}$ and $(x_1, \alpha) \in (F, A), (x_2, \alpha) \in (F^c, A), \alpha \in (G, A)$. Since $(\beta, y_1), (\beta, y_2) \in X \times Y$, by using arguments and using definition there exist $(H, A), (K, A) \in \mathcal{S}$ and $(x_1, y_1) \in (F, A), (K, A)$

$y_1 \in (K, A), y_1 \in (K, A), \beta \in (H, A)$, therefore, $(x_1, y_2) \in ((F, A), (K, A))$

$(x_2, y_2) \in ((F^c, A), (K^c, A))$. Hence $(X, Y, \tau \times \sigma, A)$ is called a binary soft $s$-$T_0$.

**Theorem 4.3.** A binary soft topological space $(X, \tau, A)$ and $(Y, \alpha, A)$ are soft $s$-$T_1$ spaces if and only if the binary soft topological space $(X, Y, \tau \times \alpha, A)$ is a soft binary $s$-$T_1$.

**Proof.** Suppose $(X, \tau, A)$ and $(X, \alpha, A)$ are soft $s$-$T_1$ space.

Let $(x_1, y_1), (x_2, y_2) \in X \times Y$ with $x_1 < x_2, y_1 < y_2$ since $(X, \tau, A)$ is a soft $s$-$T_1$, there exists soft $s$-open set such that $(F, A), (G, A) \in \mathcal{S}$ and $x_1 \in (F, A)$ and $x_2 \in (G, A)$ such that

$x_1 \notin (G, A)$ and $x_2 \notin (F, A)$ also, since $(y_1, \alpha, A)$ is soft $s$-$T_1$ space there exists soft $s$-open set such that $(H, A), (I, A) \in \mathcal{S}$ and $y_2 \in (I, A)$ such that

$y_1 \notin (I, A)$ and $y_2 \notin (H, A)$ thus $(x_1, y_1) \in ((F, A), (H, A))$ and $(x_2, y_2) \in ((G, A), (I, A))$ with $(x_1, y_1) \in ((F^c, A), (I^c, A))$ and $(x_1, y_1) \in ((F, A), (H, A))$.

This implies that $(X, Y, \tau \times \alpha, A)$ is a soft binary $s$-$T_1$ conversely assume that $(X, Y, \tau \times \alpha, A)$ is a soft binary $s$-$T_1$, let $x_1 \in X$ and $y_1 \in Y$ such that $x_1 < x_2, y_1 < y_2$.

therefore $(x_1, y_1), (x_2, y_2) \in X \times Y$ since $(X, Y, \tau \times \alpha, A)$ is a soft $s$-$T_1$, there exists soft $s$-open set $(F, A), (G, A) \in \mathcal{S}$ and $x_1 \in (F, A), (G, A)$ such that $(x_1, y_1) \in ((F^c, A), (I^c, A))$ and $(x_2, y_2) \in ((F, A), (H, A))$. Therefore $x_1 \in (F, A), x_2 \in (F, A)$.
and \(x_1 \in (H^c, A)(I^c, A)\) and \(x_2 \in (F^c, A)\) and \(y_1 \in (G^c, A)\) and \(y_1 \in (I^c, A)\) and \(y_2 \in (G^c, A)\) since \((F, A), (I, A) \in \tau \times A\) we have \((F, A), (H, A) \in \tau\) and \((G, A), (I, A) \in \tau\), this prove that 

\((\bar{X}, \tau, A)\) and \((\bar{X}, \alpha, A)\) are soft \(s\)-\(T_1\) space.

**Theorem 4.4.** A binary soft topological space \((\bar{X}, \bar{Y}, M, A)\) is a binary soft \(s - T_1\) space if and only if the binary soft point \(\varphi(X) \times \varphi(Y)\) is binary soft closed.

**Proof.** Suppose that \((\bar{X}, \bar{Y}, M, A)\) is a binary soft \(s - T_1\) space. Let \((x, y) \in X \times Y\). Let \(\{(x), \{y\}\} \in \varphi(X) \times \varphi(Y)\). We shall show that \(\{(x), \{y\}\}\) is binary soft closed. It is sufficient to show that \(\{x, y\}\) is a soft neighborhood of \((a, b)\) this implies that \(\{(x), \{y\}\}\) is binary soft closed. Conversely, suppose that \(\{(x), \{y\}\}\) is binary soft closed for every \((x, y) \in X \times Y\). Suppose \((x_1, y_1), (x_2, y_2) \in X \times Y\) with \(x_1 < x_2\), \(y_1 < y_2\). Therefore \((x_2, y_2) \in \{(x_1), \{y_1\}\}\) and \((x_1, y_1) \in \{(x_2), \{y_2\}\}\) is binary soft closed also \((x_1, y_1) \in \{(x_2), \{y_2\}\}\) and \((x_1, y_1) \in \{(x_2), \{y_2\}\}\) is binary soft set. Also \((x_1, y_1) \in \{(x_2), \{y_2\}\}\) and \((x_1, y_1) \in \{(x_2), \{y_2\}\}\) is binary soft set. This shows that \((\bar{X}, \bar{Y}, M, A)\) is a binary soft \(s - T_1\) space.

**Theorem 4.5.** A binary soft topological space \((\bar{X}, \tau, A)\) and \((\bar{Y}, \sigma, A)\) are soft \(T_2\) spaces if and only if the binary soft topological space \((\bar{X}, \bar{Y}, \tau \times \sigma, A)\) is soft binary \(s - T_2\).

**Proof.** Suppose that \((\bar{X}, \tau, A)\) and \((\bar{Y}, \sigma, A)\) are soft \(T_2\) spaces. Let \((x_1, y_1), (x_2, y_2) \in X \times Y\) with \(x_1 < x_2\), \(y_1 < y_2\). Since \((\bar{X}, \tau, A)\) is soft \(T_2\) space, there exist soft open sets such that \((F, A), (G, A) \in \tau\) and \((x_1, y_1), (F, A)\) and \((x_2, y_2), (G, A)\) also, since \((\bar{Y}, \sigma, A)\) is soft \(T_2\) space, there exist disjoint soft \(T_2\) sets such that \((H, A), (I, A) \in \sigma\) and \((y_1, y_2), (H, A)\) and \((y_2, y_2), (I, A)\) such that \((x_1, y_1), \notin (H, A)\) and \((x_2, y_2), \notin (H, A)\) and \((y_1, y_2), \notin (I, A)\) and \((y_2, y_2), \notin (I, A)\). Thus \((x_1, y_1), \notin (F, A)\) and \((x_2, y_2), \notin (G, A)\) and \((x_1, y_1), \notin (G^c, A)\) and \((x_2, y_2), \notin (I^c, A)\). Since \((F, A)\) and \((G, A)\) are disjoint, \((F, A) \cap (H, A) = (\phi, \phi)\). This implies that \((\bar{X}, \bar{Y}, \tau \times \sigma, A)\) is soft binary \(s - T_2\). Conversely assume that \((\bar{X}, \bar{Y}, \tau \times \sigma, A)\) is soft binary \(s - T_2\). Let \((x_1, x_2) \in X \times Y\) such that \(x_1 < x_2\), \(y_1 < y_2\). Therefore \((x_1, y_1), (x_2, y_2) \in X \times Y\). Since \((\bar{X}, \bar{Y}, \tau \times \sigma, A)\) is soft binary \(s - T_2\), there exist soft binary open sets \((H, A), (I, A) \in \tau \times \sigma\) and \((\bar{x}_1, \bar{y}_1), \notin (F, A)\) and \((\bar{x}_2, \bar{y}_2), \notin (F^c, A)\) and \((\bar{x}_1, \bar{y}_1), \notin (G^c, A)\) and \((\bar{x}_2, \bar{y}_2), \notin (G, A)\) and \((\bar{x}_1, \bar{y}_1), \notin (I^c, A)\) and \((\bar{x}_2, \bar{y}_2), \notin (I, A)\). Since \((F, A), (G, A) \in \tau \times \sigma\) and \((I, A) \in \sigma\), this proves that \((\bar{X}, \tau, A)\) and \((\bar{Y}, \sigma, A)\) are soft \(T_2\) spaces.

5. Binary soft structures with respect to soft points.

**Theorem 5.1.** If the binary soft topological space \((\bar{X}, \bar{Y}, \rho \times \sigma, A)\) is a binary soft \(s - T_0\), then \((\bar{X}, \rho, A)\) and \((\bar{Y}, \sigma, A)\) are soft \(T_0\).
Proof. We suppose \((\tilde{X}, \tilde{Y}, \rho, \sigma, A)\) is a binary soft \(s - T_0\) space. Suppose \(e_{G_1}, e_{G_2} \tilde{e} X_A\) and \(e_{H_1}, e_{H_2} \tilde{e} Y_A\) with such that \(e_{G_1} < e_{G_2}, e_{H_1} < e_{H_2}\). Since \((\tilde{X}, \tilde{Y}, \rho, \sigma, A)\) is a binary soft \(s - T_0\) space, accordingly there binary soft \(s - \) open set \(((F, A), (G, A))\) such that \(e_{G_1}, e_{H_1}, \tilde{e}((F, A), (G, A)); e_{G_2}, e_{H_2} \tilde{e}(F^C, A), (G^C, A)\) or \(e_{G_1}, e_{H_1}, \tilde{e}((F^C, A), (G^C, A))\). This implies that either \(e_{G_1}, e_{H_1}, \tilde{e}(F, A), (G, A)); e_{G_2}, e_{H_2} \tilde{e}(F^C, A), (G^C, A)\). This implies either \(e_{G_1}, e_{H_1}, \tilde{e}(F, A), (G, A)); e_{G_2}, e_{H_2} \tilde{e}(F^C, A), (G^C, A)\). Since \((F, A), (G, A)\) is \(\rho \times \sigma\), We have \(s - \) open \((F, A) \tilde{e} \rho\) and \(s - \) open \((F, A) \tilde{e} \sigma\), this proves that \((\tilde{X}, \rho, A)\) and \((\tilde{Y}, \sigma, A)\) are soft \(s - T_0\).

Theorem 5.2. A binary soft topological space \((\tilde{X}, \tilde{Y}, \tau \times \sigma, A)\) is binary soft \(s - T_0\) space with respect to first and second coordinates, then \((\tilde{X}, \tilde{Y}, \tau \times \sigma, A)\) is binary soft \(s - T_0\) spaces.

Proof. Let \((\tilde{X}, \tilde{Y}, \tau \times \sigma, A)\) is binary soft \(s - T_0\) space with respect to first and second coordinates. Let \(e_{G_1}, e_{H_1}\), \((e_{G_2}, e_{H_2}) \tilde{e} X \times Y\) with \(e_{G_1} < e_{G_2}, e_{H_1} < e_{H_2}\). Takes \(\alpha \tilde{e} Y\) and \(\beta \tilde{e} X\). Then \((e_{G_1}, \alpha), (e_{G_2}, \alpha) \tilde{e} X \times Y\) Since \((\tilde{X}, \tilde{Y}, \tau \times \sigma, A)\) is binary soft \(s - T_0\) space with respect to first coordinate, by using the definition, there exists \(s\)-open sets \((F, A)(G, A)\) \(\tilde{e} \tau \times \sigma\) with \(e_{G_1}, \tilde{e}(F, A), e_{G_2}, \tilde{e}(F^C, A)\). Since \((\beta, e_{H_1}), (\beta, e_{H_2}) \tilde{e} X \times Y\), by using the arguments and using the definition, \(s\)-open sets \((H, A)\) \(\tilde{e} \tau \times \sigma\) with \(e_{H_1}, \tilde{e}(K, A), e_{H_2}, \tilde{e}(K^C, A)\). Therefore, \(e_{G_1}, e_{H_1}\) \((F, A), (K, A)\) and \(e_{G_2}, e_{H_2}\) \((F^C, A), (K^C, A)\). Hence \((\tilde{X}, \tilde{Y}, \tau \times \sigma, A)\) is called a binary soft \(s - T_0\).

Theorem 5.3. A binary soft topological space \((\tilde{X}, \tilde{Y}, \tau, A)\) and \((\tilde{Y}, \sigma, A)\) are soft- \(T_3\) spaces if and only if the binary soft topological space \((\tilde{X}, \tilde{Y}, \tau, A)\) is soft binary \(s-T_1\) space

Proof. Suppose \((\tilde{X}, \tau, A)\) and \((\tilde{Y}, \sigma, A)\) are soft- \(T_1\) spaces. Let \(e_{G_1}, e_{H_1}\), \((e_{G_2}, e_{H_2}) \tilde{e} X \times Y\) with \(e_{G_1} < e_{G_2}, e_{H_1} < e_{G_2}\). Since \((\tilde{X}, \tau, A)\) is soft \(s-T_1\) space, there exists \(s\)-open sets such that \((F, A)\), \((G, A)\) \(\tilde{e} \tau\), \(e_{G_1} \tilde{e} (F, A)\) and \(e_{G_2} \tilde{e} (F, A)\) such that \(e_{G_1} \not\in (G, A)\) and \(e_{G_2} \not\in (G, A)\). Also, since \((\tilde{Y}, \sigma, A)\) is soft \(s-T_1\) space, \((I, A)\) \(\tilde{e} \sigma\), \(e_{H_1} \tilde{e} (H, A)\) and \(e_{H_2} \tilde{e} (I, A)\) such that \(e_{H_1} \not\in (I, A)\) and \(e_{H_2} \not\in (H, A)\). Thus \(e_{G_1}, e_{H_1}\) \((F, A), (H, A)\) and \(e_{G_2}, e_{H_2}\) \((G, A), (I, A)\) with \((e_{G_2}, e_{H_2}) \tilde{e} (G^C, A), (F^C, A)\) and \((e_{G_2}, e_{H_2}) \tilde{e} (F^C, A), (H^C, A)\). This implies \((\tilde{X}, \tilde{Y}, \tau, A)\) that is soft binary \(s - T_1\). Conversely assume that is soft binary \(s-T_1\). Let \(e_{G_1}, e_{H_1} \tilde{e} X \times Y\) such that \(e_{G_1}, e_{H_1} \tilde{e} Y\) is soft \(s-T_1\) there exists \(s\)-open sets \((F, A)(G, A)\) and soft \(s\)-open sets \((H, A)(I, A)\) \(\tau \times \sigma\), \((e_{G_1}, e_{H_1}) \tilde{e} (F, A), (G, A)\) and \((e_{G_2}, e_{H_2}) \tilde{e} (H, A), (I, A)\) such that \(e_{G_1}, e_{H_1} \tilde{e} (H^C, A), (F^C, A)\) and \((e_{G_2}, e_{H_2}) \tilde{e} (F^C, A), (G^C, A)\). Therefore \((e_{G_1}, e_{H_1}) \tilde{e} (F, A), e_{G_2} \tilde{e} (H, A)\) and \(e_{G_1} \tilde{e} (F^C, A)\)
and $e_{G_2} \in (F^c, A)$ and $Y \in (G^c, A)$ and $e_{H_1} \in (I, A)$ and $e_{H_1} \in (I^c, A)$ and $e_{H_2} \in (G^c, A)$ since $(F, A)(G, A)\sim (\tau \times \sigma)$, we have $(F, A)(G, A)\sim \tau$ and $(G, A)(I, A)\sim \sigma$. This proves that $(X, \tau, A)$ and $(X, \sigma, A)$ are soft $s$-T$_1$ space.

**Theorem 5.4.** A binary soft topological space $(X, Y, M, A)$ is binary soft $s$-T$_1$ space if and only if every binary soft point $\varphi(X) \times \varphi(y)$ is binary soft $s$-closed.

**Proof.** Suppose that $(X, Y, M, A)$ is binary soft $s$-T$_1$ space. Let $(x, y) \in X \times Y$. Let $\{x\}, \{e_H\}$ $\not\in \varphi(X)$ we shall show that $\{x\}, \{e_H\}$ is binary soft $s$-closed. It is sufficient to show that $\{x\} \cup \{e_H\} \in \text{binary soft } s$-open. Let $(a, b) \in \{x\} \cup \{e_H\}$. This implies that $a \not\in \{x\}$ and $b \not\in \{e_H\}$. Hence $a \not\in e_G$ and $b \not\in e_H$. That is $(a, b)$ is distinct binary soft points of $X \times Y$. Since $(X, Y, M, A)$ is binary soft $s$-T$_1$ space, there exists binary soft open sets $((F, A), (G, A))$ and $(H, A), (I, A)$ such that $(a, b) \in ((F, A), (G, A))$ and $(x, y) \in (H, A), (I, A)$ such that $(a, b) \in ((F, A), (G, A))$ and $(e_G, e_H) \in ((F^c, A), (G^c, A))$. Therefore, $(F, A), (G, A) \subseteq \{e_H\}$, $(e_G\} \in \text{is a soft neighborhood of } (a, b)$. This implies that $(\{e_H\}, \{x\})$ is binary soft $s$-closed. Conversely, suppose that $(\{e_H\}, \{e_H\})$ is binary soft $s$-closed for every $(e_G, e_H)$. Suppose $(e_G, e_H), (e_G, e_H) \in X \times Y$ with $e_{G_1} < e_{G_2}$, $e_{H_1} < e_{H_2}$. is a binary soft $s$-open. Also $(e_{G_1}, e_{H_2}) \notin \{e_G\}, \{e_H\}$ and $(\{e_G\}, \{e_{H_1}\})$ is binary soft $s$-open set. Also $(e_{G_1}, e_{H_1}), (e_{G_2}, e_{H_2})$ and $(\{e_G\}, \{e_{H_1}\})$ is binary soft $s$-open set. This shows that binary soft $s$-T$_1$ space is binary soft $s$-open set. Also $(e_{G_1}, e_{H_1}), (e_{G_2}, e_{H_2})$ and $(\{e_G\}, \{e_{H_1}\})$ is binary soft $s$-open set. This shows $(X, Y, M, A)$ is a binary soft $s$-T$_1$ space.

**Theorem 5.5.** A binary soft topological space $(X, \tau, A)$ and $(Y, \sigma, A)$ are soft $s$-T$_2$ spaces if and only if the binary soft topological space $(X, Y, \tau \times \sigma, A)$ is soft binary $s$-T$_2$ space.

**Proof.** Suppose $(X, \tau, A)$ and $(Y, \sigma, A)$ are soft $s$-T$_2$ spaces. Let $(e_{G_1}, e_{H_1}), (e_{G_2}, e_{H_2}) \in X \times Y$ with $e_{G_1} < e_{G_2}, e_{H_1} < e_{H_2}$. since $(X, \tau, A)$ is soft $s$-T$_2$ spaces and $(Y, \sigma, A)$ is soft $s$-T$_2$ spaces, there exists soft open sets such that $(F, A), (G, A) \sim \tau, e_{G_1}, e_{H_1} \in (F, A)$ and $e_{G_2}, e_{H_2} \in (G, A)$ such that $e_{G_1} \notin (G, A)$ and $e_{G_2} \notin (F, A)$. Also since $(X, \tau, A)$ is soft $s$-T$_2$ spaces, there exists disjoint soft open sets such that $(H, A), (I, A) \sim \sigma, e_{H_1}, e_{H_1} \in (H, A)$ and $e_{H_2}, e_{H_2} \in (I, A)$ such that $e_{H_1} \notin (I, A)$ and $e_{H_2} \notin (H, A)$ thus $(e_{G_1}, e_{H_1}) \in ((F, A)(G, A))$ and $(e_{G_2}, e_{H_2}) \in ((G, A)(I, A))$, with $(e_{G_1}, e_{H_1}) \in ((F^c, A)(I^c, A))$ and $(e_{G_2}, e_{H_2}) \in ((F^c, A)(I^c, A))$. $(F, A)$ and $(C, A)$ are disjoint $(F, A) \Pi (H, A) = (\varphi, \varphi)$ Also since $(H, A) \Pi (I, A) = (\varphi, \varphi)$. Thus $(F, A) \Pi (H, A), (G, A) \Pi (I, A) = (\varphi, \varphi)$, this implies that we have this implies that $(X, Y, \tau \times \sigma, A)$ is soft binary $s$-T$_2$ space. Conversely assume that $(X, Y, \tau \times \sigma, A)$ is soft binary $s$-T$_2$ space. Let $e_{G_1}, e_{G_2} \in X$ and $e_{H_1}, e_{H_2} \in Y$ such that $e_{G_1} > e_{G_2}, e_{H_1} > e_{G_2}$. Therefore $(e_{G_1}, e_{H_1}) \in X \times Y$ is soft $s$-T$_2$ there exists soft open sets $(F, A)(G, A)$ and there exists binary soft open sets $(H, A)(I, A) \sim \tau, (e_{G_1}, e_{H_1}) \sim (F, A)(G, A)$ and $(e_{G_2}, e_{H_2}) \sim (H, A)(I, A)$ such that $(e_{G_1}, e_{H_1}) \sim (H^c, A)(F, A)$ and $(e_{G_2}, e_{H_2}) \sim (H^c, A)(G^c, A)$. Therefore $(e_{G_1}, e_{H_1}) \sim (F, A)(G, A) \sim (H, A)(I, A)$ and $(e_{G_2}, e_{H_2}) \sim (F^c, A)(G^c, A)$ and
\( e_{H_1}(I, A) \) and \( e_{H_2}(I^c, A) \) and \( e_{H_2}(G^c, A) \) since \((F, A)(G,A)\sim \epsilon \times \sigma\), we have \((F, A)(G,A)\epsilon \tau \) and \((G,A)(I,A)\epsilon \sigma\). This proves that \((\tilde{X}, \tau, A)\) and \((\tilde{X}, \sigma, A)\) are soft \( s-T_2 \) space.

6. CONCLUSION

The soft binary \( S - T_0, s - T_1 \) structure with respect to first and second coordinates are introduced in this paper.

REFERENCES


