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Using a Reliable Method for Higher Dimensional of the Fractional Schrödinger Equation

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Abstract. In this article, we use modified RiemannLiouville derivative and two transformations for converting the (2+1)-dimensional fractional Schrödinger equation into corresponding ordinary differential equation. Then we apply the modified extended direct algebraic method to study the exact complex solutions of the (2+1)-dimensional fractional Schrödinger equation.

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1. Introduction

Recently, The investigation of exact solutions to nonlinear fractional differential equations plays an important role in various applications in physics, fluid flow, engineering, control theory, systems identification, biology, finance, signal processing and fractional dynamics [9]-[12].
The nonlinear fractional Schrödinger equation is one of the most important complex fractional partial differential equations. It is a fundamental equation of fractional quantum mechanics that was discovered by Nick Laskin [10]. Some numerical method have been proposed to obtain approximate solutions for fractional Schrödinger equation, such as Homotopy analysis method [6, 21], Adomian decomposition method [7, 21], and so on.

Jumarie [8] proposed a modified Riemann-Liouville derivative and some important properties for the modified Riemann- Liouville derivative. By using this properties and define two transformations, we can convert nonlinear fractional derivative partial differential equations into its nonlinear integer-order ordinary differential equations.
The modified extended direct algebraic method [14]-[18] is a powerful and efficient method for finding exact solutions of nonlinear differential equations.
In the present work, we would like to apply the modified extended direct algebraic method to establish exact complex solutions for the nonlinear (2+1)-dimensional fractional Schrödinger equation. Some conclusions are presented at the end of the paper.

2. THE MODIFIED RIEMANN-LIOUVILLE DERIVATIVE

In this section, we list some important properties for the modified Riemann- Liouville derivative of order α as follows [16]-[13]:

$$D_t^\alpha f(t) = \begin{cases} \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t (t-\xi)^{-\alpha}(f(\xi)-f(0))d\xi & 0<\alpha<1, \\ (f^n(t))^{\alpha-n} & n\le\alpha<n+1,\ n\ge 1. \end{cases}$$

$$D_t^\alpha(f(t)g(t)) = g(t)D_t^\alpha f(t) + f(t)D_t^\alpha g(t), \tag{2. 1}$$

$$D_t^\alpha t^r = \frac{\Gamma(1+r)}{\Gamma(1+r-\alpha)}t^{r-\alpha}, \tag{2. 2}$$

$$D_t^\alpha f[g(t)] = f'_g[g(t)]D_t^\alpha g(t) = D_g^\alpha f[g(t)](g'(t))^\alpha. \tag{2. 3}$$

3. THE MODIFIED EXTENDED DIRECT ALGEBRAIC METHOD

Suppose that a nonlinear fractional differential equation, in general form, is given by

$$F(u, D_t^\alpha u, u_{x_1}, u_{x_2}, u_{x_3}, \ldots, D_t^{2\alpha}u, u_{x_1x_1}, u_{x_2x_2}, u_{x_3x_3}, \ldots) = 0. \tag{3. 4}$$

Based on the properties of the modified Riemann- Liouville derivative, consider the following the variable transformation

$$u(x_1, x_2, \ldots, x_n, t) = u(\xi), \quad \xi = l_1x_1 + l_2x_2 + \ldots + l_nx_n + \frac{\lambda t^\alpha}{\Gamma(\alpha+1)}$$

where l_i and λ are constants to be determined later, the fractional differential equation (3. 4) is reduced to nonlinear ordinary differential equation1

$$F(u(\xi), \lambda u'(\xi), l_1u'(\xi), \ldots, l_nu'(\xi), \lambda^2u''(\xi), \ldots) = 0 \tag{3. 5}$$

where $"'" = \frac{d}{d\xi}$.
We assume that equation (3. 5) has a solution in the form

$$u(\xi) = \sum_{i=0}^m a_i\phi^i(\xi) + \sum_{i=1}^m b_i\phi^{-i}(\xi) \tag{3. 6}$$

where a_i and b_i $(i = 1, 2, \ldots, m)$ are real constants to be determined later. $\phi(\xi)$ expresses the solution of the auxiliary ordinary differential equation

$$\phi'(\xi) = k + \phi^2(\xi). \tag{3. 7}$$

Equation (3. 7) admits the following solution:
If $k<0$ then

$$\phi(\xi) = -\sqrt{-k}\,tanh(\sqrt{-k}\xi),$$
$$\phi(\xi) = -\sqrt{-k}\,coth(\sqrt{-k}\xi).$$

If $k = 0$ then

$$\phi(\xi) = -\frac{1}{\xi}.$$

If $k > 0$ then

$$\phi(\xi) = \sqrt{k}\, tan(\sqrt{k}\xi),$$

$$\phi(\xi) = -\sqrt{k}\, cot(\sqrt{k}\xi).$$

Integer m in (3. 6), in most cases, will be determined by balancing the highest-order nonlinear terms and the highest order partial derivatives of $u(\xi)$ in equation in (3. 5). Substituting (3. 6) into (3. 5) with (3. 7) then the left hand side of equation (3. 5) is converted into a polynomial in $\phi(\xi)$ equating each coefficient of the polynomial to zero yields a system of algebraic equations for $a_i, b_i, k, a_1, a_2, \ldots, a_n, \lambda$. By solving this system and obtaining the unknown constants and substituting the results into (3. 6) then we obtain the exact solutions of equation (3. 4).

4. Nonlinear (2+1)-dimensional fractional Schrödinger equation

Now we consider the non-linear (2+1)-dimensional fractional Schrödinger equation:

$$i\frac{\partial^\alpha u}{\partial t^\alpha} + pu_{xx} - qu_{yy} + \gamma u|u|^2 = 0,\ ,t > 0,\ 0 < \alpha < 1, \tag{4. 8}$$

where p and q are non-zero constants.
We use transformation

$$u(x, y, t) = u(\xi), \quad \xi = ax + by + \frac{ct^\alpha}{\Gamma(\alpha + 1)} \tag{4. 9}$$

where a, b and c are arbitrary constants. substituting (4. 9) into equation (4. 8) and by using (2. 1)-(2. 3), we get

$$D_t^\alpha u = D_t^\alpha u(\xi) = u'(\xi)D_t^\alpha \xi = cu_\xi,\ u_{xx} = a^2u_{\xi\xi},\ u_{yy} = b^2u_{\xi\xi},$$

then equation (4. 8) is reduced into an ordinary differential equation:

$$icu_\xi + pa^2u_{\xi\xi} - qb^2u_{\xi\xi} + \gamma u|u|^2 = 0. \tag{4. 10}$$

Function u is a complex function so we can write

$$u(\xi) = e^{\frac{ic}{2(qb^2-pa^2)}\xi}w(\xi) \tag{4. 11}$$

where $w(\xi)$ is a real function. then equation (4. 10) is reduced to:

$$c^2w + 4\gamma(pa^2 - qb^2)w^3 + 4(pa^2 - qb^2)^2w_{\xi\xi} = 0. \tag{4. 12}$$

Balancing $w_{\xi\xi}$ and w^3 (the highest-order nonlinear terms and the highest order partial derivatives of $u(\xi)$) in Eq.(4. 12), we compute $m = 1$ so the modified extended direct algebraic method in the form admits the use of the finite expansion

$$w(\xi) = A_1\phi + A_0 + B_1\phi^{-1}. \tag{4. 13}$$

Substituting (4. 13) into Eq.(4. 12) and using (3. 7), collecting the coefficients of ϕ and equal to zero, we have:

$$\begin{aligned}
\phi^3 &: 4(pa^2-qb^2)\gamma A_1^3+8(pa^2-qb^2)^2A_1=0,\\
\phi^2 &: 12(pa^2-qb^2)\gamma A_0A_1^2=0,\\
\phi^1 &: 12(pa^2-qb^2)\gamma A_0^2A_1+12(pa^2-qb^2)\gamma B_1A_1^2+8(pa^2-qb^2)^2A_1k+C^2A_1=0,\\
\phi^0 &: C^2A_0+4(pa^2-qb^2)\gamma A+24(pa^2-qb^2)\gamma AAB=0,\\
\phi^{-1} &: CB+12(pa^2-qb^2)\gamma A_0^2B+12(pa^2-qb^2)\gamma A_1B_1^2+8(pa^2-qb^2)^2B_1k=0,\\
\phi^{-2} &: 12(pa^2-qb^2)A_0B_1^2=0,\\
\phi^{-3} &: 4(pa^2-qb^2)\gamma B_1^3+8(pa^2-qb^2)^2B_1k^2=0.
\end{aligned}$$

By solving this system with Maple, we obtain:
Case ***I***:

$$A_1=\sqrt{\frac{2(qb^2-pa^2)}{\gamma}},\quad A_0=0,\quad B_1=0,\quad k=\frac{-C^2}{8(pa^2-qb^2)^2},$$

and

$$A_1=0,\quad A_0=0,\quad B_1=\frac{c^2}{4(pa^2-qb^2)^2}\sqrt{\frac{qb^2-pa^2}{2\gamma}},\quad k=\frac{-C^2}{8(pa^2-qb^2)^2}$$

Then, we get the following exact complex solution:

$$\begin{aligned}
u_1(x,y,t) &= -e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\frac{c}{2}\sqrt{\frac{1}{\gamma(qb^2-pa^2)}}\\
&\times\tanh\Big(\frac{c}{2\sqrt{2}(pa^2-qb^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})\Big),
\end{aligned}$$

or

$$\begin{aligned}
u_1(x,y,t) &= -e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\frac{c}{2}\sqrt{\frac{1}{\gamma(qb^2-pa^2)}}\\
&\times\coth\Big(\frac{c}{2\sqrt{2}(pa^2-qb^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})\Big),
\end{aligned}$$

where $k=\frac{-C^2}{8(pa^2-qb^2)^2}<0$.

Case ***II***:

$$A_1=-\sqrt{\frac{2(qb^2-pa^2)}{\gamma}},\quad A_0=0,\quad B_1=0,\quad k=\frac{-C^2}{8(pa^2-qb^2)^2},$$

and

$$A_1=0,\quad A_0=0,\quad B_1=\frac{-c^2}{4(pa^2-qb^2)^2}\sqrt{\frac{qb^2-pa^2}{2\gamma}},\quad k=\frac{-C^2}{8(pa^2-qb^2)^2}$$

Then, we get the following exact complex solution:

$$\begin{aligned} u_2(x,y,t) &= e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\frac{c}{2}\sqrt{\frac{1}{\gamma(qb^2-pa^2)}} \\ &\times \tanh(\frac{c}{2\sqrt{2}(pa^2-qb^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})), \end{aligned}$$

or

$$\begin{aligned} u_2(x,y,t) &= e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\frac{c}{2}\sqrt{\frac{1}{\gamma(qb^2-pa^2)}} \\ &\times \coth(\frac{c}{2\sqrt{2}(pa^2-qb^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})), \end{aligned}$$

where $k = \frac{-C^2}{8(pa^2-qb^2)^2} < 0$.

Case ***III***:

$$A_1 = \sqrt{\frac{2(qb^2-pa^2)}{\gamma}}, \quad A_0 = 0, \quad B_1 = \sqrt{\frac{2(qb^2-pa^2)k^2}{\gamma}}.$$

Then, we get the following exact complex solution:

When $k < 0$

$$\begin{aligned} u_3(x,y,t) &= -e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(pa^2-qb^2)k}{\gamma}} \\ &\times[\tanh(\sqrt{-k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})+\coth(\sqrt{-k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)}))]. \end{aligned}$$

When $k > 0$

$$\begin{aligned} u_3(x,y,t) &= e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(qb^2-pa^2)k}{\gamma}} \\ &\times[\tan(\sqrt{k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})+\cot(\sqrt{k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)}))], \end{aligned}$$

or

$$\begin{aligned} u_3(x,y,t) &= -e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(qb^2-pa^2)k}{\gamma}} \\ &\times[\tan(\sqrt{k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})+\cot(\sqrt{k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)}))]. \end{aligned}$$

When $k = 0$

$$u_3(x,y,t) = -e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(qb^2-pa^2)}{\gamma}}\,(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})^{-1}.$$

Case ***IV***:

$$A_1=\sqrt{\frac{2(qb^2-pa^2)}{\gamma}},\quad A_0=0,\quad B_1=-\sqrt{\frac{2(qb^2-pa^2)k^2}{\gamma}}.$$

Then, we get the following exact complex solution:

When $k<0$

$$\begin{aligned}u_4(x,y,t) &= -e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(pa^2-qb^2)k}{\gamma}}\\ &\times[\tanh(\sqrt{-k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})-\coth(\sqrt{-k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)}))],\end{aligned}$$

or

$$\begin{aligned}u_4(x,y,t) &= e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(pa^2-qb^2)k}{\gamma}}\\ &\times[\tanh(\sqrt{-k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})-\coth(\sqrt{-k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)}))].\end{aligned}$$

When $k>0$

$$\begin{aligned}u_4(x,y,t) &= e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(qb^2-pa^2)k}{\gamma}}\\ &\times[\tan(\sqrt{k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})-\cot(\sqrt{k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)}))].\end{aligned}$$

When $k=0$

$$u_4(x,y,t)=-e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(qb^2-pa^2)}{\gamma}}\,(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})^{-1}.$$

Case ***V***:

$$A_1=-\sqrt{\frac{2(qb^2-pa^2)}{\gamma}},\quad A_0=0,\quad B_1=\sqrt{\frac{2(qb^2-pa^2)k^2}{\gamma}}.$$

Then, we get the following exact complex solution:

When $k<0$

$$\begin{aligned}u_5(x,y,t) &= e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(pa^2-qb^2)k}{\gamma}}\\ &\times[\tanh(\sqrt{-k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})-\coth(\sqrt{-k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)}))],\end{aligned}$$

or

$$\begin{aligned}u_5(x,y,t) &= -e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(pa^2-qb^2)k}{\gamma}}\\ &\times[\tanh(\sqrt{-k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})-\coth(\sqrt{-k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)}))].\end{aligned}$$

When $k > 0$

$$u_5(x,y,t) = -e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(qb^2-pa^2)k}{\gamma}} \times[\tan(\sqrt{k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})-\cot(\sqrt{k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)}))].$$

When $k = 0$

$$u_5(x,y,t) = e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(qb^2-pa^2)}{\gamma}}\,(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})^{-1}.$$

Case ***VI***:

$$A_1 = -\sqrt{\frac{2(qb^2-pa^2)}{\gamma}},\quad A_0 = 0,\quad B_1 = -\sqrt{\frac{2(qb^2-pa^2)k^2}{\gamma}}.$$

Then, we get the following exact complex solution:

When $k < 0$

$$u_6(x,y,t) = e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(pa^2-qb^2)k}{\gamma}} \times[\tanh(\sqrt{-k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})+\coth(\sqrt{-k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)}))].$$

When $k > 0$

$$u_6(x,y,t) = -e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(qb^2-pa^2)k}{\gamma}} \times[\tan(\sqrt{k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})+\cot(\sqrt{k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)}))],$$

or

$$u_6(x,y,t) = e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(qb^2-pa^2)k}{\gamma}} \times[\tan(\sqrt{k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})+\cot(\sqrt{k}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)}))].$$

When $k = 0$

$$u_6(x,y,t) = e^{\frac{ic}{2(qb^2-pa^2)}(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})}\sqrt{\frac{2(qb^2-pa^2)}{\gamma}}\,(ax+by+\frac{ct^\alpha}{\Gamma(\alpha+1)})^{-1}.$$

- In all cases, a, b and c are arbitrary constants.

5. Conclusion

In this paper, based on the modified Riemann-Liouville derivative and by using two transformations, the nonlinear (2+1)-dimensional fractional Schrödinger equation turned into a nonlinear ordinary differential equation of integer orders. Then, the modified extended direct algebraic method has been successfully applied to find the exact solutions for the nonlinear (2+1)-dimensional fractional Schrödinger equation.

The method is powerful and is applicable to many nonlinear fractional partial differential equations.

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