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A Comparison of Single Server and Multiple Server Queuing Models in Different Departments of Hospitals

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Abstract. Queuing theory is the branch of Operations research in applied mathematics and deals with phenomenon of waiting lines. Queuing theory is concerned with the mathematical modeling and analysis of systems which provides service to random demands. In this paper, we have focused the applications of queuing theory in the field of healthcare (hospital) i.e. one of the biological paradigm. The health systems should have an ability to deliver safe, efficient and smooth services to the patients. We considered different departments of health care sector such as patient's registration department, outpatients department and pharmacy department and also observed different processes in the system by queuing models. Different queues and numbers of servers involved in the processes are also observed by using appropriate probability distributions. The arrival process calculated by exponential distributions and service process is measured by Poisson distributions. Single server M/M/1 and multiple server queuing models M/M/2 are used in order to analyze queuing parameters and performance measures of the system. Queuing analysis is done by using queuing simulation technique in order to compute the values of unknown parameters and performance measures. Besides this, strength of relationship between different performance measures is calculated.

AMS (MOS) Subject Classification Codes: 35S29; 40S70; 25U09
Key Words: Queuing Theory, Queuing Analysis, Queuing Simulation, Performance Measures, M/M/1 Queuing Model, M/M/2 Queuing Model.

1. INTRODUCTION

In 1930's Operations Research is taken as a discipline of science and branch of applied mathematics. It is used for applying sufficient analytical methods for assessment making. In Operations Research mechanism, firstly partitions of under study case is taken and then solved by using mathematical analysis. Different analytical methods are used in this area, for example some commonly used are as network analysis, game theory, mathematical logic, queuing theory and simulation etc. But among all of these, most widely and commonly used analytical methods in different field of life is Queuing theory. In this paper we have analyzed the health concerned system by using queuing models. The past of Queuing Theory was nearly 100 years. A Danish Telephonic Engineer A. K. Erlang developed queuing theory, in early 1920's. During studying applications of automatic telephone switching, Erlang was concerned with the capacities and utilization of the equipment and lines. Queuing theory was unmitigated in applications to furnish for a large number of situations, at the end of World War II.. Queuing theory is actually, a study of waiting lines. The theory allows the calculation and derivation of a number of representative measures which includes the estimated number of receiving service, the probability of encounters the system in certain cases, e.g. having to wait for certain time to be served or an available server, empty or full, and average waiting time in the queue. There are two basic approaches to solve the waiting lines phenomena. First one is analytical method; in this method measure of performance is computed using certain queuing models and found the values of arrival rate and service rate. Second method to solve queuing models is simulation. Simulation is used for those queuing models which are extremely difficult or impossible to solve using analytical method. Application of a queuing model is to approximate a real queuing system in a means that it can be analyzed mathematically. There are different types of queuing models, but single server and multiple server queuing models are widely applicable. Queuing models used to determine a number of performance measures of the system, some are:

i) the probability, if the queue is unfilled or filled

ii) Finding possibilities of the system in a particular state.

iii) mean number of customer in queue

iv)mean client time spent in a queue

v) statistical distribution of those numbers

The above permormance measures of queuing models are analyzed through Simulation. Simulation generally imagine that someone working artificially but showing that they deal with real thing. In this situation we have an advantage of applying simulation method because this method does not need to have too much simplifying assumptions. Systems beyond the M/M/1 queuing models often turn out to be analytically inflexible. Then stochastic discrete-event simulation is a useful tool for gaining insight [7]. In this paper, some important measures are taken to model the health concerned situation using simulation and caution about the limitations of adequate error and its effect in lost lives[3]. From birth to death, we are all part of the health care system. We rely on hospitals to get cure and treatment of our illnesses and hurts. Actually, healthcare is perhaps the stage determinant of people's quality of life and longevity [2]. With fast change in healthcare system, new methods of services and facilities used. Application of queuing model has become a prevalent analytical tool for calculating severe financial pressure and wide use of prolonged managerial expertise in changed healthcare situation [9].Mostly hospitals are of two types, "private" and the "public hospitals". Privates hospitals are run by different organizations, companies and different private funding often taken by foreign embassies, insurers or by patients themselves. While hospitals owned by government and governmental financial support are public or civil hospital. There patients get free of charge medical treatment. Khan [5] describe that medical care is an extremely scarce and expensive service in the country. The Government delivery system is not efficient enough to cover the target population. If take an overall look, we find that Pakistan, with about 187 million people in 896,096 sq km, is one of the densely populated countries. Its problems are many and health is one of them. As a result, poor and inadequate health services are acting as obstacles against the overall development of this country. Rosenquist [8] chooses staffing capacity in an outpatient radiology service with a limited waiting area by minimizing cost. He suggests scheduling patients when possible and segregating patients based on expected examination duration. Such measures would reduce variability and decrease expected waiting times. Worthington [10] suggests that increasing service capacity (the traditional method of attempting to reduce long queues) has little effect on queue length because as soon as patients realize that waiting times would reduce, the arrival rate increases, which increases the queue again. Nosek and Wilson [6] reviewd the use of queuing theory in pharmacy applications with particular attention to improving customer satisfaction. Customer satisfaction is improved by predicting and reducing waiting times and adjusting staffing. Green [1] presents the theory of queuing as applied in healthcare. She discussed the relationship amongst delays, utilization and the number of servers; the basic M/M/s model, its assumptions and extensions; and the applications of the theory to determine the required number of servers. Kembe et al.[4] studied the queuing characteristics at the Riverside Specialist Clinic Of The Federal Medical Centre Makurdi was analyzed using a multi-server queuing model and the waiting and service costs determined with a view to determining the optimal service level. data for this study was collected at the riverside specialist clinic for four weeks through observations, interviews and by administering questionnaire. The data was analyzed using TORA optimization software as well as using descriptive analysis.The results of the analysis shown that average queue length, waiting time of patients as well as overutilization of doctors at the clinic could be reduced at an optimal server level of 12 doctors and at a minimum total cost as against the present server level of 10 doctors with high total cost which include waiting and service costs.

2. Materials and Methods

Data are taken for two days of a week from a public hospital of Rawalpindi. Three departments of this hospital are focused which are “patient’s registration department”, “outpatients department” and “pharmacy department”. Data is collected only from patients and doctors and not any other medical personals in outpatient department. The data collection is primary. The method are used for data collection are, “questionnaire”, “direct observation” and “interviews”. The following assumptions are fulfilled by the data for queuing models.

i. It is assumed that the patients’ onset follow a Poisson probability distribution.

ii. Time of inter-arrival of patients is independent and exponentially distributed.

iii. Service time is also exponentially distributed.

iv. It is assumed that patients are served by any server on first-come first-served basis.

v. It is also assumed that no patient will leave the queue without getting service.

vi. The queue is infinite.

vii. For outpatient department doctors were only servers.

viii. Rate of serving was not dependent on the queue length. Serving rate remained moderate despite of queue length.

2.1. Patient's Registration Department and Queuing Model. In any hospital before being paid services from doctors patient must have to register himself from front office i.e. patients registration window. This window is provided a computer slip to the patient of whom department he visit. In public hospital this department starts working in the morning almost 7:00:00 am. In this study it is observed that there are a large number of patients who are waiting early in the morning before opening this window. For this study data is collected for only 35 mints i.e. 7:50:00 am to 8:25:00 am. The basic queuing model observed in this department is multiple-queue multipleservers, and labeled as M/M/s, where M/M represents the Poisson probability distribution of arrivals and departures and s (positive integer) symbolize number of servers. Diagrammatically expressed as:

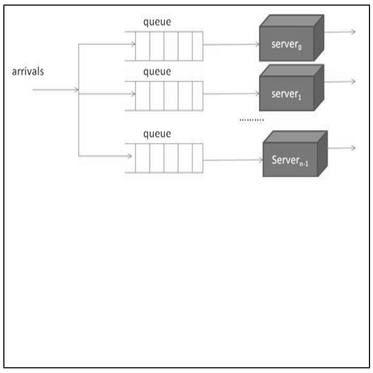

figure1

FIGURE 1. **Multiple queues with multiple servers queuing model**

2.2. Outpatients Department (OPD) and Queuing Model. Some specialty areas of treatment served in outpatient department are: Pediatrics, Surgery, Orthopedics, Joint Replacement and Reconstructive surgery, Cardiology, Dermatology, Chest, Medicine, Eye, Dentistry, ENT, Obstetrics & Gynecology, Neurosciences, Psychiatry, Pre-anesthetic and Pain clinic and Physiotherapy. Dispensaries also provide outpatient department services. Medicine outpatient department is considered for this study. In this public hospitals medicine OPD starts working from 8:00:00 am to 14:00:00 pm during weekdays. In this paper, we have taken observations only for two hours from 9:00:00 am to 11:00:00 am. The basic queuing model observed in this department with queuing process is single-queue with multiple-servers, and labeled as M/M/s, where M/M represents the Poisson probability distribution of arrivals and departures and s (positive integer) symbolize number of servers and under study department observed servers were 2 i.e. s=2s. Schematically represent as:

2.3. Pharmacy Department and Queuing Model. Service in pharmacy department in a hospital is provided under the direction of a professionally competent, legally qualified pharmacist, and from this department all medications are supplied to the nursing units and other services, where special prescriptions are filled for patients in the hospital, for ambulatory patients and out-patients as well. For this study in a public hospital pharmacy department is observed for 45 minutes only i.e. from 12:00:00 pm to 12:45:00 pm. The basic queuing model with basic queuing process observed in this department was single-queue

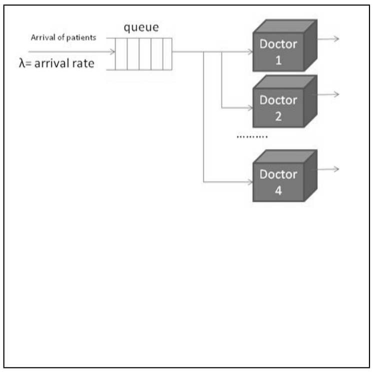

figure 2

FIGURE 2. Multiple queues with multiple servers queuing model

with single-servers, and labeled as M/M/1, where M/M represents the Poisson probability distribution of arrivals and departures and 1 symbolize number of server.

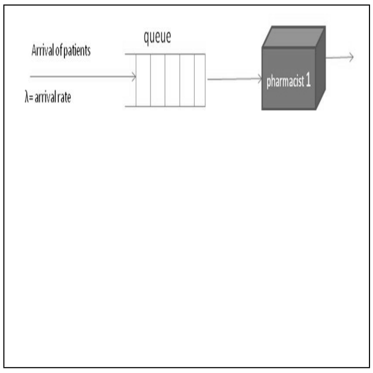

figure 3

FIGURE 3. **Single Queue with Single Server Queuing Model**

2.4. **Parameters and Performance Measures.** Parameters and performance measures of these queuing models are following:

n = total number of patients in a system.
s = number of servers
λ = Arrival rate/hour
μ = Serving rate/hour
$s\mu$= Service rate when $s > 1$
ρ = The probability that the system is busy or utilization factor, calculated as,
Utilization factor= $\lambda/s\mu$
That shows, service competence being utilized on the average arriving patient.
Performance measures were calculated to estimate the

Expected queue length of waiting patients in a queue i.e. L_q and measured as:

$$L_q = \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\mu\lambda}{(\mu s-\lambda)^2}\right] p_0$$

Expected waiting time of the patients in the queue i.e. W_q and measured as

$$W_q = \frac{L_q}{\lambda}$$

Expected queue length of waiting patients in the system i.e. L_s and measured as:

$$L_s = L_q + \frac{\lambda}{\mu}$$

Expected waiting time of the patients in the system i.e. W_s and measured as:

$$W_s = \frac{L_s}{\lambda}$$

3. Analysis of Data

We have used WinQsb software to compute the performance measures using queuing analysis and queuing simulation of single server and multiple server queuing model at a public hospital using arrival rate λ, service rate μ and number of servers. Queuing simulation performed for only patient's registration department.

Table.1 Parameters and performance measures of queuing model using queuing analysis for 1st day.

Performance measure	Patient's registration		Outpatients		Pharmacy
	m/m/2,	m/m/1	m/m/2,	m/m/1	m/m/1
Arrival rate(λ) of patients	75/hour	75/hour	19/hour	19/hour	38/hour
Service rate(μ) of patients	41/hour	41/hour	13/hour	13/hour	47/hour
Overall system utilization ρ	90.01%	90.01%	73.07%	73.07%	90.47%
Probability that all server are idle(po)	4.46%	8.53%	15.55%	26.92%	9.56%
Average no. of patients in the system (L_s)	11.19	10.71	3.13	2.71	9.50
Average no. of patients in the queue (L_q)	9.36	9.80	1.67	2.00	8.59
Average time patients spends in the system(W_s- hour)	0.149	0.286	0.165	0.285	0.250
Average time patients spends in the queue(W_q- hour)	0.125	0.261	0.088	0.208	0.226

Table.2 Parameters and performance measures of queuing model using queuing simulation for patient's registration department (1st day).

Performance measure	Server 1	Server 2	Overall for two servers
Mean inter-arrival time	0.8 min	0.8 min	0.8 min
Mean Serving time	1.3 min	1.3 min	1.3 min
Server utilization	98.80%	98.80%	98.80%
Average number of patients in the queue (L_q)	25.97	37.96	63.94
Average no. of patients spendsin the queue (W_q)	20.12	28.07	24.09

Table.3 parameters and performance measures of queuing models using queuing analysis for 2nd day.

Performance measure	Patient's registration		Outpatients		Pharmacy
	m/m/2,	m/m/1	m/m/2,	m/m/1	m/m/1
Arrival rate(λ) of patients	68/hour	68/hour	17/hour	17/hourr	33/hour
Service rate(μ) of patients	38/hour	38/hour	12/hour	12/hour	38/hour
Overall system utilization ρ	89%	89%	70.83%	70.83%	86.84%
Probability that all server are idle(po)	5.56%	10.52%	17.07%	29.16%	13.16%
Average no. of patients in the system (L_s)	8.97	8.50	2.84	2.42	6.60
Average no. of patients in the queue (L_q)	7.00	7.61	1.42	1.72	5.73
Average time patients spends in the system(W_s- hour)	0.132	0.250	0.167	0.285	0.200
Average time patients spends in the queue(W_q- hour)	0.105	0.223	0.084	0.202	0.174

Table.4 Parameters and performance measures of queuing model using queuing simulation for patient's registration department (2nd day).

Performance measure	Server 1	Server 2	Overall two servers
Mean inter-arrival time	0.88 min	0.88 min	0.88 min
Mean Serving time	1.56 min	1.56 min	1.56 min
Server utilization	99.06%	99.06%	99.06%
Average number of patients in the queue (L_q)	26.49	38.59	65.09
Average no. of patients spendsin the queue (W_q)	23.09	30.61	26.82

Correlation between utilization factor, Lq and Wq

As result shown by using queuing analysis and queuing simulation that utilization factor increased or decreased as average waiting time and average number of patients waiting in the queue varies. We used SPSS software to measure the strength of relationship between these characteristics of queuing theory.

Table.5 Strength of relationship between utilization factor $\mathbf{L_q}$ and $\mathbf{W_q}$.

Performance Measure		Utilization Factor	Average No. of Patients	Waiting Time
			Wq.	in queue .
Utilization Factor	Pearson Correlation (Sig2-tailed)	1	0.966^{**} 0.002	0.667 0.148
Average No. of Patients Wq	Pearson Correlation (Sig2-tailed)	0.966^{**} 0.002	1	0.626 0.184
Waiting Time in Queue	Pearson Correlation (Sig2-tailed)	0.667 0.148	0.626 0.184	1

4. Conclusion

The queuing characteristics at a public hospital of Rawalpindi are analyzed using queuing analysis and queuing simulation in three departments. Single server and multiple server queuing models have been used for these analysis.Analysis shown that waiting time of patients in queue is greater in pharmacy department in both days as compared to other departments. From the analysis it has also been observed that waiting time of patients could be reduced by using multiple servers rather than single server queuing model. Furthermore, using the correlation analysis it is found that utilization factor and average number of patients in the queue are significant at both 0.05% and 0.01% level of significance. Strength of relationship between utilization factor and Lq is 0.966 which shows that they are highly correlated.

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