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Triangular Cubic Power Aggregation Operators and Their Application to Multiple Attribute Group Decision Making

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Abstract. In this paper, we develop a new method for multiple attribute group decision making for triangular cubic numbers, which is the extension of cubic numbers. In this paper, we define triangular power aggregation operator such that triangular cubic power weighted averaging (TCPWA) operator, triangular cubic power weighted geometric (TCPWG) operator and triangular cubic power weighted quadratic averaging (TCPWQA) operator and then applied in order to develop some methods for multiple attribute group decision $(MAGD)$ making problem. Finally, a numerical example illustrates the applicability and effectiveness of the proposed method.

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1. Introduction

In 1965 Zadeh [22] presented the idea of fuzzy set theory and it has been studied in various fields such as, medical diagnosis, information science, fuzzy algebra and decision making problem. But fuzzy set (Fs) has not explain due to not available of non-membership. Therefore in 1986 [4] new idea of intuitionistic fuzzy (IFs) set was defined by Atanassove.

It is generalization of the fuzzy set (Fs) theory. In different fields intuitionistic fuzzy set theory has been studied such as, logic program, algebra, topology, medical diagnosis and decision making problems. In [14, 15, 17] has been studied intuitionistic fuzzy aggregation operator, intuitionistic fuzzy geometric (IFG) and operator intuitionistic fuzzy ordered weighted $(IFOW)$ operator. The uncertainty problem does not explain by means of intuitionistic fuzzy set. So therefor Jun et al [7, 8] defined the concept of cubic set in 2012. Jun defined a new theory which is known as cubic set theory. This theory is able to deal with uncertain problem, cubic set theory also explain the satisfied, unsatisfied and uncertain information. Cubic set theory applied in many area's such that BCK/BCI algebra and other structure [7, 8]. Like the other scholars in [3,7,13,15,17,19] discuss various aggregation operators

The aggregation operator information are in interesting topic for research works [18, 19] the odered weighted averaging (OWA) operator is a reordering step having input arguments are re-arranged in descending order [15, 16] the ordered weighted geometric operator is the aggregation operator is happening on the (OWA) operator. In this paper, we develop different types of operators such that triangular cubic power weighted averaging $(TCPWA)$ operator, triangular cubic power weighted geometric $(TCPWG)$ operator and triangular cubic power weighted quadratic averaging $(TCPWQA)$ operator. By applying these operators we construct a numerical example to verify the validity of our results.

The paper is arranged as follows; In section 2, we evaluate and described basic definitions of fuzzy set, cubic numbers numbers, triangular cubic numbers and power aggregation operators. In section 3, we develop different type of operators such that triangular cubic power weighted averaging $(TCPWA)$ operator, triangular cubic power weighted geometric $(TCPWG)$ operator and triangular cubic power weighted quadratic averaging $(TCPWQA)$ operator. In section 4, different steps are define which are used to construct a numerical application. In section 5, we give a numerical application. Finally conclusion is given by section 6.

2. PRELIMINARIES

We defined some of the fundamental concepts and definitions which are necessary for this paper fuzzy set and cubic set, the idea of fuzzy set and IFs defined as follows in this section such that,

Definition 2.1. [5] Let L be a fixed set. An IFs P in L is an object having the form: $P = \{\langle l, \mu_A(l), \nu_A(l)\rangle \mid l \in L\}$, (1) here $\nu_A : L \to [0,1]$ and $\mu_A : L \to [0,1]$ represent the degree of non-membership,and the degree of membership of the element $l \in L$ to P, respectively and for every $l \in L$ $0 \le \mu_A(l) + \nu_A(l) \le 1$. (2) For each IFs P in L,

$\pi_A(l) = 1 - \mu_A(l) - \nu_A(l)$, for all $l \in L$ (3) $\pi_A(l)$ is said the degree of indeterminacy of l to P.

Definition 2.2. [10] Let L be a fixed non empty set. A cubic set is an object of the form such that, $C = \{\langle a, A(a), \xi(a)\rangle : a \in L\}$,here A is an interval-valued fuzzy (IVF) set and ξ is a fuzzy set in L. A cubic set $\tilde{C} = \langle a, A(a), \xi(a)\rangle$ is simply denoted by $\zeta = \left\langle \tilde{A}, \xi \right\rangle$. The collection of all cubic set is denoted by $C(L)$ such that,

(a) if $\xi \in \tilde{A}(l)$ for all $l \in L$ so it is called interval cubic set,

(b) If $\xi \notin \tilde{A}(l)$ for all $l \in L$ so it is called external cubic set,

(c) If $\xi \in \tilde{A}(l)$ or $\lambda \notin \tilde{A}(l)$ its called cubic set for all $l \in L$.

Definition 2.3. [10] Let $A = \left\langle \tilde{A}, \xi \right\rangle$ and $B = \left\langle \tilde{B}, \mu \right\rangle$ be any two cubic set in L such that,

(a) (Equality) $A = B \Leftrightarrow A = B$ and $\xi = \mu$.

(b) $(P-$order$)$ $A \subseteq_A B \Leftrightarrow A \subseteq B$ and $\xi \leq \mu$.

(c) $(R-$order$)$ $A \subseteq_R B \Leftrightarrow A \subseteq B$ and $\xi \geq \mu$.

Definition 2.4. [10] The complement of $A = \left\langle \tilde{A}, \xi \right\rangle$ is defined to be the cubic set as follows; $A^c = \{\langle x, A^c(l), 1 - \xi(l)\rangle \, | l \in L\}$.

Definition 2.5. [18] A triangular fuzzy numbers $\tilde{\Psi}$ can be defined by a triplet $\left(\Psi^L, \Psi^M, \Psi^U\right)$ where Ψ^L and Ψ^U stand for the lower and upper values of the support of $\tilde{\Psi}$, respectively, and Ψ^M for the model value.

Let $\tilde{\Psi} = \begin{bmatrix} [\overset{-}{\Psi}, \overset{+}{\Psi}]^L, [\overset{-}{\Psi}, \overset{+}{\Psi}]^M, \\ [\overset{-}{\Psi}, \overset{+}{\Psi}]^U, (\xi^L, \xi^M, \xi^U) \end{bmatrix}$, and $\tilde{\Upsilon} = \begin{bmatrix} [\overset{-}{\Upsilon}, \overset{+}{\Upsilon}]^L, [\overset{-}{\Upsilon}, \overset{+}{\Upsilon}]^M], \\ [\overset{-}{\Upsilon}, \overset{+}{\Upsilon}]^U, (\Phi^L, \Phi^M, \Phi^U) \end{bmatrix}$ be any two triangular cubic numbers. Then we define the following properties such that,

$$(a)\ \tilde{\Psi} \oplus \tilde{\Upsilon} = \left\{ \begin{array}{c} [\overset{-}{\Psi} + \overset{-}{\Upsilon} - \overset{-}{\Psi}\overset{-}{\Upsilon}, \overset{+}{\Psi} + \overset{+}{\Upsilon} - \overset{+}{\Psi}\overset{+}{\Upsilon}]^L, \\ [\overset{-}{\Psi} + \overset{-}{\Upsilon} - \overset{-}{\Psi}\overset{-}{\Upsilon}, \overset{+}{\Psi} + \overset{+}{\Upsilon} - \overset{+}{\Psi}\overset{+}{\Upsilon}]^M, \\ [\overset{-}{\Psi} + \overset{-}{\Upsilon} - \overset{-}{\Psi}\overset{-}{\Upsilon}, \overset{+}{\Psi} + \overset{+}{\Upsilon} - \overset{+}{\Psi}\overset{+}{\Upsilon}]^U, \\ (\xi^L\Phi^L, \xi^M\Phi^M, \xi^U\Phi^U) \end{array} \right\}$$

$$(b)\ \tilde{\Psi} \otimes \tilde{\Upsilon} = \left\{ \begin{array}{c} [\overset{-}{\Psi}\overset{-}{\Upsilon}, \overset{+}{\Psi}\overset{+}{\Upsilon}]^L, [\overset{-}{\Psi}\overset{-}{\Upsilon}, \overset{+}{\Psi}\overset{+}{\Upsilon}]^M, [\overset{-}{\Psi}\overset{-}{\Upsilon}, \overset{+}{\Psi}\overset{+}{\Upsilon}]^U, \\ (\xi^L + \Phi^L - \xi^L\Phi^L, \xi^M + \Phi^M - \xi^M\Phi^M, \\ \xi^U + \Phi^U - \xi^U\Phi^U). \end{array} \right\}$$

$$(c)\ \Phi \otimes \tilde{\Psi} = \left\{ \begin{array}{c} [1 - \left(1 - \overset{-}{\Psi}\right)^{\Phi}, 1 - \left(1 - \overset{+}{\Psi}\right)^{\Phi}]^L, \\ [1 - \left(1 - \overset{-}{\Psi}\right)^{\Phi}, 1 - \left(1 - \overset{+}{\Psi}\right)^{\Phi}]^M, \\ [1 - \left(1 - \overset{-}{\Psi}\right)^{\Phi}, 1 - \left(1 - \overset{+}{\Psi}\right)^{\Phi}]^U, (\xi^{\Phi}, \xi^{\Phi}, \xi^{\Phi}) \end{array} \right\}$$

$$(d)\ \tilde{\Psi}^{\Phi} = \left\{ \begin{array}{c} [\overset{-}{\Psi}^{\Phi}, \overset{+}{\Psi}^{\Phi}]^L, [\overset{-}{\Psi}^{\Phi}, \overset{+}{\Psi}^{\Phi}]^M, [\overset{-}{\Psi}^{\Phi}, \overset{+}{\Psi}^{\Phi}]^U, \\ (1 - ((1 - \xi)^{\Phi})^L, \\ (1 - ((1 - \xi)^{\Phi})^M, (1 - ((1 - \xi)^{\Phi})^U. \end{array} \right\}$$

Example 2.6. Let $\tilde{\Psi} = \begin{pmatrix} [0.11, 0.12], \\ [0.32, 0.35], \\ [0.44, 0.55], \\ 0.11, 0.12, 0.13 \end{pmatrix}$ and $\tilde{\Upsilon} = \begin{pmatrix} [0.31, 0.32], \\ [0.46, 0.48], \\ [0.56, 0.70], \\ 0.35, 0.40, 0.50 \end{pmatrix}$ be any two triangular cubic fuzzy numbers, and let $\Phi = 0.3$. Then, we verify the above results as follows;

$$(a)\ \tilde{\Psi}\oplus\tilde{\Upsilon}=\begin{pmatrix}[0.11+0.31-0.11\times 0.31, 0.12+0.32-0.12\times 0.32],\\ [0.32+0.46-0.32\times 0.46, 0.35+0.48-0.35\times 0.48],\\ [0.44+0.56-0.44\times 0.56, 0.55+0.70-0.55\times 0.70],\\ (0.11\times 0.35, 0.12\times 0.40, 0.13\times 0.50)\end{pmatrix}$$

$$=\begin{pmatrix}([0.3859, 0.4016], [0.6340, 0.6620], [0.7536, 0.48650],\\ 0.0385, 0.0482, 0.0654\end{pmatrix}$$

$$(b)\ \tilde{\Psi}\otimes\tilde{\Upsilon}=\begin{pmatrix}[0.11\times 0.31, 0.12\times 0.32],\\ [0.32\times 0.46, 0.35\times 0.48],\\ [0.44\times 0.56, 0.55\times 0.70],\\ (0.11+0.35-0.11\times 0.35),\\ [0.12+0.40-0.12\times 0.40],\\ [0.13+0.50-0.13\times 0.50],\end{pmatrix}$$

$$=\begin{pmatrix}([0.0341, 0.0384], [0.4172, 0.1680],\\ [0.2464, 0.3850], 0.42, 0.47, 0.056\end{pmatrix}.$$

$$(c)\ \Phi\otimes\tilde{\Psi}=\begin{pmatrix}([1-(1-0.11)^{0.3}, 1-(1-0.12)^{0.3}],\\ ([1-(1-0.32)^{0.3}, 1-(1-0.35)^{0.3}],\\ ([1-(1-0.44)^{0.3}, 1-(1-.0.55)^{0.3}],\\ (0.11)^{0.3}\times(0.12)^{0.3}\times(0.13)^{0.3}\end{pmatrix}$$

$$=\begin{pmatrix}([0.041, 0.034], [0.412, 0.180],\\ [0.244, 0.380], 0.5157, 0.5293, 0.5422\end{pmatrix}.$$

$$(d)\ \tilde{\Psi}^{\Phi}=\begin{pmatrix}([(0.11)^{0.3}, (0.12)^{0.3}], ([(0.32)^{0.3},\\ (0.35)^{0.3}], ([(0.44)^{0.3}, (0.55)^{0.3}],\\ (1-(1-0.11)^{0.3}, (1-(1-0.12)^{0.3}\\ (1-(1-0.13)^{0.3}\end{pmatrix}$$

$$=\begin{pmatrix}(0.5157, 0.5293, 0.5422),\\ ([0.041, 0.034], [0.412, 0.180],\\ [0.244, 0.380]\end{pmatrix}.$$

Definition 2.7. If $\tilde{A}$ is a triangular cubic variable such that

$\tilde{A}=\tilde{\Psi}=[[\overset{-}{\Psi},\overset{+}{\Psi}]^{L},[\overset{-}{\Psi},\overset{+}{\Psi}]^{M},[\overset{-}{\Psi},\overset{+}{\Psi}]^{U},(\xi^{L},\xi^{M},\xi^{U})]$.Then the expected value of $\tilde{A}$ is define as follows,

$$E\left(\tilde{A}\right)=\frac{[\left(\bar{\Psi}+\overset{+}{\Psi}\right)^{L}+4(\bar{\Psi}+\overset{+}{\Psi})^{M}+\left(\bar{\Psi}+\overset{+}{\Psi}\right)+\xi^{L}+4\xi^{M}+\xi^{U}]}{18}. \quad (4)$$

Example 2.8. Let $\tilde{\Psi}=([0.31, 0.39], [0.40, 0.45], [0.79, 0.80], 0.55, 0.65, 0.80)$ be any triangular cubic number. Then, we verify the above results as follows; $E\left(\tilde{A}\right)=0.5355$.

2.1. **Power aggregation operators.**

In [26] weighted averaging aggregation operator called power aggregation (PA) as follows;

$$PA\left(\Psi_{1},\Psi_{2},...,\Psi_{n}\right)=\frac{\sum(1+\Lambda(\Psi_{i}))\Psi_{i}}{\sum(1+\Lambda(\Psi_{i}))}, \quad (5)$$

here $\Lambda\left(\Psi_{i}\right)=\sum_{\substack{(j=1\\ j\neq i)}}^{n} Supt(\Psi_{i},\Psi_{j})$,and Supt $(\Psi,\tilde{\Upsilon})$ is the support for Ψ from Υ satisfied the conditions such that,

(a) Supt $(\Psi,\Upsilon)\in[0,1]$,

(b) Supt (Ψ, Υ) =Supt (Υ, Ψ),

(c) Supt $(\Psi, \Upsilon) \geq$ Supt (x, y), if $|\Psi - \Upsilon| < |x - y|$.

Xu and Yager [27] further defined a power geometric (PG) operator based on the PA operator and geometric mean such that, $PG(\Psi_1, \Psi_2, ..., \Psi_n) = \prod_{i=1}^{n} \Psi_i^{\frac{(1+\Lambda(\Psi_i))}{\sum_{i=1}^{n}(1+\Lambda(\zeta_i))}}$. (6)

3. Triangular Cubic Power Weighted Averaging $(TCPWA)$ Operator, $TCPWG$ Operator and $TCPWQA$ Operator

In the following section, we present triangular cubic power aggregation operators which we used through out this paper.

Definition 3.1. Let $\tilde{\zeta}_i = \left([\overset{-}{\zeta}_i, \overset{+}{\zeta}_i]^L, [\overset{-}{\zeta}_i, \overset{+}{\zeta}_i]^M, [\overset{-}{\zeta}_i, \overset{+}{\zeta}_i]^U, \lambda_i^L, \lambda_i^M, \lambda_i^U \right)$ be a set of triangular cubic numbers and $w = (w_1, w_2, ..., w_n)^T$ be the weighting vector of $\tilde{\zeta}_i$ $(i = 1, 2, ..., n)$ and $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. Then, we define triangular cubic power weighted average operator as follows,

$$TCPWA_w\left(\tilde{\zeta}_1, \tilde{\zeta}_2, ..., \tilde{\zeta}_n\right) =$$

$$\begin{bmatrix} 1 - \prod_{j=1}^{n}(1 - \overset{-}{\zeta}_i)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}, 1 - \prod_{j=1}^{n}(1 - \overset{+}{\zeta}_i)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}} \end{bmatrix}^L$$
$$\begin{bmatrix} 1 - \prod_{j=1}^{n}(1 - \overset{-}{\zeta}_i)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}, 1 - \prod_{j=1}^{n}(1 - \overset{+}{\zeta}_i)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}} \end{bmatrix}^M$$
$$\begin{bmatrix} 1 - \prod_{j=1}^{n}(1 - \overset{-}{\zeta}_i)^{\frac{s_k\left(1+T(\tilde{r}_{ij}^{(k)})\right)}{\sum_{k=1}^{t} s_k\left(1+T(\tilde{r}_{ij}^{(k)})\right)}}, 1 - \prod_{j=1}^{n}(1 - \overset{+}{\zeta}_i)^{\frac{s_k\left(1+T(\tilde{r}_{ij}^{(k)})\right)}{\sum_{k=1}^{t} s_k\left(1+T(\tilde{r}_{ij}^{(k)})\right)}} \end{bmatrix}^{(U)}$$
$$\left(\prod_{j=1}^{n} (\xi_{\zeta_i})^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}} \right)^L, \left(\prod_{j=1}^{n} (\xi_{\zeta_i})^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}} \right)^M,$$
$$\left(\prod_{j=1}^{n} (\xi_{\zeta_i})^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}} \right)^U \quad (7)$$

$$T(\tilde{\zeta}_i) = \sum_{\substack{(j=1 \\ j \neq i)}}^{n} w_j \text{Supt}\,(\tilde{\zeta}_i, \tilde{\zeta}_j), \quad (8)$$

and Supt $(\tilde{\zeta}_i, \tilde{\zeta}_j)$ is the support for $\tilde{\zeta}_i$ from $\tilde{\zeta}_j$ satisfied the following conditions such that,

(a) Supt $(\tilde{\zeta}_i, \tilde{\zeta}_j) \in [0, 1]$,

(b) Supt $(\tilde{\zeta}_i, \tilde{\zeta}_j)$ =Supt $(\tilde{\zeta}_i, \tilde{\zeta}_j)$,
(c) Supt $(\tilde{\zeta}_i, \tilde{\zeta}_j) \geq$ Supt $(\tilde{\zeta}_s, \tilde{\zeta}_t)$,
if $d(\tilde{\zeta}_i, \tilde{\zeta}_j) < d(\tilde{\zeta}_s, \tilde{\zeta}_t)$ In this case d denotes the distance measure. If $w = (\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n})^T$, then the $TCPWA$ operator reduces to a triangular cubic power average $(TCPA)$ operator.

$$TCPA_w\left(\tilde{\zeta}_1, \tilde{\zeta}_2, ..., \tilde{\zeta}_n\right) =$$

$$\left[1-\prod_{j=1}^{n}(1-\bar{\zeta}_i)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}, 1-\prod_{j=1}^{n}(1-\overset{+}{\zeta}_i)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right]^L$$
$$\left[1-\prod_{j=1}^{n}(1-\bar{\zeta}_i)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}, 1-\prod_{j=1}^{n}(1-\overset{+}{\zeta}_i)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right]^M$$
$$\left[1-\prod_{j=1}^{n}(1-\bar{\zeta}_i)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}, 1-\prod_{j=1}^{n}(1-\overset{+}{\zeta}_i)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right]^{(U)},$$
$$\left(\prod_{j=1}^{n}(\xi_{\zeta_i})^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right)^L, \left(\prod_{j=1}^{n}(\xi_{\zeta_i})^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right)^M,$$
$$\left(\prod_{j=1}^{n}(\xi_{\zeta_i})^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right)^U \quad (9)$$

here

$$T(\tilde{\zeta}_i) = \frac{1}{n}\sum_{\substack{(j=1\\ j\neq i)}}^{n} \text{supt } (\tilde{\zeta}_i, \tilde{\zeta}_j). \quad (10)$$

The $TCPWA$ operator we construct the proprties as follows;

Theorem 3.2. (Idempotency) Let $\tilde{\zeta} = \left(\tilde{\zeta}_1, \tilde{\zeta}_2, ..., \tilde{\zeta}_n\right)$. Then,

$$TCPWA_w\left(\tilde{\zeta}_1, \tilde{\zeta}_2, ..., \tilde{\zeta}_n\right) = \tilde{\zeta}. \quad (11)$$

Theorem 3.3. (Boundedness) $\min_i \tilde{\zeta}_i \leq TCPWA_w\left(\tilde{\zeta}_1, \tilde{\zeta}_2, ..., \tilde{\zeta}_n\right) \leq \max_i \tilde{\zeta}_i$.

Based on the $TCPWA$ operator and the geometric mean, we define a triangular cubic power weighted geometric $(TCPWG)$ operator:

$$TCPWG_w\left(\tilde{\zeta}_1, \tilde{\zeta}_2, ..., \tilde{\zeta}_n\right) =$$

$$\left\langle \begin{array}{c} \left[\prod_{j=1}^{n} \left(\overset{-}{\zeta_i}\right)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}, \prod_{j=1}^{n} \left(\overset{+}{\zeta}_i\right)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}} \right]^{L} \\ \left[\prod_{j=1}^{n} \left(\overset{-}{\zeta_i}\right)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}, \prod_{j=1}^{n} \left(\overset{+}{\zeta}_i\right)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}} \right]^{M} \\ \left[\prod_{j=1}^{n} (\overset{-}{\zeta_i})^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}, \prod_{j=1}^{n} \left(\overset{+}{\zeta}_i\right)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}} \right]^{(U)}, \\ \left(1 - \prod_{j=1}^{n} (1-\xi_i)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right)^{L}, \left(1 - \prod_{j=1}^{n} (1-\xi_i)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right)^{M}, \\ \left(1 - \prod_{j=1}^{n} (1-\xi_i)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right)^{U} \end{array} \right\rangle. \quad (12)$$

Especially, If $w = (\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n})$. Then the $TCPWG$ operator reduce to a triangular cubic power geometric $TCPG$ operator,

$$TCPG_w\left(\tilde{\zeta}_1,\tilde{\zeta}_2,...,\tilde{\zeta}_n\right)=$$

$$\left[\prod_{j=1}^{n}\left(\overset{-}{\zeta_i}\right)^{\frac{\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u}\left(1+\Lambda(r_{ij}^{(v)})\right)}},\prod_{j=1}^{n}\left(\overset{+}{\zeta}_i\right)^{\frac{\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u}\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right]^L$$

$$\left[\prod_{j=1}^{n}\left(\overset{-}{\zeta_i}\right)^{\frac{\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u}\left(1+\Lambda(r_{ij}^{(v)})\right)}},\prod_{j=1}^{n}\left(\overset{+}{\zeta}_i\right)^{\frac{\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u}\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right]^M$$

$$\left[\prod_{j=1}^{n}\left(\overset{-}{\zeta_i}\right)^{\frac{\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u}\left(1+\Lambda(r_{ij}^{(v)})\right)}},\prod_{j=1}^{n}\left(\overset{+}{\zeta}_i\right)^{\frac{\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u}\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right]^{(U)},$$

$$\left(1-\prod_{j=1}^{n}\left(1-\xi_i\right)^{\frac{\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u}\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right)^L,\left(1-\prod_{j=1}^{n}\left(1-\xi_i\right)^{\frac{\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u}\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right)^M,$$

$$,\left(1-\prod_{j=1}^{n}\left(1-\xi_i\right)^{\frac{\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u}\left(1+\Lambda(r_{ij}^{(v)})\right)}}\right)^U \quad (13)$$

It can be easily proved that the $TCPWG$ operator has the following properties similar to $TCPWA$ operator.

Theorem 3.4. (Idempotency) Let
$\left(\tilde{\zeta}_1,\tilde{\zeta}_2,...,\tilde{\zeta}_n\right)=\tilde{\zeta}$, then $TCPWG_w\left(\tilde{\zeta}_1,\tilde{\zeta}_2,...,\tilde{\zeta}_n\right)=\tilde{\zeta}$.

Theorem 3.5. (Boundedness).
$\min_i\tilde{\zeta}_i\leq TCPWA_w\left(\tilde{\zeta}_1,\tilde{\zeta}_2,...,\tilde{\zeta}_n\right)\leq\max_{uni}\tilde{\zeta}_i$.

From the definitions of the $TCPWA$ and $TFPWG$ operators, it can be seen that the fundamental characteristics of these two operators is that they weight all the given triangular cubic numbers,

Definition 3.6. [21] (WQA): $R^n\to R$, if WQA

$$WQA_w\left(\Psi_1,\Psi_2,...,\Psi_n\right)=\left(\sum_{i=1}^{n}w_i\left(\Psi_i\right)^2\right)^{\frac{1}{2}}. \quad (14)$$

Then (WQA) is called a weighted quadratic averaging operator, where
$w=(w_1,w_2,...,w_n)^T$ be the weighting vector of $(\Psi_1,\Psi_2,...,\Psi_n)$ and $w_i\in[0,1]$ $\sum_{i=1}^{n}w_i=1$, where R is set of all real numbers.

Definition 3.7. An ordered weighted quadratic averaging $(OWQA)$ operator of dimension n is a mapping $OWQA : R^n \to R$, that has an associated vector $w = (w_1, w_2, ..., w_n)^T$ and $\sum_{i=1}^{n} w_i = 1$, such that,

$$OWQA_w(\Psi_1, \Psi_2, ..., \Psi_n) = \left(\sum_{i=1}^{n} w_i \left(\Psi_{\alpha(i)}\right)^2\right)^{\frac{1}{2}}, \quad (15)$$

here $(\alpha(1), \alpha(2), ..., \alpha(n))$ is a permutation of $(1, 2, ..., n)$, such that $\Psi_{\alpha(i-1)} \geq \Psi_{\alpha(i)}$ for all $i = 1, 2, ..., n$.

In the following, based on the quadratic average operator and triangular cubic power weighted average $(TCPWA)$operator we define triangular cubic power weighted quadratic averaging operator $(TCPWQA)$ operator, which allows the input data to support each other in the aggregating process.

Definition 3.8. Let $\tilde{\zeta}_i = \left\langle \begin{array}{c} [\overset{-}{\zeta}_i, \overset{+}{\zeta}_i]^L, [\overset{-}{\zeta}_i, \overset{+}{\zeta}_i]^M, \\ [\overset{-}{\zeta}_i, \overset{+}{\zeta}_i]^U, \xi_i^L, \xi_i^M, \xi_i^U \end{array} \right\rangle (i = 1, 2, ..., n)$ be a set of triangular cubic numbers and $w = (w_1, w_2, ..., w_n)^T$ be the weighting vector of $\tilde{\zeta}_i\,(i = 1, 2, ..., n)$ and $w_i \in [0, 1]$, $\sum_{i=1}^{n} w_i = 1$, then we define the triangular cubic power weighted quadratic average operator as follows.

$$TCPWQA_w\left(\tilde{\zeta}_1, \tilde{\zeta}_2, ..., \tilde{\zeta}_n\right) =$$

$$\left[\sqrt{1-\prod_{j=1}^{n}(1-(\bar{\zeta}_i)^2)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}, \sqrt{1-\prod_{j=1}^{n}(1-(\overset{+}{\zeta}_i)^2)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}\right]^L,$$

$$\left[\sqrt{1-\prod_{i=1}^{n}(1-((\bar{\zeta}_i)^2)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}, \sqrt{1-\prod_{i=1}^{n}(1-(\overset{+}{\zeta}_i)^2)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}\right]^M,$$

$$\left[\sqrt{1-\prod_{j=1}^{n}(1-((\bar{\zeta}_i)^2)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}, \sqrt{1-\prod_{j=1}^{n}(1-(\overset{+}{\zeta}_i)^2)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}\right]^{(U)},$$

$$\left(\sqrt{\prod_{j=1}^{n}((\bar{\zeta}_i)^{2\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}, \sqrt{\prod_{j=1}^{n}(\overset{+}{\zeta}_i)^{2\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}\right)^L,$$

$$\left(\sqrt{\prod_{j=1}^{n}((\bar{\zeta}_i)^{2\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum\limits_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}, \sqrt{\prod_{j=1}^{n}(\overset{+}{\zeta}_i)^{2\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum\limits_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}\right)^M,$$

$$\left(\sqrt{\prod_{j=1}^{n}((\bar{\zeta}_i)^{2\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}, \sqrt{\prod_{j=1}^{n}(\overset{+}{\zeta}_i)^{2\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}\right)^U \quad (16)$$

From the definitions of $TCPWA$, $TCPWG$, and $TCPWQA$ operators, it can be seen that all these operators not only depend upon the input arguments and allow values being aggregated to support and reinforce each other.

4. Models For Multiple Attribute Group Decision Making With Triangular Cubic Information

The above power aggregation operators utilize to multiple attribute group decision making problem in this section. For $(MAGD)$ with triangular cubic information. Let $P = \{P_1, P_2, ..., P_m\}$ be the set of alternatives, $Q = \{Q_1, Q_2, ..., Q_n\}$ be the set of attributes. Let $w = (w_1, w_2, ..., w_n)^T$ be the weighting vector and $w_j \geq 0$, $j = 1, 2, ..., n$ here $\sum_{i=1}^{n} w_j = 1$. Let $B = \{B_1, B_2, ..., B_t\}$ be the set of decision maker's whose weighting vector is $b = (b_1, b_2, ..., b_u) \in H$, with $b_u \geq 0$, $v = 1, 2, ..., u$ $\sum_{v=1}^{u} b_u = 1$. Consider that $\tilde{A}_k = \left(\zeta_{ij}^{(v)}\right)_{m\times n} = [\left\langle \begin{array}{c} [\bar{\zeta}_i, \overset{+}{\zeta}_i]^L, [\bar{\zeta}_i, \overset{+}{\zeta}_i]^M, \\ [\bar{\zeta}_i, \overset{+}{\zeta}_i]^U, \xi_i^L, \xi_i^M, \xi_i^U \end{array} \right\rangle]_{m\times n}$, is the multiple attribute group decision making matrix, where $\tilde{\zeta}_{ij}^{(v)}$ is an attribute value which takes the form of triangular cubic numbers, given by the decision maker's $B_k \in B$, for the alternative $P_i \in P$ with respect to the attribute $Q_j \in Q$. Then, we utilize the $TCPWA$, $TCPWG$ and $TCPWQA$ operators to develop an approach to $MAGD$ making problems with triangular cubic information, which can be described as following steps;

Approach 1

Step 1. In this step, we normalize each attribute value $\tilde{\zeta}_{ij}^{(v)}$ in the matrix $\tilde{A}_v$ into a corresponding element in the matrix

$$\tilde{R}_v = \left(\tilde{r}_{ij}^{(v)}\right)_{m\times n}, = \left[\left(r_{ij}^{(L)}\right)^{(k)}, \left(r_{ij}^{(M)}\right)^{(k)}, \left(r_{ij}^{(U)}\right)^{(k)}\right],$$

using the following equation such that,

$$\left\{ \begin{array}{c} [1-[\overset{-}{\zeta}+\overset{+}{\zeta}]^U,\ 1-[\overset{-}{\zeta}+\overset{+}{\zeta}]^M,\ 1-[\overset{-}{\zeta}+\overset{+}{\zeta}]]^L, \\ 1-\xi^U,\ 1-\xi^M,\ 1-\xi^L] \end{array} \right\} \quad (17)$$

Step 2. Calculate the support measure as follows:

$$\text{Supt}\left(\tilde{r}_{ij}^{(l)}, \tilde{r}_{ij}^{(v)}\right) = 1 - d\left(\tilde{r}_{ij}^{(k)}, \tilde{r}_{ij}^{(l)}\right) \; l = 1, 2, ..., t, \qquad (18)$$

which satisfies the support conditions. We calculate $d\left(\tilde{r}_{ij}^{(l)}, \tilde{r}_{ij}^{(v)}\right)$ with the distance as follows:

$$d\left(\tilde{r}_{ij}^{(v)}, \tilde{r}_{ij}^{(l)}\right) = \frac{\left(\begin{array}{c} \left|\left(r_{ij}^{(L)}\right)^{(k)} - \left(r_{ij}^{(L)}\right)^{(l)}\right| + \\ \left|\left(r_{ij}^{(M)}\right)^{(k)} - \left(r_{ij}^{(M)}\right)^{(l)}\right| \\ + \left|\left(r_{ij}^{(U)}\right)^{(k)} - \left(r_{ij}^{(U)}\right)^{(l)}\right| \end{array} \right)}{9} \qquad (19)$$

Step 3. We apply the weights $b = (b_1, b_2, ..., b_u)$ of the decision maker's B_v $(v = 1, 2, ..., u)$ to calculate the weighted support $\Lambda\left(\tilde{r}_{ij}^{(v)}\right)$ of the triangular cubic preference value $\tilde{r}_{ij}^{(v)}$ by the other triangular cubic preference value $\tilde{r}_{ij}^{(l)}$ for the preference value

$\tilde{r}_{ij}^{(l)}$ $(l = 1, 2, ..., t, \text{and } l \neq u)$

$$\Lambda\left(\tilde{r}_{ij}^{(v)}\right) = \sum_{\substack{l=1 \\ l\neq k}}^{u} \text{supt}\left(\tilde{r}_{ij}^{(v)}, \tilde{r}_{ij}^{(l)}\right), \qquad (20)$$

and calculate the weights $\nabla_{ij}^{(v)}$ $(v = 1, 2, ..., u)$ of the triangular cubic preference value $\tilde{r}_{ij}^{(v)}$ $(v = 1, 2, ..., u)$ such that,

$$\nabla_{ij}^{(v)} = \frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum\limits_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}, v = 1, 2, ..., u, \qquad (21)$$

where $\nabla_{ij}^{(v)} \geq 0$, $(v = 1, 2, ..., u)$ and $\sum_{v=1}^{u} \nabla_{ij}^{(v)} = 1$.

Step 4. In this step, we apply the decision information given in the matrix $\tilde{R}_v$, $TCPWA$, $TCPWG$ and $TCPWQA$ operators such that,

$$
\begin{aligned}
&\tilde{r}_{ij} = \left(\tilde{r}_{ij}^{(L)}, \tilde{r}_{ij}^{(M)}, \tilde{r}_{ij}^{(U)}\right) = TCPWQA\left(\tilde{r}_{ij}^{(1)}, \tilde{r}_{ij}^{(2)}, ..., \tilde{r}_{ij}^{(v)}\right) \\
&\left[\sqrt{1-\prod_{j=1}^{n}\left(1-(\bar{\zeta_i})^2\right)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}, \sqrt{1-\prod_{j=1}^{n}\left(1-(\overset{+}{\zeta}_i)^2\right)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}\right]^{L}, \\
&\left[\sqrt{1-\prod_{j=1}^{n}\left(1-(\bar{\zeta_i})^2\right)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}, \sqrt{1-\prod_{j=1}^{n}\left(1-(\overset{+}{\zeta}_i)^2\right)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}\right]^{M}, \\
&\left[\sqrt{1-\prod_{j=1}^{n}\left(1-(\bar{\zeta_i})^2\right)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}, \sqrt{1-\prod_{j=1}^{n}\left(1-(\overset{+}{\zeta}_i)^2\right)^{\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}\right]^{(U)}, \\
&\left(\sqrt{\prod_{j=1}^{n}(\bar{\zeta_i})^{2\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}, \sqrt{\prod_{j=1}^{n}(\overset{+}{\zeta}_i)^{2\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}\right)^{L}, \\
&\left(\sqrt{\prod_{j=1}^{n}(\bar{\zeta_i})^{2\frac{s_k\left(1+T(\tilde{r}_{ij}^{(k)})\right)}{\sum_{k=1}^{t} s_k\left(1+T(\tilde{r}_{ij}^{(k)})\right)}}}, \sqrt{\prod_{j=1}^{n}(\overset{+}{\zeta}_i)^{2\frac{s_k\left(1+T(\tilde{r}_{ij}^{(k)})\right)}{\sum_{k=1}^{t} s_k\left(1+T(\tilde{r}_{ij}^{(k)})\right)}}}\right)^{M}, \\
&\left(\sqrt{\prod_{j=1}^{n}(\bar{\zeta_i})^{2\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}, \sqrt{\prod_{j=1}^{n}(\overset{+}{\zeta}_i)^{2\frac{b_v\left(1+\Lambda(\tilde{r}_{ij}^{(v)})\right)}{\sum_{v=1}^{u} b_v\left(1+\Lambda(r_{ij}^{(v)})\right)}}}\right)^{U} \qquad (22)
\end{aligned}
$$

To aggregate all the individual decision matrices $\tilde{R}_v$ $(v = 1, 2, ..., u)$ into the collective decision matrix $\tilde{R}_v = (\tilde{r}_{ij})_{m\times n} = \left[\Psi_{ij}^{L}, \Psi_{ij}^{M}, \Psi_{ij}^{U}\right]_{m\times n}$, where $b = (b_1, b_2, ..., b_v)$ be the weighting vector of decision maker's.

Step 5. Aggregate all triangular cubic preference value $\tilde{r}_{ij}$ $(j = 1, 2, ..., n)$ by applying the triangular cubic power weighted average operator where $\sum_{i=1}^{n} w_j = 1$

$$\tilde{r}_{ij} = \left(\tilde{r}_{ij}^{(L)}, \tilde{r}_{ij}^{(M)}, \tilde{r}_{ij}^{(U)}\right) = TCPWA\left(\tilde{r}_{ij}^{(1)}, \tilde{r}_{ij}^{(2)}, ..., \tilde{r}_{ij}^{(v)}\right) = \left(\begin{array}{c} \left[1-\prod_{j=1}^{n}\left(1-\bar{\zeta}_i\right)^{w_j}, 1-\prod_{j=1}^{n}\left(1-\overset{+}{\zeta}_i\right)^{w_j}\right]^L, \\ \left[1-\prod_{j=1}^{n}\left(1-\bar{\zeta}_i\right)^{w_j}, 1-\prod_{j=1}^{n}\left(1-\overset{+}{\zeta}_i\right)^{w_j}\right]^M, \\ \left[1-\prod_{j=1}^{n}\left(1-\bar{\zeta}_i\right)^{w_j}, 1-\prod_{j=1}^{n}\left(1-\overset{+}{\zeta}_i\right)^{w_j}\right]^U, \\ \left(\prod_{j=1}^{n}((\lambda_i)^{w_j})^L, \prod_{j=1}^{n}((\lambda_i)^{w_j})^M, \prod_{j=1}^{n}((\lambda_i)^{w_j})^U\right) \end{array}\right) \quad (23)$$

or the triangular cubic power weighted geometric $(TCPWG)$ operator :

$$\tilde{r}_{ij} = \left(\tilde{r}_{ij}^{(L)}, \tilde{r}_{ij}^{(M)}, \tilde{r}_{ij}^{(U)}\right) = TCPWG\left(\tilde{r}_{ij}^{(1)}, \tilde{r}_{ij}^{(2)}, ..., \tilde{r}_{ij}^{(v)}\right)$$

$$\left[\prod_{j=1}^{n}(\bar{\zeta}_{ij})^{w_j}, \prod_{j=1}^{n}\left(\overset{+}{\zeta}_{ij}\right)^{w_j}\right]^{(L)} \left[\prod_{j=1}^{n}(\bar{\zeta}_{ij})^{w_j}, \prod_{j=1}^{n}\left(\overset{+}{\zeta}_{ij}\right)^{w_j}\right]^{(M)}$$
$$\left[\prod_{j=1}^{n}(\bar{\zeta}_{ij})^{w_j}, \prod_{j=1}^{n}\left(\overset{+}{\zeta}_{ij}\right)^{w_j}\right]^{(U)},$$
$$\left(\begin{array}{c} 1-\prod_{j=1}^{n}((1-\xi_{ij})^{w_j})^{(L)}, \\ 1-\prod_{j=1}^{n}((1-\xi_{ij})^{w_j})^{(M)}, 1-\prod_{j=1}^{n}((1-\xi_{ij})^{w_j})^{(U)} \end{array}\right) \quad (24)$$

or the triangular cubic power weighted quadratic average $(TCPWQA)$ operator;

$$\tilde{r}_{ij} = \left(\tilde{r}_{ij}^{(L)}, \tilde{r}_{ij}^{(M)}, \tilde{r}_{ij}^{(U)}\right) = TCPWQA\left(\tilde{r}_{ij}^{(1)}, \tilde{r}_{ij}^{(2)}, ..., \tilde{r}_{ij}^{(v)}\right) =$$
$$\left(\begin{array}{c} \left[\sqrt{1-\prod_{j=1}^{n}(1-(\bar{\zeta})^2)^{w_j}}, \sqrt{1-\prod_{j=1}^{n}(1-(\overset{+}{\zeta})^2)^{w_j}},\right]^L \\ \left[\sqrt{1-\prod_{j=1}^{n}(1-(\bar{\zeta})^2)^{w_j}}, \sqrt{1-\prod_{j=1}^{n}(1-(\overset{+}{\zeta})^2)^{w_j}},\right]^M \\ \left[\sqrt{1-\prod_{j=1}^{n}(1-(\bar{\zeta})^2)^{w_j}}, \sqrt{1-\prod_{j=1}^{n}(1-(\overset{+}{\zeta})^2)^{w_j}},\right]^U \\ \left(\sqrt{\prod_{j=1}^{n}((\xi^2)^{w_j}),}\right)^L, \left(\sqrt{\prod_{j=1}^{n}((\xi^2)^{w_j}),}\right)^L, \\ \left(\sqrt{\prod_{j=1}^{n}((\xi^2)^{w_j}),}\right)^U \end{array}\right). \quad (25)$$

To derive the overall triangular cubic values $\tilde{r}_i\,(i = 1, 2, ..., m)$ of the alternative A_i where $w = (w_1, w_2, ..., w_n)^T$ be the weighting vector of the attributes.

Step 6. To rank these collective overall preference values $\tilde{r}_i\,(i = 1, 2, ..., m)$, there should be first compared each $\tilde{r}_i$ with all the $\tilde{r}_j\,(j = 1, 2, ..., m)$ by applying E.q. 4.

Step 7. In this step, we find the rank of all the alternatives.

Flow Chart

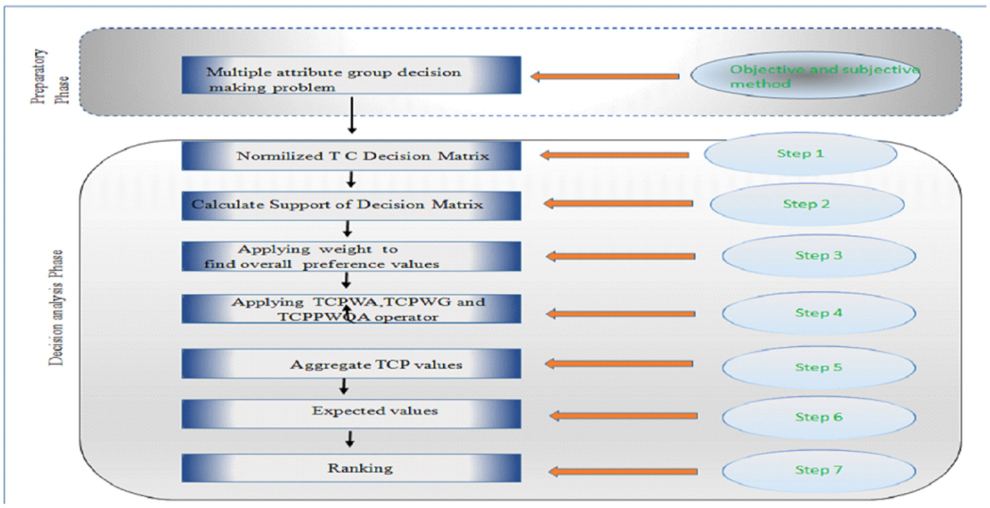

Fig (a)

In this section, we have proposed one approach to solve the triangular cubic multiple attribute group decision making problems with the known weights information of decision makers.

5. Numerical Example

Consider a company wants to invest company, and found the best option. For this purpose they consider the set of four possible alternatives to invest the money such that, A_1 is a mobile company, A_2 is a laptop company, A_3 is a light charger company, A_4 is an arms company. The investment company must take a decision according to the following four attributes such that, Q_1 is the risk analysis, Q_2 is the growth analysis, Q_3 is the social-political impact analysis, Q_4 is the environmental impact analysis. The four possible alternatives $A_i(i = 1, 2, 3, 4)$ are to be constructed by applying triangular cubic numbers. Let $B_k(k = 1, 2, 3)$ be the set of decision maker's having weighting vector $s = (0.4, 0.3, 0.3)$ under the four attributes having weighting vectors $w = (0.1, 0.2, 0.3, 0.4)^T$, the triangular cubic decision matrix are shown in Tables $(1, 2, 3)$ such that,

Table 1. Decision matrix $\tilde{A}_1$

	Q_1
P_1	$\langle[0.11, 0.12], [0.32, 0.35], [0.44, 0.55], 0.11, 0.12, 0.13\rangle$
P_2	$\langle[0.23, 0.24], [0.33, 0.36], [0.56, 0.57], 0.21, 0.22, 0.23\rangle$
P_3	$\langle[0.34, 0.38], [0.41, 0.45], [0.62, 0.63], 0.25, 0.35, 0.45\rangle$
P_4	$\langle[0.16, 0.18], [0.58, 0.60], [0.88, 0.92], 0.16, 0.19, 0.21\rangle$
	Q_2
P_1	$\langle[0.13, 0.14], [0.38, 0.42], [0.45, 0.85], 0.13, 0.39, 0.87\rangle$
P_2	$\langle[0.15, 0.25], [0.25, 0.35], [0.35, 0.45], 0.35, 0.45, 0.55\rangle$
P_3	$\langle[0.18, 0.22], [0.24, 0.36], [0.38, 0.49], 0.19, 0.29, 0.33\rangle$
P_4	$\langle[0.58, 0.68], [0.72, 0.77], [0.81, 0.83], 0.71, 0.81, 0.91\rangle$
	Q_3
P_1	$\langle[0.13, 0.23], [0.33, 0.43], [0.45, 0.85], 0.23, 0.33, 0.43\rangle$
P_2	$\langle[0.43, 0.53], [0.58, 0.62], [0.66, 0.71], 0.18, 0.19, 0.24\rangle$
P_3	$\langle[0.31, 0.35], [0.45, 0.49], [0.52, 0.58], 0.42, 0.44, 0.46\rangle$
P_4	$\langle[0.42, 0.52], [0.55, 0.58], [0.68, 0.72], 0.29, 0.33, 0.49\rangle$
	Q_4
P_1	$\langle[0.31, 0.32], [0.34, 0.35], [0.41, 0.42], 0.91, 0.92, 0.93\rangle$
P_2	$\langle[0.15, 0.25], [0.25, 0.35], [0.35, 0.45], 0.28, 0.32, 0.35\rangle$
P_3	$\langle[0.15, 0.25], [0.25, 0.35], [0.35, 0.45], 0.31, 0.41, 0.56\rangle$
P_4	$\langle[0.15, 0.25], [0.25, 0.35], [0.35, 0.45], 0.42, 0.58, 0.71\rangle$

Table 2 Decision matrix $\tilde{A}_2$

	Q_1
P_1	$\langle[0.18, 0.28], [0.35, 0.38], [0.44, 0.54], 0.38, 0.48, 0.52\rangle$
P_2	$\langle[0.22, 0.32], [0.36, 0.41], [0.52, 0.58], 0.27, 0.39, 0.47\rangle$
P_3	$\langle[0.29, 0.36], [0.47, 0.58], [0.59, 0.69], 0.37, 0.49, 0.58\rangle$
P_4	$\langle[0.44, 0.49], [0.52, 0.59], [0.69, 0.72], 0.71, 0.73, 0.75\rangle$
	Q_2
P_1	$\langle[0.29, 0.38], [0.42, 0.68], [0.72, 0.88], 0.19, 0.29, 0.36\rangle$
P_2	$\langle[0.26, 0.46], [0.57, 0.61], [0.68, 0.77], 0.94, 0.05, 0.96\rangle$
P_3	$\langle[0.71, 0.72], [0.81, 0.82], [0.88, 0.89], 0.12, 0.22, 0.33\rangle$
P_4	$\langle[0.47, 0.52], [0.54, 0.64], [0.65, 0.66], 0.26, 0.38, 0.52\rangle$
	Q_3
P_1	$\langle[0.19, 0.22], [0.24, 0.38], [0.68, 0.78], 0.61, 0.63, 0.68\rangle$
P_2	$\langle[0.21, 0.28], [0.33, 0.39], [0.42, 0.56], 0.81, 0.85, 0.91\rangle$
P_3	$\langle[0.31, 0.42], [0.48, 0.57], [0.62, 0.77], 0.28, 0.68, 0.98\rangle$
P_4	$\langle[0.46, 0.51], [0.82, 0.88], [0.91, 0.97], 0.19, 0.49, 0.89\rangle$
	Q_4
P_1	$\langle[0.28, 0.33], [0.38, 0.48], [0.54, 0.58], 0.12, 0.16, 0.19\rangle$
P_2	$\langle[0.41, 0.61], [0.64, 0.79], [0.88, 0.95], 0.88, 0.89, 0.94\rangle$
P_3	$\langle[0.13, 0.15], [0.19, 0.21], [0.22, 0.23], 0.21, 0.31, 0.44\rangle$
P_4	$\langle[0.31, 0.42], [0.53, 0.64], [0.75, 0.86], 0.86, 0.88, 0.93\rangle$

Table 3 Decision matrix $\tilde{A}_3$

	Q_1
P_1	$\langle[0.28, 0.39], [0.58, 0.62], [0.68, 0.77], 0.75, 0.85, 0.89\rangle$
P_2	$\langle[0.61, 0.66], [0.69, 0.71], [0.88, 0.89], 0.72, 0.88, 0.98\rangle$
P_3	$\langle[0.68, 0.69], [0.73, 0.75], [0.81, 0.83], 0.28, 0.32, 0.48\rangle$
P_4	$\langle[0.48, 0.49], [0.59, 0.69], [0.74, 0.83], 0.82, 0.92, 0.96\rangle$
	Q_2
P_1	$\langle[0.16, 0.26], [0.28, 0.39], [0.42, 0.56], 0.55, 0.58, 0.64\rangle$
P_2	$\langle[0.22, 0.24], [0.26, 0.28], [0.32, 0.48], 0.31, 0.38, 0.57\rangle$
P_3	$\langle[0.21, 0.38], [0.39, 0.55], [0.59, 0.91], 0.66, 0.76, 0.81\rangle$
P_4	$\langle[0.19, 0.81], [0.83, 0.88], [0.89, 0.97], 0.61, 0.71, 0.88\rangle$
	Q_3
P_1	$\langle[0.29, 0.39], [0.39, 0.49], [0.51, 0.59], 0.66, 0.71, 0.88\rangle$
P_2	$\langle[0.22, 0.44], [0.61, 0.64], [0.66, 0.78], 0.79, 0.80, 0.89\rangle$
P_3	$\langle[0.81, 0.82], [0.84, 0.86], [0.89, 0.96], 0.81, 0.88, 0.97\rangle$
P_4	$\langle[0.56, 0.59], [0.68, 0.77], [0.79, 0.88], 0.39, 0.48, 0.55\rangle$
	Q_4
P_1	$\langle[0.66, 0.78], [0.81, 0.88], [0.92, 0.98], 0.97, 0.98, 0.99\rangle$
P_2	$\langle[0.69, 0.79], [0.89, 0.92], [0.96, 0.99], 0.91, 0.95, 0.96\rangle$
P_3	$\langle[0.42, 0.44], [0.46, 0.48], [0.49, 0.50], 0.52, 0.55, 0.61\rangle$
P_4	$\langle[0.91, 0.92], [0.93, 0.95], [0.96, 0.98], 0.19, 0.59, 0.99\rangle$

Step 1. In this step, we constructing the normalized decision matrix $\tilde{R}_k$, by using E.q. 17 the result shown in Tables $(4 - 6)$.

Table 4 Decision matrix $\tilde{R}_1$

	Q_1
P_1	$\langle[0.45, 0.56], [0.65, 0.68], [0.88, 0.89], 0.87, 0.88, 0.89\rangle$
P_2	$\langle[0.43, 0.44], [0.64, 0.67], [0.76, 0.77], 0.77, 0.78, 0.79\rangle$
P_3	$\langle[0.37, 0.38], [0.55, 0.59], [0.62, 0.66], 0.55, 0.65, 0.75\rangle$
P_4	$\langle[0.08, 0.12], [0.40, 0.42], [0.82, 0.84], 0.79, 0.81, 0.84\rangle$
	Q_2
P_1	$\langle[0.13, 0.14], [0.38, 0.42], [0.45, 0.85], 0.13, 0.39, 0.87\rangle$
P_2	$\langle[0.15, 0.25], [0.25, 0.35], [0.35, 0.45], 0.35, 0.45, 0.55\rangle$
P_3	$\langle[0.18, 0.22], [0.24, 0.36], [0.38, 0.49], 0.19, 0.29, 0.33\rangle$
P_4	$\langle[0.58, 0.68], [0.72, 0.77], [0.81, 0.83], 0.71, 0.81, 0.91\rangle$
	Q_3
P_1	$\langle[0.13, 0.23], [0.33, 0.43], [0.45, 0.85], 0.23, 0.33, 0.43\rangle$
P_2	$\langle[0.43, 0.53], [0.58, 0.62], [0.66, 0.71], 0.18, 0.19, 0.24\rangle$
P_3	$\langle[0.31, 0.35], [0.45, 0.49], [0.52, 0.58], 0.42, 0.44, 0.46\rangle$
P_4	$\langle[0.42, 0.52], [0.55, 0.58], [0.68, 0.72], 0.29, 0.33, 0.49\rangle$
	Q_4
P_1	$\langle[0.31, 0.32], [0.34, 0.35], [0.41, 0.42], 0.91, 0.92, 0.93\rangle$
P_2	$\langle[0.15, 0.25], [0.25, 0.35], [0.35, 0.45], 0.28, 0.32, 0.35\rangle$
P_3	$\langle[0.15, 0.25], [0.25, 0.35], [0.35, 0.45], 0.31, 0.41, 0.56\rangle$
P_4	$\langle[0.15, 0.25], [0.25, 0.35], [0.35, 0.45], 0.42, 0.58, 0.71\rangle$

Table5. Decision matrix $\tilde{R}_2$

	Q_1
P_1	$\langle[0.46, 0.56], [0.62, 0.65], [0.72, 0.82], 0.48, 0.52, 0.62\rangle$
P_2	$\langle[0.42, 0.48], [0.59, 0.64], [0.68, 0.78], 0.53, 0.61, 0.73\rangle$
P_3	$\langle[0.31, 0.41], [0.42, 0.53], [0.64, 0.71], 0.42, 0.51, 0.63\rangle$
P_4	$\langle[0.28, 0.31], [0.41, 0.48], [0.51, 0.56], 0.25, 0.27, 0.29\rangle$
	Q_2
P_1	$\langle[0.29, 0.38], [0.42, 0.68], [0.72, 0.88], 0.19, 0.29, 0.36\rangle$
P_2	$\langle[0.26, 0.46], [0.57, 0.61], [0.68, 0.77], 0.94, 0.05, 0.96\rangle$
P_3	$\langle[0.71, 0.72], [0.81, 0.82], [0.88, 0.89], 0.12, 0.22, 0.33\rangle$
P_4	$\langle[0.47, 0.52], [0.54, 0.64], [0.65, 0.66], 0.26, 0.38, 0.52\rangle$
	Q_3
P_1	$\langle[0.19, 0.22], [0.24, 0.38], [0.68, 0.78], 0.61, 0.63, 0.68\rangle$
P_2	$\langle[0.21, 0.28], [0.33, 0.39], [0.42, 0.56], 0.81, 0.85, 0.91\rangle$
P_3	$\langle[0.31, 0.42], [0.48, 0.57], [0.62, 0.77], 0.28, 0.68, 0.98\rangle$
P_4	$\langle[0.46, 0.51], [0.82, 0.88], [0.91, 0.97], 0.19, 0.49, 0.89\rangle$
	Q_4
P_1	$\langle[0.28, 0.33], [0.38, 0.48], [0.54, 0.58], 0.12, 0.16, 0.19\rangle$
P_2	$\langle[0.41, 0.61], [0.64, 0.79], [0.88, 0.95], 0.88, 0.89, 0.94\rangle$
P_3	$\langle[0.13, 0.15], [0.19, 0.21], [0.22, 0.23], 0.21, 0.31, 0.44\rangle$
P_4	$\langle[0.31, 0.42], [0.53, 0.64], [0.75, 0.86], 0.86, 0.88, 0.93\rangle$

Table 6 Decision matrix $\tilde{R}_3$

	Q_1
P_1	$\langle[0.23, 0.32], [0.38, 0.42], [0.61, 0.72], 0.11, 0.15, 0.25\rangle$
P_2	$\langle[0.11, 0.12], [0.29, 0.31], [0.34, 0.39], 0.02, 0.12, 0.28\rangle$
P_3	$\langle[0.17, 0.19], [0.25, 0.27], [0.31, 0.32], 0.52, 0.68, 0.72\rangle$
P_4	$\langle[0.17, 0.26], [0.31, 0.41], [0.51, 0.52], 0.04, 0.08, 0.18\rangle$
	Q_2
P_1	$\langle[0.16, 0.26], [0.28, 0.39], [0.42, 0.56], 0.55, 0.58, 0.64\rangle$
P_2	$\langle[0.22, 0.24], [0.26, 0.28], [0.32, 0.48], 0.31, 0.38, 0.57\rangle$
P_3	$\langle[0.21, 0.38], [0.39, 0.55], [0.59, 0.91], 0.66, 0.76, 0.81\rangle$
P_4	$\langle[0.19, 0.81], [0.83, 0.88], [0.89, 0.97], 0.61, 0.71, 0.88\rangle$
	Q_3
P_1	$\langle[0.29, 0.39], [0.39, 0.49], [0.51, 0.59], 0.66, 0.71, 0.88\rangle$
P_2	$\langle[0.22, 0.44], [0.61, 0.64], [0.66, 0.78], 0.79, 0.80, 0.89\rangle$
P_3	$\langle[0.81, 0.82], [0.84, 0.86], [0.89, 0.96], 0.81, 0.88, 0.97\rangle$
P_4	$\langle[0.56, 0.59], [0.68, 0.77], [0.79, 0.88], 0.39, 0.48, 0.55\rangle$
	Q_4
P_1	$\langle[0.66, 0.78], [0.81, 0.88], [0.92, 0.98], 0.97, 0.98, 0.99\rangle$
P_2	$\langle[0.69, 0.79], [0.89, 0.92], [0.96, 0.99], 0.91, 0.95, 0.96\rangle$
P_3	$\langle[0.42, 0.44], [0.46, 0.48], [0.49, 0.50], 0.52, 0.55, 0.61\rangle$
P_4	$\langle[0.91, 0.92], [0.93, 0.95], [0.96, 0.98], 0.19, 0.59, 0.99\rangle$

Step 2. We apply Eq. $(18\text{-}21)$ to calculate the weight $\nabla_{ij}^{(v)}$ associated with the attribute values $r_{ij}^{(v)}$ which are expressed in the matrix $\nabla^{(v)} = (\nabla_{ij}^{(v)})_{4\times 4}$ $(v = 1, 2, 3)$ which are shown in Table $(7\text{-}9)$ respectively.

Table 7, Weight matrix $\nabla^{(1)}$

	Q_1	Q_2	Q_3	Q_4
P_1	0.3840	0.3905	0.3853	0.3996
P_2	0.3939	0.3805	0.3806	0.3804
P_3	0.3899	0.3916	0.3891	0.3905
P_4	0.3782	0.3908	0.3858	0.3966

Table 8, Weight matrix $\nabla^{(2)}$

	Q_1	Q_2	Q_3	Q_4
P_1	0.3163	0.3037	0.3086	0.3019
P_2	0.3184	0.3133	0.3051	0.3139
P_3	0.3093	0.3016	0.3165	0.3033
P_4	0.3127	0.3037	0.3043	0.3093

Table 9, Weight matrix $\nabla^{(3)}$

	Q_1	Q_2	Q_3	Q_4
P_1	0.2961	0.3056	0.3059	0.2984
P_2	0.2876	0.3060	0.3142	0.3056
P_3	0.3007	0.3065	0.2942	0.3060
P_4	0.3086	0.3053	0.3098	0.2970

Step 3. In this step, we apply the $TCPWA$, $TCPWG$ and $TCPWQA$ operators to aggregate all the individual decision matrix into the collective decision matrix, the aggregating results are shown in Table $(10-12)$ respectively.

Table10 Decision matrix $\tilde{R}$ $(TCPWA)$

	Q_1
P_1	$\langle[0.39, 0.49], [0.57, 0.60], [0.77, 0.82], 0.39, 0.44, 0.56\rangle$
P_2	$\langle[0.35, 0.38], [0.54, 0.58], [0.65, 0.70], 0.23, 0.42, 0.57\rangle$
P_3	$\langle[0.29, 0.34], [0.43, 0.49], [0.55, 0.60], 0.50, 0.61, 0.70\rangle$
P_4	$\langle[0.17, 0.22], [0.37, 0.46], [0.66, 0.69], 0.22, 0.28, 0.37\rangle$
	Q_2
P_1	$\langle[0.19, 0.26], [0.36, 0.51], [0.54, 0.81], 0.22, 0.40, 0.61\rangle$
P_2	$\langle[0.21, 0.32], [0.37, 0.42], [0.47, 0.58], 0.46, 0.54, 0.66\rangle$
P_3	$\langle[0.40, 0.46], [0.53, 0.60], [0.66, 0.81], 0.24, 0.35, 0.43\rangle$
P_4	$\langle[0.44, 0.69], [0.72, 0.78], [0.80, 0.87], 0.50, 0.61, 0.76\rangle$
	Q_3
P_1	$\langle[0.20, 0.28], [0.32, 0.43], [0.94, 0.67], 0.42, 0.50, 0.83\rangle$
P_2	$\langle[0.30, 0.43], [0.52, 0.56], [0.60, 0.69], 0.453, 0.47, 0.54\rangle$
P_3	$\langle[0.52, 0.57], [0.62, 0.66], [0.71, 0.82], 0.44, 0.61, 0.72\rangle$
P_4	$\langle[0.48, 0.54], [0.69, 0.76], [0.80, 0.89], 0.16, 0.41, 0.60\rangle$
	Q_4
P_1	$\langle[0.43, 0.54], [0.55, 0.63], [0.69, 0.79], 0.50, 0.31, 0.34\rangle$
P_2	$\langle[0.53, 0.66], [0.74, 0.81], [0.90, 0.95], 0.57, 0.61, 0.65\rangle$
P_3	$\langle[0.25, 0.28], [0.31, 0.40], [0.57, 0.58], 0.32, 0.41, 0.53\rangle$
P_4	$\langle[0.64, 0.69], [0.73, 0.78], [0.84, 0.90], 0.41, 0.66, 0.85\rangle$

Table11. Decision matrix $\tilde{R}$ $(TCPWG)$

	Q_1
P_1	$\langle[0.37, 0.47], [0.54, 0.58], [0.74, 0.81], 0.64, 0.66, 0.82\rangle$
P_2	$\langle[0.28, 0.30], [0.48, 0.52], [0.57, 0.63], 0.48, 0.60, 0.68\rangle$
P_3	$\langle[0.27, 0.31], [0.39, 0.44], [0.50, 0.53], 0.50, 0.62, 0.71\rangle$
P_4	$\langle[0.37, 0.64], [0.65, 0.81], [0.82, 0.89], 0.58, 0.69, 0.83\rangle$
	Q_2
P_1	$\langle[0.17, 0.22], [0.35, 0.47], [0.50, 0.76], 0.30, 0.43, 0.28\rangle$
P_2	$\langle[0.19, 0.29], [0.32, 0.38], [0.41, 0.53], 0.36, 0.39, 0.54\rangle$
P_3	$\langle[0.28, 0.36], [0.39, 0.52], [0.55, 0.69], 0.37, 0.48, 0.55\rangle$
P_4	$\langle[0.14, 0.19], [0.36, 0.42], [0.60, 0.62], 0.50, 0.55, 0.59\rangle$
	Q_3
P_1	$\langle[0.22, 0.30], [0.32, 0.40], [0.65, 0.70], 0.60, 0.62, 0.70\rangle$
P_2	$\langle[0.27, 0.40], [0.48, 0.52], [0.56, 0.66], 0.66, 0.68, 0.78\rangle$
P_3	$\langle[0.40, 0.46], [0.54, 0.59], [0.63, 0.72], 0.55, 0.70, 0.71\rangle$
P_4	$\langle[0.45, 0.52], [0.65, 0.71], [0.76, 0.83], 0.44, 0.69, 0.78\rangle$
	Q_4
P_1	$\langle[0.37, 0.46], [0.45, 0.50], [0.54, 0.59], 0.87, 0.89, 0.91\rangle$
P_2	$\langle[0.50, 0.63], [0.68, 0.77], [0.81, 0.91], 0.46, 0.64, 0.78\rangle$
P_3	$\langle[0.23, 0.28], [0.31, 0.37], [0.46, 0.47], 0.34, 0.43, 0.50\rangle$
P_4	$\langle[0.41, 0.54], [0.63, 0.70], [0.82, 0.90], 0.63, 0.72, 0.87\rangle$

$Table12$.Decision matrix $\tilde{R}$ $(TCPWQA)$

	Q_1
P_1	$\langle[0.09, 0.13], [0.17, 0.19], [0.30, 0.34], 0.39, 0.40, 0.43\rangle$
P_2	$\langle[0.37, 0.41], [0.56, 0.59], [0.66, 0.67], 0.23, 0.42, 0.57\rangle$
P_3	$\langle[0.30, 0.35], [0.44, 0.55], [0.56, 0.61], 0.49, 0.61, 0.70\rangle$
P_4	$\langle[0.19, 0.23], [0.38, 0.43], [0.67, 0.70], 0.21, 0.27, 0.37\rangle$
	Q_2
P_1	$\langle[0.18, 0.26], [0.35, 0.50], [0.54, 0.78], 0.26, 0.43, 0.61\rangle$
P_2	$\langle[0.21, 0.33], [0.39, 0.44], [0.49, 0.60], 0.45, 0.54, 0.58\rangle$
P_3	$\langle[0.46, 0.47], [0.57, 0.64], [0.68, 0.87], 0.24, 0.35, 0.44\rangle$
P_4	$\langle[0.46, 0.69], [0.72, 0.78], [0.80, 0.88], 0.50, 0.61, 0.76\rangle$
	Q_3
P_1	$\langle[0.21, 0.28], [0.32, 0.43], [0.58, 0.67], 0.48, 0.51, 0.61\rangle$
P_2	$\langle[0.31, 0.42], [0.53, 0.57], [0.60, 0.70], 0.45, 0.47, 0.54\rangle$
P_3	$\langle[0.16, 0.17], [0.20, 0.23], [0.26, 0.34], 0.18, 0.19, 0.26\rangle$
P_4	$\langle[0.11, 0.14], [0.17, 0.16], [0.33, 0.40], 0.03, 0.08, 0.187\rangle$
	Q_4
P_1	$\langle[0.45, 0.56], [0.58, 0.66], [0.72, 0.82], 0.50, 0.55, 0.58\rangle$
P_2	$\langle[0.53, 0.66], [0.75, 0.73], [0.84, 0.95], 0.57, 0.61, 0.65\rangle$
P_3	$\langle[0.27, 0.30], [0.32, 0.41], [0.60, 0.61], 0.32, 0.41, 0.53\rangle$
P_4	$\langle[0.68, 0.71], [0.74, 0.79], [0.85, 0.90], 0.41, 0.66, 0.82\rangle$

Step 4. By applying the decision information given $(10-12)$ $TCPWA$, $TCPWG$ and $TCPWQA$ operators, and $w = (0.1, 0.2, 0.3, 0.4)^T$ be the weighting vector of the attributes, we find the overall preference values of the alternatives the aggregation results are shown in Table 13.

Table13. The overall preference values of the alternatives

	$TCPWA$
P_1	$\langle[0.34, 0.43], [0.69, 0.79], [0.81, 0.78], 0.38, 0.39, 0.43\rangle$
P_2	$\langle[0.42, 0.54], [0.61, 0.67], [0.79, 0.85], 0.45, 0.51, 0.60\rangle$
P_3	$\langle[0.39, 0.44], [0.49, 0.55], [0.65, 0.69], 0.34, 0.38, 0.53\rangle$
P_4	$\langle[0.53, 0.63], [0.70, 0.76], [0.78, 0.92], 0.29, 0.50, 0.66\rangle$
	$TCPWG$
P_1	$\langle[0.22, 0.35], [0.39, 0.46], [0.60, 0.67], 0.95, 0.97, 0.99\rangle$
P_2	$\langle[0.32, 0.43], [0.50, 0.57], [0.61, 0.71], 0.62, 0.69, 0.83\rangle$
P_3	$\langle[0.28, 0.34], [0.39, 0.46], [0.52, 0.58], 0.85, 0.93, 0.96\rangle$
P_4	$\langle[0.33, 0.34], [0.57, 0.64], [0.75, 0.81], 0.96, 0.98, 0.99\rangle$
	$TCPWQA$
P_1	$\langle[0.54, 0.57], [0.60, 0.64], [0.69, 0.72], 0.11, 0.51, 0.19\rangle$
P_2	$\langle[0.59, 0.65], [0.67, 0.68], [0.76, 0.85], 0.13, 0.17, 0.21\rangle$
P_3	$\langle[0.69, 0.71], [0.73, 0.78], [0.75, 0.88], 0.05, 0.08, 0.13\rangle$
P_4	$\langle[0.87, 0.89], [0.91, 0.93], [0.95, 0.97], 0.01, 0.05, 0.11\rangle$

Step 5. According to the aggregating results shown in Table 13 and the expected value of triangular cubic variable by applying Eq. 4, the ordering of the alternatives are shown in Table 14.

Table 14. Ordering of the alternatives

$TCPWA$		$TCPWG$		$TCPWQA$	
$E(P_4)$	0.6472	$E(P_4)$	0.7188	$E(P_4)$	0.6311
$E(P_1)$	0.5916	$E(P_1)$	0.6144	$E(P_3)$	0.5316
$E(P_2)$	0.6005	$E(P_3)$	0.5916	$E(P_1)$	0.5455
$E(P_3)$	0.3516	$E(P_2)$	0.5866	$E(P_2)$	0.5150

Step 6. Now we arrange the expected values and select the highest one such that,

(a) $TCPWA = E(P_4) > E(P_1) > E(P_2) > E(P_3)$. Thus the most wanted alternative is p_4.

(b) $TCPWG = E(P_4) > E(P_1) > E(P_3) > E(P_2)$. Thus the most wanted alternative is p_4.

(c) $TCPWQA = E(P_4) > E(P_1) > E(P_3) > E(P_2)$. Thus the most wanted alternative is p_4.After ranking we find that p_4 is the best alternative. Graphical representation of these operators are shown Fig 1, Fig 2 and Fig 3 as below such that,

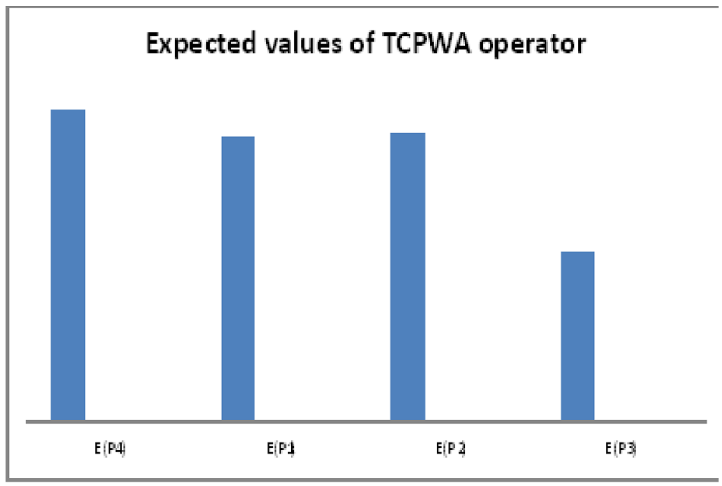

Fig (1)

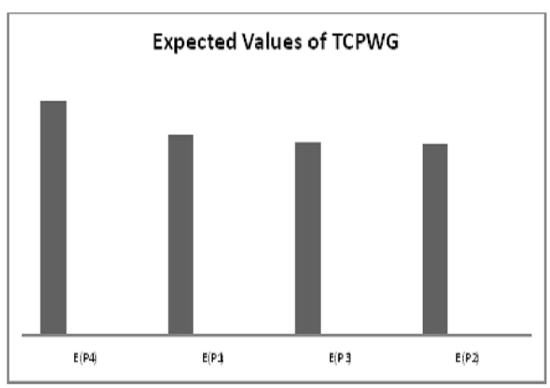

Fig 2.

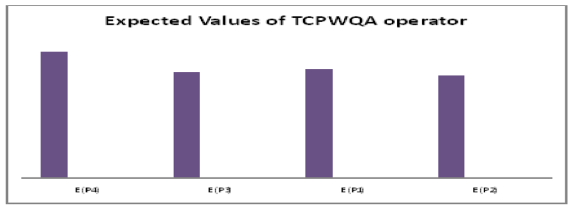

Fig 3.

6. Conclusion

In this paper, we developed three kinds of power aggregation operators such that, the triangular cubic power weighted averaging $(TCPWA)$ operator, triangular cubic power weighted geometric $(TCPWG)$ operator and triangular cubic power weighted quadratic average $(TCPWQA)$ operator. Then, these operators were utilized to develop a approach to solve the triangular cubic multiple attribute group decision making problems with the known weights, we also discussed some basic properties of these operators. Finally we take a numerical application and applying these operators to verify the validity of our results.

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