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Application of Fuzzy Numbers to Assessment of Human Skills

Michael Gr. Voskoglou
Department of Mathematical Sciences,
Graduate Technological Educational
Institute of Western Greece,
Patras, Greece.
Email: voskoglou@teiwest.gr, mvosk@hol.gr

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Abstract. Fuzzy numbers play an important role in fuzzy mathematics analogous to the role played by the ordinary numbers in crisp mathematics. A fuzzy number is a special form of a fuzzy set on the set of real numbers. In the paper at hands two of the simpler forms of them, the triangular and the trapezoidal fuzzy numbers are used together with the centre of gravity defuzzification technique to develop two methods for assessing human skills. Applications of them are presented to assessment of student skills and of football player performance illustrating our results. The advantages and disadvantages of the two methods are also discussed leading to the analogous conclusions about their usefulness.

AMS (MOS) Subject Classification Codes: 03E72, 68T20, 97C30
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1. INTRODUCTION

The fuzzy sets theory was created in response of expressing mathematically real world situations in which definitions have not clear boundaries. For example, “the high mountains of a country”, “the young people of a city”, “the good players of a team”, etc. The notion of a *fuzzy set* (FS) was introduced by Zadeh in 1965 as follows:

Definition 1: A FS on the universal set *U* of the discourse (or a fuzzy subset of *U*) is a set of ordered pairs of the form $A = \{(x, m_A(x)): x \in U\}$, defined in terms of a *membership function* $m_A : U \rightarrow [0,1]$ that assigns to each element of *U* a real value from the interval [0,1].

The value $m_A(x)$ is called the *membership degree* of *x* in *A*. The greater is $m_A(x)$, the better *x* satisfies the characteristic property of *A*. Note that, for reasons of simplicity, many authors identify a FS with its membership function. For general facts on FSs we refer to the book [1]

Fuzzy Logic (FL), based on FS theory, constitutes a generalization and complement of the classical bi-valued logic that finds nowadays many applications to almost all sectors of human activities (e.g. [1, Chapter 6, 2-12], etc). Due to its nature of characterizing the ambiguous real life situations with multiple values, FL offers, among others, rich resources for assessment purposes, which are more realistic than those of the classical logic ([2-3], etc).

The FL approach for a problem's solution involves the following steps:

- *Fuzzification* of the problem's data by representing them with properly defined FSs.
- *Evaluation of the fuzzy data* by applying principles and methods of FL in order to express the problem's solution in the form of a unique FS.
- *Defuzzification* of the problem's solution in order to "translate" it in our natural language for use with the original real-life problem.

One of the most popular defuzzification methods is the *Center of Gravity (COG) technique*. When using it, the fuzzy outcomes of the problem's solution are represented by the coordinates of the COG of the membership's function graph of the FS involved in the solution [13].

In the paper at hands *Fuzzy Numbers (FNs)* are used for assessing human skills. The rest of the paper is organized as follows: In Section 2 the background on FNs is presented, which is necessary for the understanding of the rest of the paper. In Section 3 two methods are developed in which *Triangular Fuzzy Numbers* (*TFNs*) and *Trapezoidal Fuzzy Numbers* (*TpFNs*) are used respectively for assessing human skills. Examples concerning assessment of student and player performance are developed in Section 4 illustrating the use of our results, while our final conclusions and our proposal for future research are included in the last Section 5.

2. FUZZY NUMBERS

A FN is a special form of FS on the set $\boldsymbol{R}$ of real numbers defined as follows:

Definition 2: A FN is a FS A on the set $\boldsymbol{R}$ of real numbers with membership function m_A: $\boldsymbol{R} \rightarrow [0, 1]$, such that:

- A is *normal*, i.e. there exists x in $\boldsymbol{R}$ such that $m_A(x) = 1$.
- A is *convex*, i.e. all its *a-cuts*

$$A^a = \{x \in U: m_A(x) \geq a\},$$

with a in [0, 1], are closed real intervals.

- Its membership function $y = m_A(x)$ is a *piecewise continuous* function.

For general facts on FNs we refer to the book [14]

Note that one can define the *four arithmetic operations on FNs* as we do for the ordinary numbers with two different but equivalent to each other methods [14]. However, none of these methods is frequently used in practical applications, because both of them are laborious, involving complicated calculations. What is usually

preferred is the use of simpler forms of FNs instead of their general form, where these operations can be performed in a simple way.
The simplest form of FNs are the TFNs. Roughly speaking a TFN (*a*, *b*, *c*), with *a, b* and *c* real numbers, expresses mathematically the fuzzy statement *"approximately equal to b"* or equivalently that *"b lies in the interval [a, c]"*. The membership function's *graph* of a TFN (*a, b, c*) in the interval [*a, c*] is the union of two straight line segments forming a triangle with the X-axis, while out of [*a, c*] its value is constantly zero (Figure 1).

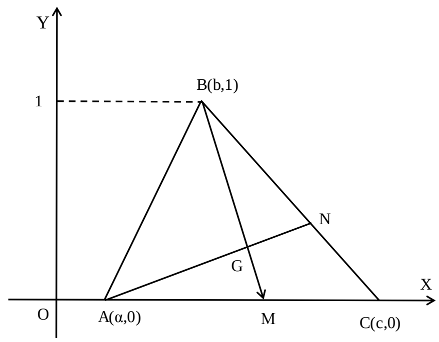

Figure 1: Graph and COG of the TFN (*a, b, c*)

Consequently, the analytic definition of a TFN is given as follows:
Definition 3: Let *a, b* and *c* be real numbers with $a < b < c$. Then the TFN A = (*a*, *b*, *c*) is a FN with membership function:

$$y = m(x) = \begin{cases} \frac{x-a}{b-a} \quad , & x \in [a,b] \\ \frac{c-x}{c-b}, & x \in [b,c] \\ 0, & x < a \ and \ x > c \end{cases}$$

Obviously we have that *m(b)=1*. Note also that *b* need not be in the "middle" of *a* and *c*.
The TpFNs, which are actually generalizations of the *TFNs,* is another simple form of FNs. A TpFN (*a, b, c, d*) with *a, b, c, d* in ***R*** expresses mathematically the fuzzy statement *"approximately in the interval [b, c]"*. Its membership function *y=m(x)* is constantly 0 out of the interval [*a, d*], while its graph in [*a, d*] is the union of three straight line segments forming a trapezoid with the X-axis (Figure 2),

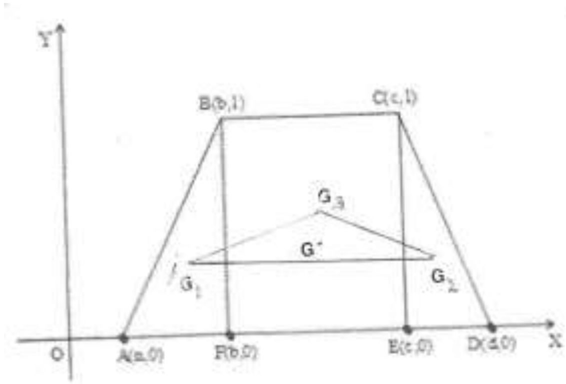

Figure 2: Graph and COGs of the TpFN *(a, b, c, d)*

Consequently, the analytic definition of a TpFN is given as follows:

Definition 4: Let $a < b < c < d$ be real numbers. Then the TpFN (a, b, c, d) is a FN with membership function:

$$y = m(x) = \begin{cases} \frac{x-a}{b-a} & , \quad x \in [a,b] \\ x = 1, & , \quad x \in [b,c] \\ \frac{d-x}{d-c}, & \quad x \in [c,d] \\ 0, & \quad x < a \ and \ x > d \end{cases}$$

It is easy to observe that the TFN (a, b, d) can be considered as a special case of the TpFN (a, b, c, d) with $c=b$..

It can be shown [14] that the two general methods for performing operations on FNs mentioned above lead to the following simple rules for the *addition* and *subtraction* of TpFNs, while the same rules hold also for the TFNs:

Definition 5: Let A = (a_1, a_2, a_3, a_4) and B = (b_1, b_2, b_3, b_4) be two TpFNs. Then

- The *sum*

$$A + B = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4).$$

- The *difference*

$$A - B = A + (-B) = (a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1),$$

where

$$-B = (-b_4, -b_3, -b_2, -b_1)$$

is defined to be the *opposite* of B.

Nevertheless, whereas the sum and the difference of two TpFNs/TFNs as well as the

opposite of a TpFN/TFN are also TpFNs/TFNs, their *product* and *quotient,* although they are FNs, they are not always TpFNs/TFNs [14].
One can define also the following two *scalar operations* on TpFNs/FNs:
Definition 6: Let A = (a_1, a_2, a_3, a_4) be a TpFN and let k be a real number. Then:

$$k + A = (k+a_1, k+a_2, k+a_3, k+a_4)$$
$$kA = (ka_1, ka_2, ka_3, ka_4),$$

if k>0 and

$$kA = (ka_4, ka_3, ka_2, ka_1),$$

if k<0.
We introduce now the following definition, which will be used later in this paper for assessing, with the help of TpFNs/TFNs, the overall performance of human groups during several activities:
Definition 7: Let A_i = (a_{1i}, a_{2i}, a_{3i}, a_{4i}), i = 1, 2, …, *n* be a finite number of TpFNs/TFNs, where *n* is a non negative integer, $n \geq 2$. Then we define their *mean value* to be the TpFN/TFN:

$$A = \frac{1}{n}(A_1 + A_2 + \dots + A_n).$$

We shall close our brief account on FNs by using the *COG technique* for defuzzifying TFNs/TpFNs.
Proposition 8: The coordinates (*X*, *Y*) of the COG of the graph of a TFN (*a*, *b*, *c)* are calculated by the formulas:

$$X = \frac{a+b+c}{3}, Y = \frac{1}{3}.$$

Proof: The graph of the TFN (*a*, *b*, *c)* is the triangle ABC of Figure 1, where A (*a*, 0), B (*b*, 1) and C (*c*, 0). Then, the COG, say G, of ABC is the intersection point of its medians AN and BM, where N ($\frac{b+c}{2}$, $\frac{1}{2}$) and M ($\frac{a+c}{2}$, 0). It is then a routine task of Analytic Geometry to find the equations of AN and BM and to determine the coordinates of G by solving the linear system of them.
Next, Proposition 8 will be used as a Lemma for the defuzzification of TpFNs. The corresponding result is the following:
Proposition 9: The coordinates (*X*, *Y*) of the COG of the graph of the TpFN (*a, b, c, d*) are calculated by the formulas

$$X = \frac{c^2+d^2-a^2-b^2+dc-ba}{3(c+d-a-b)},\ Y = \frac{2c+d-a-2b}{3(c+d-a-b)}.$$

Proof: We divide the trapezoid forming the graph of the TpFN (*a*, *b*, *c*, *d*) in three parts, two triangles and one rectangle (Figure 2). The coordinates of the three vertices of the triangle ABE are (*a*, 0), (*b*, 1) and (*b*, 0) respectively, therefore by Proposition 8 the COG of this triangle is the point G_1 ($\frac{a+2b}{3}$, $\frac{1}{3}$).

Similarly one finds that the COG of the triangle FCD is the point G_2 ($\frac{d+2c}{3},\frac{1}{3}$). Also, it is easy to check that the COG of the rectangle BCFE, being the point G_3 of the intersection of its diagonals, has coordinates G_3($\frac{b+c}{2},\frac{1}{2}$). Further, the areas of the two triangles are equal to S_1 = $\frac{b-a}{2}$ and S_2 = $\frac{d-c}{2}$ respectively, while the area of the rectangle is equal to S_3 = c- b.

It is well known [15] then that the coordinates of the COG of the trapezoid, being the resultant of the COGs G_i (x_i, y_i), for i=1, 2, 3, are calculated by the formulas

$$X=\frac{1}{S}\sum_{i=1}^{3}S_i x_i,\quad Y=\frac{1}{S}\sum_{i=1}^{3}S_i y_i \qquad (1)$$

where $S = S_1 + S_2 + S_3 = \frac{c+d-b-a}{2}$ is the area of the trapezoid.

The proof of the Proposition is completed by replacing the above found values of S, S_i, x_i and y_i, i = 1, 2, 3, to formulas (1) and by performing the corresponding operations.

An alternative method of defuzzifying TpFNs is given by the following result:

Proposition 10: Consider the graph of the TpFN (a, b, c, d) (Figure 2). Let G_1 and G_2 be the COGs of the rectangular triangles AEB and CFD and let G_3 be the COG of the rectangle BEFC respectively . Then $G_1G_2G_3$ is always a triangle, whose COG G has coordinates

$$x=\frac{2(a+d)+7(b+c)}{18},\quad y=\frac{7}{18}.$$

Proof: By Proposition 8 one finds that G_1 ($\frac{a+2b}{3},\frac{1}{3}$) and G_2 ($\frac{d+2c}{3},\frac{1}{3}$). Further, it is easy to check that the GOG G_3 of the rectangle BCFD, being the intersection of its diagonals, has coordinates ($\frac{b+c}{2},\frac{1}{2}$). The y – coordinates of all points of the straight line containing the line segment G_1G_2 are equal to $\frac{1}{3}$, therefore the point G_3, having y – coordinate equal to $\frac{1}{2}$, does not belong to this line. Hence, by Proposition 8, the COG G of the triangle $G_1G_2G_3$, has coordinates

$$x = \left(\frac{a+2b}{3}+\frac{d+2c}{3}+\frac{b+c}{2}\right):3=\frac{2(a+d)+7(b+c)}{18}$$

and

$$y = (\frac{1}{3} + \frac{1}{3} + \frac{1}{2}) : 3 = \frac{7}{18}.-$$

Since the COGs G_1, G_2 and G_3 are the balancing points of the triangles AEB and CFD and of the rectangle BEFC respectively, the COG G of the triangle $G_1G_2G_3$, being the balancing point of those COGs, could be considered instead of the COG of the trapezoid ABCD (Proposition 9) for defuzzifying the TpFN (*a, b, c, d*).

3. USING TFNs AND TPFNs AS ASSESSMENT TOOLS

In this section we are going to utilize TFNs and TpFNs respectively as tools for the development of two methods evaluating the mean performance of a group's objects participating in a certain activity. For this, let us consider a group G of *n* objects, where *n* is a natural number grater than 1. The individual performance of each object is evaluated by assigning to it a score within the range 0-100, which is characterized as follows:

- A (85 - 100) = Excellent, B (75 - 84) = Very good, C (60 – 74) = Good, D (50 – 59) = Fair and F (0, 49) = Unsatisfactory.(Failed).

i) Utilizing TFNs: We assign to each of the above fuzzy characterizations a TFN denoted by the same letter as follows:

- A (85, 92.5, 100), B (75, 79.5, 84), C (60, 67, 74), D (50, 54.5, 59) and F (0, 24.5, 49).

Observe that the middle entry of each of the above TFNs is equal to the mean value of the other two entries. In other words, if (*a, b, c*) is any of the above TFNs, then

$$b = \frac{a+c}{2} \quad (2).$$

Therefore, it is logical to consider the *mean value* (Definition 7) of all those TFNs, denoted for simplicity by the same letter G, for assessing the group's *mean performance*.

Consequently, one can compare the mean performance of two different groups, say G_1 and G_2, by defuzzifying the TFNs G_1 and G_2.

Note that, since the mean values G_1 and G_2 are linear combinations of the TFNs A, B, C, D, F with non negative rational coefficients, the following result facilitates their defuzzification:

Proposition 11: Let

$$M(a, b, c) = k_1A+k_2B+k_3C+k_4D+k_5F$$

be a TFN, with k_i non negative real numbers, i=1, 2, 3, 4, 5. Then the x-coordinate of the COG of the graph of M is equal to

$$X(M) = b$$

Proof: If M is one of the TFNs A, B, C, D, F, then combining Proposition 8 with equality (2) one finds that

$$x(M)=\frac{a+\frac{a+c}{2}+c}{3}=\frac{3(a+c)}{6}=b\cdot$$

In general, if A (a_1, b_1, c_1), B (a_2, b_2, c_2),…., F(a_5, b_5, c_5) and M(a, b, c), then

$$M=\sum_{i=1}^{5}k_i(a_i,b_i,c_i)=(\sum_{i=1}^{5}k_ia_i,\sum_{i=1}^{5}k_ib_i,\sum_{i=1}^{5}k_ic_i)\,.$$

Therefore,

$$X(M)=\frac{\sum_{i=1}^{5}k_ia_i+\sum_{i=1}^{5}k_ib_i+\sum_{i=1}^{5}k_ic_i}{3}=\sum_{i=1}^{5}k_i\,\frac{a_i+b_i+c_i}{3}=\sum_{i=1}^{5}k_ib_i=b\,.$$

Remark 12: An alternative way for defuzzifying a TFN T = (a, b, c) is to use the *Yager Index Ya* (T), introduced in [16] in terms of the a-cuts of T, with a in [0, 1], in order to help the ordering of FSs. It can be shown ([17], p. 62) that

$$Ya(T)=\frac{2b+a+c}{4}\cdot$$

Observe now that

$$X(T)=Ya(T)\Leftrightarrow\frac{a+b+c}{3}=\frac{2b+a+c}{4}\Leftrightarrow 4\left(a+b+c\right)=3\left(2b+a+c\right)\Leftrightarrow a+c=2b$$

.

The last equality is not true in general for $a<b<c$; e.g. take a=1, b=2.5 and c=3. In other words we have in general that

$$X(T)\neq Ya(T).$$

Nevertheless, by (2) the above equality holds for the TFNs A, B, C, D and F. Therefore, it obviously holds also for any linear combination of those TFNs. Thus, the above two defuzzification techniques provide the same outcomes when used in the above developed assessment method with TFNs.

ii) Utilizing TpFNs: We assign to each member M of the group G a TpFN denoted by the same letter M, as follows:

Assume that the performance of M was evaluated by a numerical score, say s, lying in the subinterval [a_1, b_1] of [0, 100]. Consequently a_1 and b_1 are numerical scores assigned to the fuzzy characterizations Q and T, which are equal to one (the same or different) of the fuzzy characterizations A, B, C, D, F. Then, we choose M to be equal to the TpFN (a, a_1, b_1, b), where a is the lower numerical score assigned to Q and b is the upper numerical score assigned to T. For example, if s lies in the interval [73, 87], then Q = C and T = A, therefore P = (60, 73, 87, 100).

The *mean value* S of all those TpFNs, could be also considered here for assessing the group's mean performance, which, after defuzzifying S with the help either of Proposition 2 or of Proposition 3, can be characterized in terms of the linguistic characterizations A, B, C, D and F.

4. EVALUATING HUMAN SKILLS

The examples that will be presented here illustrate the use of the fuzzy assessment methods developed in the previous section for evaluating human skills.

Example 13: Table 1 depicts the learning skills of the students of two different classes, say C_1 and C_2, in terms of the linguistic grades A, B, C, D and F

Table 1: Student Performance

Grade	C_1	C_2
A	20	20
B	15	30
C	7	15
D	10	15
F	8	5
Total	60	85

Considering the TFNs A, B, C, D, F introduced in the previous section one observes that in Table 1 they actually appear 60 in total TFNs representing the individual performances of the students of C_1 and 85 TFNs representing the individual performances of the students of C_2.

Then, calculating the mean values C_1 and C_2 of those TFNs for each class one finds that

$$C_1 = \frac{1}{60}(20A+15B+7C+10D+8F) \approx (62.42, 70.88, 79.33)$$

and

$$C_2 = \frac{1}{85}(20A+30B+15C+15D+5F) \approx (65.88, 72.71, 79.53).$$

Further, by Proposition 11 one finds that $X(C_1) \approx 70.88$ and $X(C_2) \approx 72.71$, which shows that both classes demonstrated a good (C) mean performance, with the performance of C_2 being better.

Remark 14: The evaluation of the class mean performance using TFNs was obtained from the linguistic grades A, B, C, D and F, while no further information is given about the numerical scores assigned to each student. This means that the traditional method of calculating the mean value of the student scores could not applied in this case for assessing the student group mean performance.

Further, even if the student scores were given, the application of the method with TpFNs could be difficult in practice, involving in general laborious calculations with a great number of different TpFNs.

Example 15: The performance of five players of a soccer club in a football match was assessed by six different specialists using a numerical scale from 0 to 100 as follows: M_1 (player 1)**:** 44, 47, 49, 50, 51, 52, M_2**:** 82, 83. 86, 88, 92, 95, M_3: 79, 82, 85, 93, 93, 95, M_4: 86, 86, 87, 87, 87, 88 and M_5: 38, 40, 44, 53, 59, 61.
Here we shall use both methodss with TFNs and TpFNs for assessing the mean performance of those players.
Utilizing TFNs: Inspecting the given data one observes that the 30 in total scores assigned to the five players by the six specialists correspond to 14 characterizations for excellent (A) performance, to 4 for very good (B), to 6 for fair (D) and to 6 characterizations for unsatisfactory (F) performance. Therefore, the mean player performance can be assessed by calculating the TFN

$$M = \frac{1}{30}(14A + 4B + 6D + 6F) \approx (59.67, 69.57, 79.47).$$

Therefore, by Proposition 11, $X(M) = 69.57$, which shows that the above five players demonstrated a good (C) mean performance
Utilizing TpFNs: We assign to each player a TpFN as follows: $M_1 = (0, 44, 52, 59)$, $M_2 = (75, 82, 95, 100)$, $M_3 = (75, 79, 95, 100)$, $M_4 = (85, 86, 88, 100)$ and $M_5 = (0, 38, 61, 74)$.
We calculate the mean value of the TpFNs M_i , i =1, 2, 3, 4, 5, which is equal to

$$M = \frac{1}{5}\sum_{i=1}^{5} M_i = (47, 65.8, 78.2, 86.6).$$

Applying Proposition 9 one finds that

$$X(M) = \frac{(78.2)^2 + (86.6)^2 - (65.8)^2 - 47^2 + 78.2*86.6 - 47*(65.8)}{3(78.2 + 86.6 - 47 - 65.8)} \approx 68.95$$

which shows that the above five players demonstrated a good (C) mean performance.
Alternatively, applying Proposition 10 one finds that

$$x(M) = \frac{2(47 + 86.6) + 7(65.8 + 78.2)}{18} \approx 70.84,$$

showing again that the five players demonstrated a good mean performance.
Remark 16: Defuziffying the TpFNs M_i , i =1, 2, 3, 4, 5, as we did for M, one defines a total order among the individual performances of the five players. On the contrary, such an order cannot be defined when using TFNs instead of TpFNs. In fact, for two players to whom the same TFN corresponds, we don't know which performed better, if the numerical scores assigned to them are not known.

5. CONCLUSIONS

The COG defuzzification technique was combined in this paper with the use of TFNs and TpFNs respectively to develop two fuzzy methods for assessing human skills. The first method with TFNs is particularly useful when the individuals are assessed with qualitative grades only, because in such cases the calculation of the mean value of their scores cannot be applied for assessing their mean performance. On the other hand, the second method with TpFNs, although, in contrast to the first one with TFNs, is not always easy to be applied in practice due to the laborious calculations needed, it has the advantage that through it one can define a total order among the individual performances of the members of the group under assessment.

Both methods have a general character, which means that they could be applied for the assessment of several other human or even machine activities (e.g. computers), apart from the assessment of student and player performance presented here and this is our main proposal for future research on the subject.

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