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A Numerical Approach of Slip Conditions Effect on Nanofluid Flow over a Stretching Sheet under Heating Joule Effect

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Abstract. In this analysis, we considered the effects of Joule heating and partial slip boundary conditions on time dependent mixed convective nanofluid flow over a stretching sheet along with heat source/sink. The governing model is transformed into the system of nonlinear ODE's by using the well known transformations. In order to calculate the physical quantities of the problem, we use the higher order convergence method, called shotting method followed by Runge-Kutta Fehlberg method. The importance of different physical parameters on velocity, temperature and concentration profiles are calculated numerically. The parameters of engineering interest i.e, skin fraction, Nusselt and Sherwood numbers are also calculated. Finally, we concluded that the velocity profiles decrease by increasing values of A and M. Moreover the variation of temperature, velocity and concentration profiles are analyzed for the different physical parameters.

AMS (MOS) Subject Classification Codes: 35S29; 40S70; 25U09
Key Words: Shooting method; MHD boundary layer flow; nanofluid flow; Joule heating; partial slip conditions.

1. Introduction

The investigation of the boundary layer pseudo plastic fluids has been of a great interest because it has many practical usage in industry such as emulsion coated sheets like photographic film, extrusion of polymer sheets, e tc. In order to examine the rheological assents of fluids, the Navier-Stokes equations are insufficient alone. Therefore, rheological models are implemented to reduced this problem. The description of non-Newtonian fluids does not exist in single constitutive relationship between stress and strain. Due to their industrial and physiological applications, the non-Newtonian fluids have gained tremendous attraction. A plenty of applications in various built-up processing and biological fluids, an interest in boundary layer non-Newtonian fluid is increased significantly. A Few examples are drilling mud, plastic polymer, hot rolling, optical fibers, metal spinning, paper production and cooling of metallic plates. The investigation of the flow due to extending surface in a moving liquid is important in advanced industry. For example, the expulsion of metals and plastics, glass blowing, cooling or drying of papers, etc. The problems of linear stretching sheet for many events of fluid have also been investigated by many researchers. Sakiadis *et al.* [34] examined the boundary layer flow over a stretching surface. Various authors have discussed the different feathers of the flow over a movable plats. Vleggaar [36] studied the boundary layer on the stretching surface almost proportional to the distance from the orifice. Carn [14] studied Newtonian fluid flow layer over a stretching sheet on the uniform stress.

The magnetohydrodynomics of an electrically conducting fluid is an important phenomena used in metallurgical and modern metal-working. The electrical furnace metal can be fused by using the magnetic field and In a nuclear reactor containment boat, the wall of nuclear reactor is cooled down by applying magnetic field. MHD term was first proposed by Swedish electrical engineer, Alfve [5] in 1942. MHD equations are the combination of Maxwell's equations of electromagnetism, continuity equation and Navier-Stokes equations. If the electrically conducting liquid placed in a static magnetic field, the fluid movement pushes currents that create forces on the fluid. Equations that describe the MHD flow are a mixture of continuity equation, Navier Stokes equations and Maxwell's equations in fluid dynamics. Zeb *et al.* [39] we studied the effect of thermal radiation on time dependent fluid flow over a stretching sheet with variable thermal conductivity. Zeb *et al.* [40] studied the effect of thermal radiation and slip boundary condition on time dependent fluid flow over a stretching sheet along with variable thermal conductivity. Hussain *et al.* [20]investigated the impact of thermal radiation on bioconversions model for magnetohydrodynamics squeezing flow of nanofluid with heat and mass transfer between parallel surface.Time dependent MHD oscillating and rotating flow of Maxwell fluid in a cylinder subject to shear stress on the boundary was analyzed by Zafar *et al.* [38]. Mukhopadhyay *et al.* [25] investigated the effects of Joule heating on magnetohydrodynamics Newtonian fluid flow over a stretching sheet by placing between suction and junction. Raju *et al*. [30] analyzed the MHD free convective with pours medium by taking horizontal channel assuming insolated and incompressible bottom wall with the effect of heating joule and viscous dissipation. Ahmad *et al*. [4] examined quasi-linearization for unsteady MHD in heat and mass transfer flow of nanofluid overastreachin sheet. Huichu [19] analyzed the unsteady MHD boundary layer flow over a stretching sheet along with frictional and Ohmic heating. Malik, *et al*. [23] investigated the magnetohydrodynamics stagnation point flow over a stretching sheet by assuming the convective boundary condition. MHD time dependent flow of a burger fluid past a circular cylinder along with porous medium was studied Safadar *et al.* [32] . the chemically reacting of incompressible fluid over an vertical plates was discussed by Awan *et al.* [9]. Sadiq *et al.* [33] discussed the exact solution of unsteady flow of Oldroyed-B fluid placed in a circular cylinder. Awan [8] analysed the exact solution for the time dependent flow of Maxwell fluid is placed in coaxial cylinder. Ali *et al.* [6]

investigated the influence of Magnetohydrodynamic electrically conducting oscillating and Rotating flows of Maxwell Fluids in a porous medium.

Another type of fluid is nanofluids which is measured by dispersing of small sized materials such as nanotubes, nanofibers, nanowires, droplets, nanosheet and nanorods. The nano-fluids are nanoscale colloidal suspensions containing condensed nanomaterials. The study of nanofluids has been a topic of intense research during the last one decade due to their interesting thermophysical properties and anticipated applications in heat transfer. The thermal conductivity of equal sample colloidally stable dispersed of nanofluids were obtained by using different experimental methods reported by the International Nanofluid Property Benchmark Exercise (INPBE). Nanotechnology has been commonly used in engineering since materials with size of nanometers possess unique chemical and physical properties. Tested nanofluids in the above study were based on aqueous and nonaqueous base-fluids and many others. In the above analysis, the data has been taken from most of the organization with in a relativity narrow based ($\pm 10\%$ or below) about the sample mean with little outliers. It is found that the thermal conductivity of nano-fluids enlarges with increase in particle concentration and aspect ratio, as expected from classical theory, among various experimental approaches; small systematic differences in the absolute values of the nano-fluid thermal conductivity are obtained while such differences tend to disappear when the data are normalized to measure thermal conductivity of the base-fluids. Further explanation can be found in work by Jacopo *et al.* [11]. The evaluation of convective boundary conditions on MHD boundary layer nanofluid over a stretching sheet was found by Ishak *et al*. [10]. The steady flow of a third grade fluid in a porous half space was found by Karimi *et al*. [21] via rational Bernstein collocation method.

The phenomena of velocity slip has been discussed under different cases by using non-adherence of the fluid to solid boundary in [37]. Time dependent rotational flow of a second grad fluid with caputo time fractional deriavtive was examined by Raza *et al.* [31]. The viscous fluid is normally sticks to the boundary, for example, particulate fluid, rare field gas and many more which are discussed in [35]. The consequences of the slip condition plays a vital rule in the field of scientific, industrial and biological applications such as the internal cavities and artificial polishing of heart valves [24]. The most important study taken into the account is the slip boundary conditions over stretching sheets were carried out by Anderson [7]. Ibrahim *et al*. [17] analyzed manatohydrodynamics boundary layer flow and heat transfer of nanofluid by assuming permeable stretching sheet taking the effect of slip boundary conditions. Poornima *et al*. [28] find out the effect of radiation on convection boundary layer flow due to a non linear stretching sheet. Rmaa Bhargava and Mania Goyal [12] simulate the consequences of velocity slip on MHD nanofluid with heat generation over a stretching sheet. The event of entropy generation on magnetohydrodynamics nanofluid flow over a stretching sheet under the considering velocity slip condition with heat generation was found out by Govindaraju *et al*. in [16]. Malik *et al*. [22] explained the manatohydrodynamics and mixed convection flow of Eyring-Powell nanofluid by assuming the stretching sheet. Ndeem *et al*. [26] proposed a numerical studied viscoelastic nanofluid for two-dimensional stagnation point flow. They found influence of the embedded parameters. They used viscoelastic nanofluid for the controlling heat transfer from the sheet. Therefore, many ways were taken for improving the thermal conductivity of these fluids through suspension nano/micro or large-sized material particles in the fluid. Duwairi

[15] discussed the influence of Joule heating on the forced convection flow with thermal radiation. Partha *et al*. [27] reported that the events of the radiation on mixed convection heat transfer from an exponentially stretching surface under the consideration of viscous dissipation. Abro *et al*. [2] analyzed MHD generalized burger fluid over a permeable plates. Reddy [29] studied the viscous dissipation and thermal radiation on MHD flow due to a stretching sheet. MHD second grade unsteady flow of heat transfer with porous medium by using caputo-fabrizoi fractional derivative was found by Abro *et al.* [1]. Abelman *et al.* [3]. analyzed the analytical solution of magnatohydrodynomics rotating and oscillating flow of a Maxwell fluid electrically conducting in a porous medium. Bhuiyan *et al*. [13] reported the Joule heating effects on MHD natural convection flows being with viscous dissipation from a horizontal circular cylinder. Ishaq *et al*. [18] examined time dependent MHD flow nanofluid film of an eyring Powell Fluid over a porous Stretching Sheet

From the above literature review, it is confirmed that no attempt has been made to the effect of heat source/sink on time dependent mixed convective nanofluid and heat transfer over a stretching sheet with Joule heating. We have successfully computed the solution of the coupled ordinary differential equations via numerical scheme through shooting method followed by Runge-Kutta Fehlberg method. The variation of different physical aspects are presented through graphs. Also, we obtained the numerical results for local skin fraction, heat transfer rate and Sherwood number by various different parameters (discussed in tables).

2. Mathematical model

Let us consider an unsteady MHD incompressible mixed convective Nano-fluid flow over a stretching sheet along with partial slip condition. The flow is produced by a stretching sheet. The flow is in the region $y > 0$ and subjected to a non-uniform magnetic field of strength $B = B_0\sqrt{1-\gamma_1\bar{t}}$ applied normally to the sheet, B_0 is the initial strength of the magnetic field; see in the Fig 1. The fluid and heat flows are initiated at time zero. The sheet emerges out of a slit at origin $x = 0,\ y = 0$ and moves with non-uniform velocity $u_w(x,\bar{t}) = \frac{a_1 x}{1-\gamma_1\bar{t}}$, where a_1 and γ_1 are positive constants with dimensions of $\bar{t}^{-1}$, and a_1 is the initial stretching rate.

$$\frac{\partial\bar{u}}{\partial x} + \frac{\partial\bar{v}}{\partial y} = 0, \tag{2. 1}$$

$$\frac{\partial\bar{u}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{u}}{\partial x} + \bar{v}\frac{\partial\bar{v}}{\partial y} = \nu\frac{\partial^2\bar{u}}{\partial^2 y} - \frac{\sigma\beta^2\bar{u}}{\rho} + [\beta_T(\bar{T}-\bar{T}_\infty) + \beta_C(C-C_\infty)]g, \tag{2. 2}$$

$$\frac{\partial\bar{T}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{T}}{\partial x} + \bar{v}\frac{\partial\bar{T}}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2\bar{T}}{\partial y^2} + \sigma\frac{B_0^2}{\rho}\bar{u}^2 + \tau[D_B(\frac{\partial C}{\partial y}\frac{\partial C}{\partial y}) + (\frac{D_{\bar{T}}}{\bar{T}_\infty})(\frac{\partial\bar{T}}{\partial y})^2]$$
$$+ \frac{Q}{\rho c_p}(T-T_\infty), \tag{2. 3}$$

$$\frac{\partial C}{\partial\bar{t}} + \bar{u}\frac{\partial C}{\partial x} + \frac{\partial\bar{T}}{\partial y} = D_B(\frac{\partial^2 C}{\partial^2 y}) + \tau[D_B(\frac{D_{\bar{T}}}{\bar{T}_\infty})(\frac{\partial^2\bar{T}}{\partial y^2}) \tag{2. 4}$$

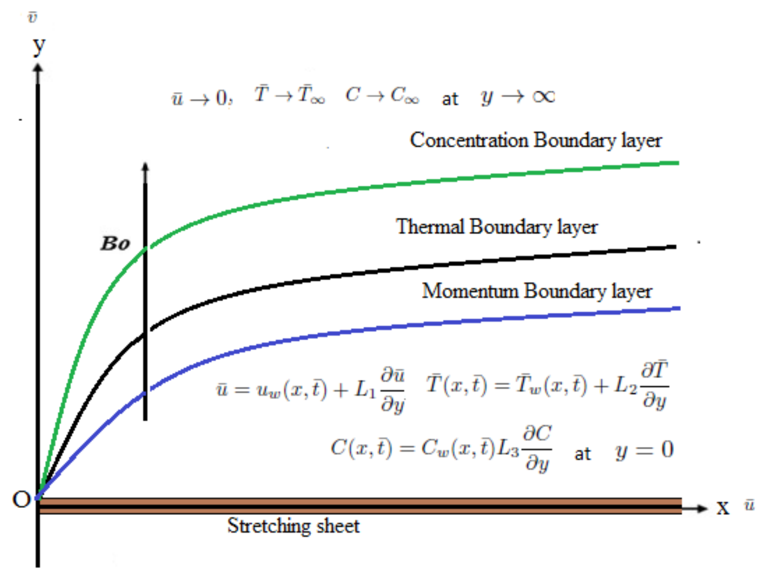

FIGURE 1. Geometry of the problem

subjected to the boundary conditions

$$\bar{u} = u_w(x,\bar{t}) + L_1 \frac{\partial \bar{u}}{\partial y},\ v = 0,\ \bar{T}(x,\bar{t}) = \bar{T}_w(x,\bar{t}) + L_2 \frac{\partial \bar{T}}{\partial y}, C(x,\bar{t}) = C_w(x,\bar{t}) L_3 \frac{\partial C}{\partial y}$$
$$\text{as} \quad y = 0, \bar{u} \to 0, \quad \bar{T} \to \bar{T}_\infty \quad C \to C_\infty \quad \text{at} \quad y \to \infty. \tag{2. 5}$$

Here, $\bar{u}$ and $\bar{v}$ the velocity components along the $\bar{x}$ and $\bar{y}$ - directions, respectively, μ denotes viscosity, ρ represents the density, β_T - the coefficient of volumetric thermal expansion, $\bar{T}$ denotes the temperature, C represents the concentration, β_C denotes coefficient of volumetric concentration expansion, $\bar{T}_w$ and C_w- the temperature and concentration along the stretching sheet, $\bar{T}_\infty$ and C_∞- the ambient temperature and concentration, $D_{\bar{T}}$ - the thermophoresis coefficient, D_B - the Brownian diffusion coefficient, k - the thermal conductivity, and τ represents the ratio of effective heat capacity and heat capacity of the fluid. Using the stream function $\varphi(x,y)$ the continuity equation equation (2. 1) is satisfied identically for the velocity component $\bar{u}$ and $\bar{v}$ specified as

$$\bar{u} = \frac{\partial \psi}{\partial y},\ \bar{v} = -\frac{\partial \psi}{\partial x}. \tag{2. 6}$$

The similarity variables are defined as follows:

$$\eta = \sqrt{\frac{a_1}{v(1-\gamma_1 \bar{t})}}\, y, \tag{2. 7}$$

$$\theta(\eta) = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, \tag{2. 8}$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \tag{2. 9}$$

$$= \sqrt{\frac{a_1 \upsilon}{(1 - \gamma_1 \bar{t})}} x h(\eta). \tag{2. 10}$$

By substituting the above similarity transformation equations (2. 1)–(2. 4) are reduced at the form:

$$h''' + hh'' - h'^2 - Mh' - A(h' + \frac{1}{2}\eta h'') + \lambda_1 \theta + \lambda_2 \phi = 0, \tag{2. 11}$$

$$\frac{1}{Pr}\theta'' + h'\theta - h\theta' - A(\theta + \frac{1}{2}\eta\theta') + \delta\theta + N_b \phi'\theta' + N_t(\theta')^2 + EcM^2(h')^2 = 0, \tag{2. 12}$$

$$\phi'' + Leh'\phi - Leh\phi' - ALe(\phi + \frac{1}{2}\eta\phi') + \frac{N_b}{N_t}(\theta'') = 0. \tag{2. 13}$$

Subject to boundary conditions

$$h(0) = 0,\ h'(0) = 1 + k_1 h''(0), \quad \theta(0) = 1 + k_2\theta'(0), \quad \phi^(0) = 1 + k_3\phi'(0),$$

$$h'(\infty) = 0,\ \theta(\infty) = 0,\ \phi(\infty) = 0,$$

Where g' is the dimensionless velocity, θ denotes the temperature, ϕ represents the concentration η denotes the similarity variables where $A = \frac{\gamma_1}{a}$ is the unsteady parameter, $M = \frac{\sigma(B_o)^2}{\rho a}$ is magnetic parameter, $Pr = \frac{\mu C_p}{k_\infty}$ is Prandtl number, $Re_y = \frac{u_w\sqrt{y}}{\nu}$ Reynolds number, $Le = \frac{\nu}{D_b}$ the Lewis number, $K_r = \frac{k_o}{b}$ the reaction rate parameter, $N_b = \frac{\tau D_B}{\nu}(C - C_\infty)$ the Brownian motion parameter, $\lambda_1 = \frac{Gr}{Re^3/2_x}$, the local thermal Grashof number $\lambda_2 = \frac{Gm}{Re^3/2_x}$, -the local concentration Grashof number, $\bar{Re}_x = \frac{\bar{u}_w\bar{x}}{\nu}$ is the local Reynolds number, $k_1 = L_1\sqrt{\frac{a_1}{1-\gamma_1 t}}$ represents the velocity slip factor, L_1 represents the initial value of velocity slip factor, $k_2 = L_2\sqrt{\frac{a_1}{1-\gamma_1 t}}$ denotes the thermal slip factor, L_2 is the initial value of thermal slip factor, $k_3 = L_3\sqrt{\frac{a_1}{1-\gamma_1 t}}$ represents the mass slip factor, L_3 is the initial value of mass slip factor, if Q is negative it will be a heat sink if Q is positive then it will be heat source. The condition of the no-slip case is attained when $k_1 = k_2 = k_3 = 0$. The quantities C_f, $Nu_{\bar{x}}$ and $Sh_{\bar{x}}$ are define by

$$C_f = \frac{\tau_w}{\rho u_w^2} \tag{2. 14}$$

$$N_{u_x} = \frac{xq_w}{k(\bar{T}_w - \bar{T}_\infty)}, \tag{2. 15}$$

$$Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)}, \tag{2. 16}$$

where τ_w is the skin friction or shear stress along the stretching surface, q_w the heat flux and j_m the concentration flux from the surface and are given by

$$\tau_x = \mu\frac{d\bar{u}}{dy}_{y=0} \quad q_w = [-k\frac{d\bar{T}}{dy}]_{y=0} \quad j_m = [-D_B\frac{dC}{dy}]_{y=0}. \tag{2. 17}$$

where u_w q_m and q_w, are the wall shear stress, mass fluxes and heat transfer respectively. In dimensionless form, In dimensionless form, the reduced local Nusselt and Sherwood numbers can be written as

$$C_f\sqrt{Re_x} = h''(0), \quad \frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0), \quad \frac{Sh_x}{\sqrt{Re_x}} = -\phi'(0). \tag{2. 18}$$

3. SHOOTING METHOD FOR THE PROPOSED MODEL

Eqs (2. 11) – (2. 13) are the system of nonlinear, 3rd order in h, 2nd order in θ and 2nd order in ϕ respectively. First of all these non-linear ODE's are reduce into a system of first order ODE's and then solved by using shooting method. The equations (2. 11) – (2. 13) can be written as:

$$h''' = -hh'' + Mh' + h'^2 + A(h' + \frac{1}{2}\eta h'') - \lambda_1\theta - \lambda_2\phi, \tag{3. 19}$$

$$\theta'' = Pr[-h'\theta + h\theta' + A(\theta + \frac{1}{2}\eta\theta') - \delta\theta - N_b\phi'\theta' - N_t(\theta')^2 - EcM^2(h')^2], \tag{3. 20}$$

$$\phi'' = Le(-h'\phi + h\phi' + A(\phi + \frac{1}{2}\eta\phi')) - \frac{N_b}{N_t}\theta''. \tag{3. 21}$$

To convert these higher order nonlinear ODE's into system of first order ODE's, let

$$h = u_1,\ h' = u_2,\ h' = u_3,\ h'' = u_3 \text{ and } h''' = u_3', \tag{3. 22}$$

$$\theta = u_4,\ \theta' = u_5 \text{ and } \theta'' = u_5' \tag{3. 23}$$

$$\phi = u_6,\ \phi' = u_7 \text{ and } \phi'' = u_7' \tag{3. 24}$$

The nonlinear coupled ODE's are converted into a system o first order simultaneous algebraic form, which can be defined as form a

$$u_1' = u_2, \tag{3. 25}$$

$$u_2' = u_3, \tag{3. 26}$$

$$u_3' = -u_1u_3 + u_2^2 + Mu_2 + A(u_2 + \frac{1}{2}\eta u_3) - \lambda_1 u_4 - \lambda_2 u_6, \tag{3. 27}$$

$$u_4' = u_5, \tag{3. 28}$$

$$u_5' = -u_2u_4 + u_1u_5 + A(u_4 + \frac{1}{2}\eta u_5) - \delta u_4 - N_b u_7 u_5 - N_t u_5^2 - EcMu_2^2 \tag{3. 29}$$

$$u_6' = u_7, \tag{3. 30}$$

$$u_7' = -Leu_2u_6 + Leu_1u_7 + ALe(u_6 + \frac{1}{2}\eta u_7) - \frac{N_b}{N_t}[u_5']. \tag{3. 31}$$

In the above equations, the prime denotes the derivative with respect to η. The boundary conditions are

$$\begin{aligned} &u_1(0) = 0,\ u_2(0) = 1 + k_1u_3(0),\ u_4(0) = 1 + k_2u5(0), \quad u_6(0) = 1 + k_3u7(0) \\ &u_2(\infty) = 0,\ u_4(\infty) = 0,\ u_6(\infty) = 0 \end{aligned} \tag{3. 32}$$

To determine the solution of system of seven ODE's (3. 25) – (3. 31) by using shooting method, seven initial assumptions are required, but in system (3. 32), two initial guesses are given in h, one in θ and one in ϕ and the other three conditions are defined as $\eta \to \infty$.

These three conditions generate result in three unknowns. The subsequent and foremost step of this method is choosing the estimated values of η at ∞. The solution process is initiated with certain initial guesses and finding out the solution of (BVP) including governing model. The method of solution with a new values of η at $\eta \to \infty$ and the method is repeated until two consecutive values of $g''(0)$, $\theta'(0)$ and $\phi'(0)$ are different only after the significant digits. Thus final values of η are considered as $\eta \to \infty$.

4. Results and discussion

The variation of different physical aspects are presented through graphs (as shown in Figs 2–19). The influence of unsteady parameter is shown in Fig: 2 on the velocity and temperature profiles. Fig 3 is for the variation of velocity and temperature profiles for values of M. A decrease in variation of velocity and temperature profiles increases by enlarge values of M. Fig: 4 plotted for the distribution of concentration profile for the distinct values of M and A respectively. The decreases the concentration profile by increasing values of M and A. Figure 5 plotted for the distribution of concentration and temperature profiles for the distinct values of Pr. The temperature profile decreases and concentration profile increases by increasing values of Pr. Figure 6 designated for the velocity and temperature profiles for the different values of λ_1. The increase in the velocity profile and decrease in temperature profile by increasing the values of λ_1. Figure 7 indicates the variation of velocity and temperature profiles for the various values of λ_2. The result has shown the step down the velocity and temperature profiles by the step up values of λ_2. Figure 8 shows the distribution of the temperature profile for the various values of the N_t and N_b. The result has show that the temperature profile decreases by increasing values of thermophoresis N_t and Brownian motion parameters N_b. The Figure 9 shows that the influence of Ecklet number E_c on temperature and concentration profiles. The result shows that the temperature profile increases and reduced the concentration profile by the step up values of Ec. Figure 10 designated for the distribution of the concentration profile for values of L_e on the concentration profile. The result shows that a decrease in concentration profile as the L_e increases. This is due to the fact that there is a decrease in the nanoparticle volume fraction boundary layer thickness with the increase in the Lewis number. Figure 11 shows the variation of the concentration profile for the distinct values of N_b. It is noticed that the concentration profile is reduced by incrementing value in Brownian motion N_b. The effect of the thermophoresis parameter N_t on the concentration of the flow field is show in Figure 12. We observe that the behavior of N_t indicates a cold surface while negative to a hot surface. It is seen that the concentration decreases, as the thermophoresis parameter increases. The variation of velocity slip parameter k_1 on the velocity profile is shown in Figure 13. The result shows that the velocity graph is reduced with the step up values of velocity slip parameter k_1. Figure 14 is plotted for the distinction of the temperature profile for the different values thermal slip parameter k_2. On observing this figure, the temperature graph is reduced as the value of temperature slip parameter k_2 increase. Figure 15 is plotted for the influence of the solutal slip parameter k_3 on the concentration profile. The result has shown that the concentration profile is reduced by the step up value of solutal slip parameter k_3. Figure 16 represents the temperature profile with using different value for the heat source parameter. If we increase the value of the heat source, then the temperature profile increases. Heat source provides extra heat to the sheet which increases its temperature.

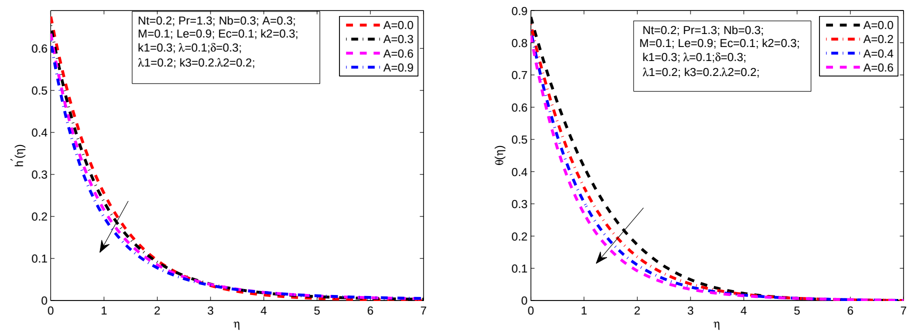

FIGURE 2. The variation of h' and θ for distinct values of A.

This increment is responsible for the increment of the thickness of the thermal boundary layer. Figure 17 is plotted for the temperature profile with the use of different values for the heat sink parameter. If we increase the values of the heat sink, then the temperature profile decreases. This implies that it removes heat from the sheet which decreases thickness of the boundary layer. Figure 18 represents the concentration profile by using different values for the heat source parameter. If we step up the value of the heat source, then the concentration profile decreases. Figure 19 is plotted for the concentration profile with the use of different values for the heat sink parameter. If we increase the value of the heat sink then concentration profile increases. Table 1–2 is prepared for the influence of various parameters on skin friction C_f, Nusselt number Nu_x and Sherwood numbers Sh. Table 1 shows that heat skin friction increases with an increase in M, A, Pr, δ, L_e and λ_1. The skin friction steps up by the incrementing values of N_b. Nu_x increases with an increase in M, δ, and L_e. Nu_x steps down by increasing values of Pr, A, L_e and λ_1. Sherwood number Sh_x increases by the incrementing values of A, Pr, N_b, and λ_1. Sherwood number Sh_x is reduced by increasing values of M, δ and L_e. The Table 2 shows that the skin friction steps up by incrementing the values of λ_2, N_t, k_1 and k_3. Skin friction increments by the incrementing the values of k_2. Nu_x increases with an increase in λ_2, N_t k_2 and Ec. Nu_x reduces by increasing values of k_1 and k_3. Sherwood number Sh_x steps up with an increment in k_1 and k_3. Sherwood number Sh_x decreases by increasing values of λ_2, N_t, k_2 and Ec.

5. CONCLUSIONS AND FUTURE WORK

The present work is the analysis of an unsteady incompressible magnetohydrodynamics boundary layer flow heat transfer and a nanofluid over a stretching sheet under the effect of slip boundary conditions and heating joule in the existence of heat source or sink has been considered. We use similarity transformation for transforming the nonlinear coupled ordinary differential equations. We successfully computed the solution of coupled ordinary differential equations via numerical scheme through shooting method followed by Runge-Kutta Fehlberg method. The behaviour of various parameters on velocity, temperature and

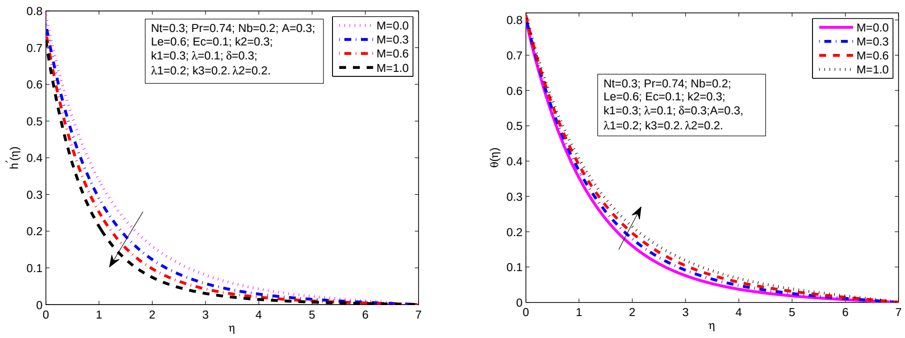

FIGURE 3. The variation of h' and θ for distinct values of M.

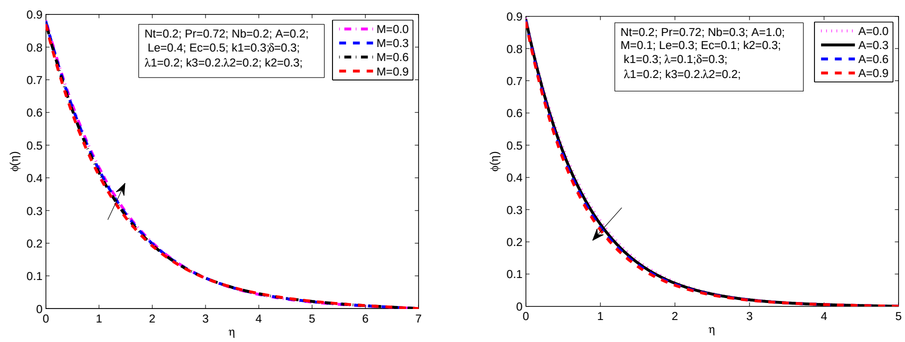

FIGURE 4. The variation of ϕ for distinct values of M and A.

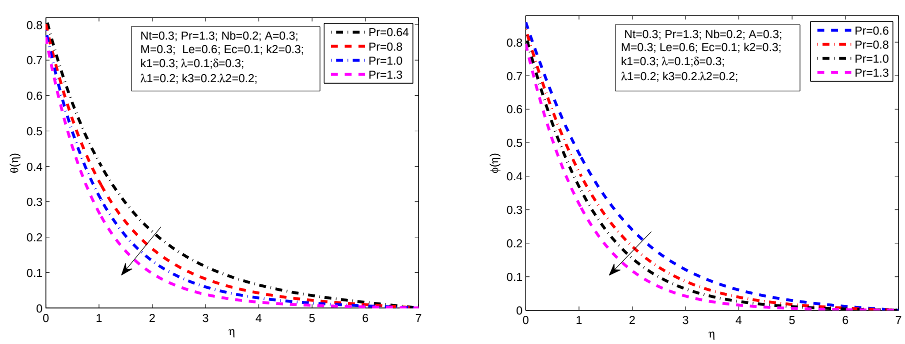

FIGURE 5. The variation of θ and ϕ for distinct values of Pr.

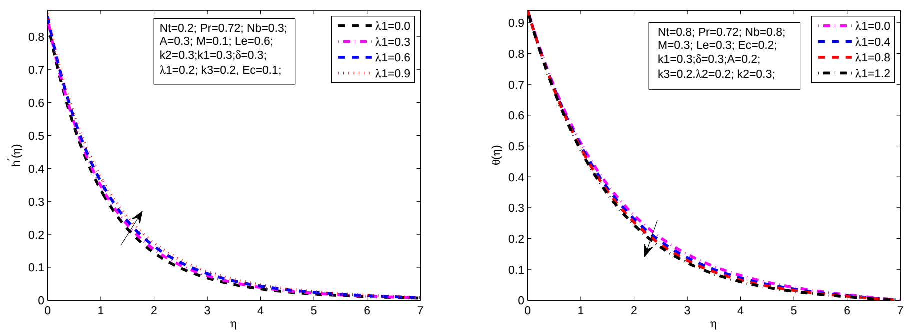

FIGURE 6. The variation of h' and θ for distinct values of λ_1.

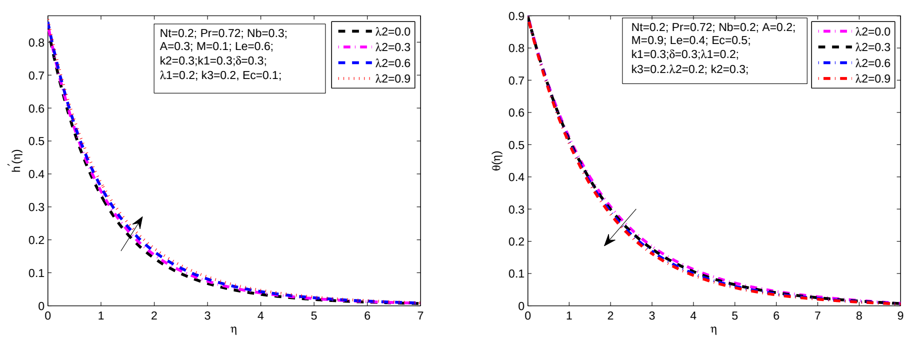

FIGURE 7. The variation of h' and θ for distinct values of λ_2

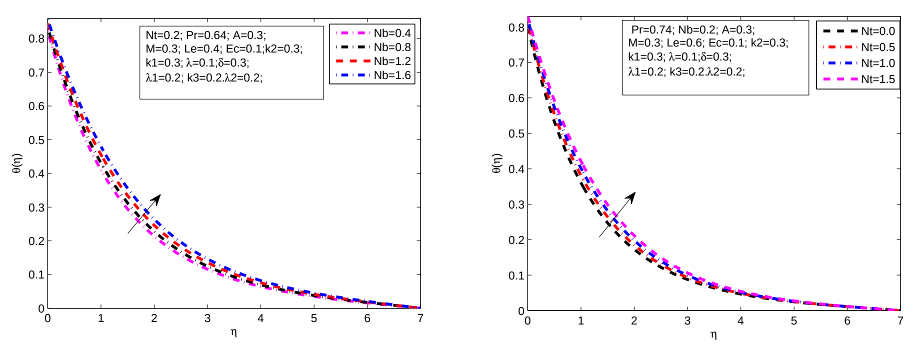

FIGURE 8. The variation of θ for distinct values of N_b and N_t on θ.

concentration profiles are shown graphically. The behaviour of the local skin friction coefficient, local Nusselt number and local Sherwood number are shown numerically through table. The major conclusions are listed bellow;

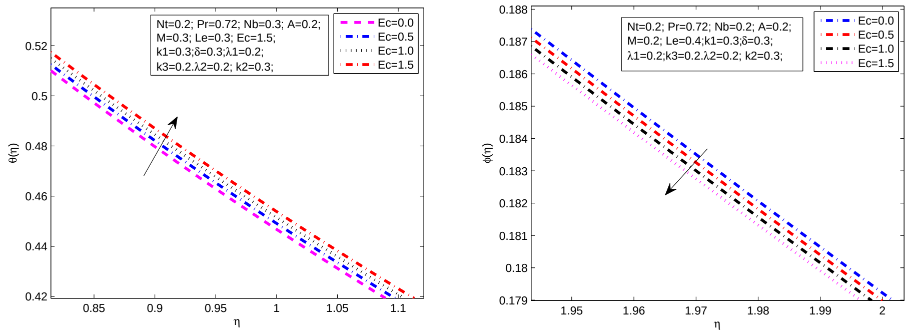

FIGURE 9. The variation θ and ϕ for distinct values of Ec.

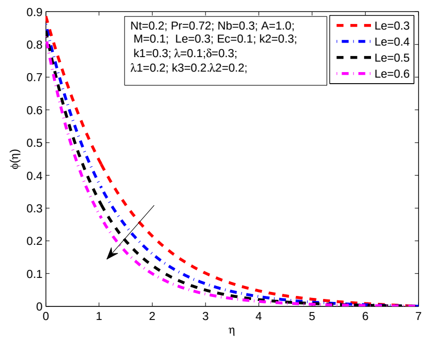

FIGURE 10. The variation of ϕ for distinct values of Le.

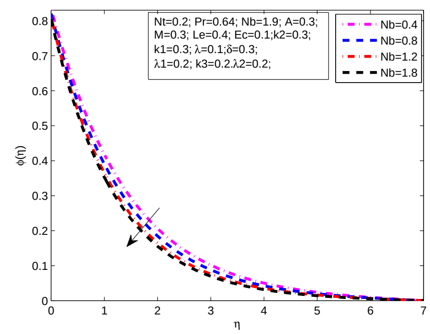

FIGURE 11. The variation of ϕ for distinct values of N_b.

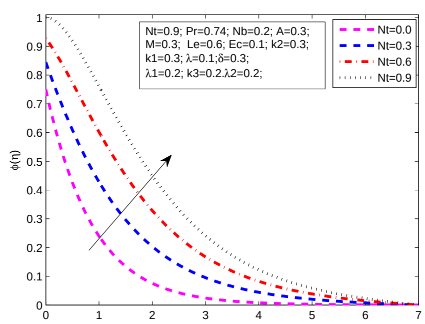

FIGURE 12. The variation of ϕ for distinct values of N_t.

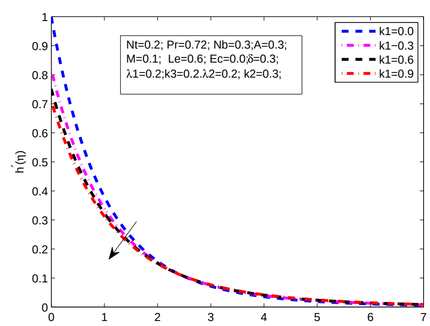

FIGURE 13. The variation of h' for distinct values of k_1.

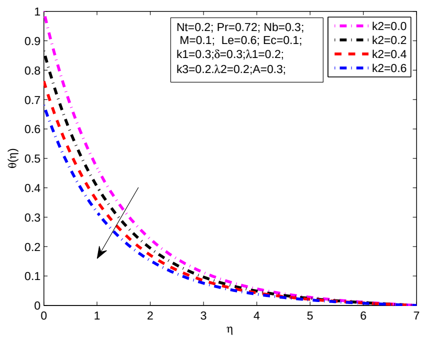

FIGURE 14. Influence of k_2 on θ.

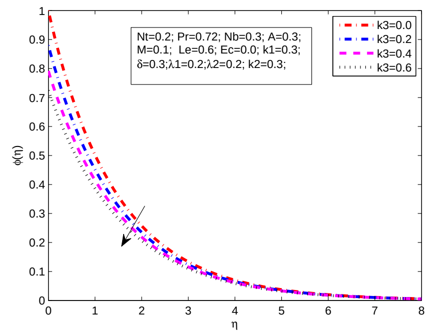

FIGURE 15. The variation of ϕ for distinct values of $k3$.

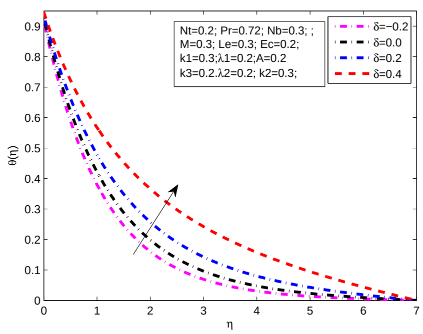

FIGURE 16. The variation of θ for distinct values of δ.

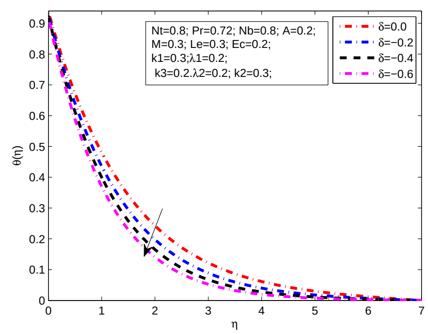

FIGURE 17. The variation of θ for distinct values of δ.

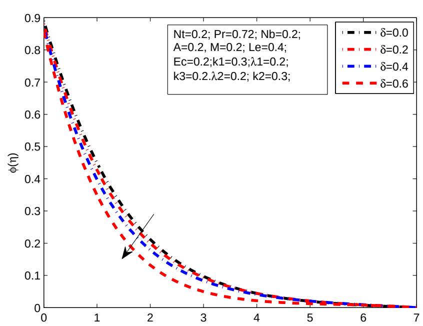

FIGURE 18. The behaviour of δ on ϕ.

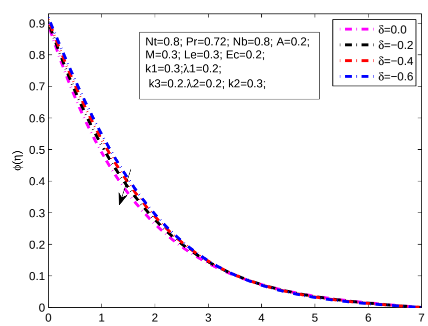

FIGURE 19. The behaviour of δ on ϕ.

A	M	Pr	δ	Le	N_b	λ_1	$-h''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.8	1.0	0.72	0.5	0.5	0.5	0.5	0.12222	0.57535	0.1318
0.0							0.11130	0.48859	0.1543
0.2							0.11417	0.51136	0.1480
0.4							0.11695	0.53340	0.1422
	0.0						0.10736	0.58869	0.1285
	0.2						0.11053	0.58578	0.1292
	0.4						0.11359	0.58300	0.1299
		1.0					0.12099	0.53798	0.1417
		1.3					0.11989	0.50762	0.1495
		1.6					0.11900	0.48595	0.1548
			-0.4				0.12480	0.68052	0.1023
			-0.2				0.12427	0.65881	0.1084
			0.0				0.12372	0.63620	0.1148
			0.2				0.12314	0.61264	0.1214
				0.0			0.12247	0.57527	0.1269
				0.2			0.12232	0.57531	0.1299
				0.4			0.12217	0.57537	0.1328
					0.2		0.12000	0.61117	0.1638
					0.4		0.12189	0.68796	0.1373
					0.4		0.12241	0.66397	0.1282
						0.0	0.13071	0.57135	0.1327
						0.2	0.12730	0.57297	0.1324
						0.4	0.12391	0.57456	0.1320

TABLE 1. Effect of A, M, Pr, δ, Le, N_b and λ_1 on $-h''(0)$, $-\theta'(0)$ and $-\phi'(0)$

- There is a decrease in velocity profile $h'(0)$ with increasing values of A, λ_2 and k_1
- The velocity $h'(0)$ profile increases by increasing values of the λ_1.
- There is a increase in temperature profile $\theta(\eta)$ with the increasing value of M, λ_2, $N_b,\ N_t$.
- There is a decrease in temperature $\theta(\eta)$ profile with the increasing value of Pr, λ_1, Le and k_2.
- By increasing heat sink δ temperature profile $\theta(\eta)$ decrease and alternatively concentration $\phi(\eta)$ profiles are increases.
- The gradient of the temperature increases with the increase in heat source parameter δ and alternatively concentration $\phi(\eta)$ profiles are decreases.
- The concentration profile $\phi(\eta)$ increment with the incrementing values in magnetic parameter M, k_1, N_t, Ec, λ_1 and Pr.
- The concentration profile $\phi(\eta)$ reduced withe the step up values of $N_b,\ Le$, A, k_3 and λ_2.
- In future we will try to implement other numerical method for comparison of our analysis.

λ_2	N_t	k_1	k_2	k_3	Ec	$-h''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.8	1.0	0.72	0.5	0.5	0.5	0.12222	0.57535	0.1318
0.0						0.11801	0.57698	0.1315
0.2						0.12054	0.57600	0.1317
0.4						0.12306	0.57502	0.1319
	0.0					0.11576	0.51269	0.7546
	0.4					0.11650	0.48669	0.6464
	0.8					0.11697	0.46321	0.5599
		0.0				0.12235	0.58113	0.1302
		0.1				0.12222	0.57535	0.1318
		0.2				0.12209	0.59936	0.1335
			0.0			0.17092	0.59901	0.1268
			0.2			0.12222	0.57535	0.1318
			0.3			0.10736	0.56731	0.1336
				0.0		0.10336	0.57051	0.1578
				0.2		0.10558	0.51713	0.1441
				0.4		0.10736	0.56731	0.1336
					0.0	0.10874	0.57244	0.2149
					0.2	0.10637	0.60568	0.1620
					0.4	0.10494	0.62642	0.1298

TABLE 2. Effect of λ_2, N_t, k_1, k_2, k_3, and Ec on $-h''(0)$, $-\theta'(0)$ and $-\phi'(0)$

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